A TREATISE

ON

INDUSTRIAL PHOTOMETRY

WITH SPECIAL APPLICATION TO

ELECTRIC LIGHTING

BY

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TRANSLATORS' PREFACE.

Dr. Palaz's *Traité de Photométrie Industrielle Spécialement Appliquée à L'Éclairage Électrique* was published in Paris in 1892. It is in great part a compilation of facts and experiments by the best authorities, and contains valuable data for the student in electricity and for those specially interested in the subject of electric lighting.

No work to be compared with it has, as far as we know, appeared up to this time, and the great need of such a work, both in colleges and technical schools as a text-book and in the interests of electric lighting in general, seems to warrant its translation into English.

In some cases, where different methods, employed at the University of Michigan, could also be used with advantage, the translators have noted the fact by references to the Appendix, which further contains some additional information on the general subject.

Besides this work, Professor Palaz is the author of a work on Industrial Electricity (*Cours d'Électricité Industrielle*), being a series of lectures given to the engineers of the Jura-Simplon Railway Company in 1892.

Adrien Palaz was born July 20, 1863, at Riex, in French Switzerland. After studying in the gymnasia of Burgdorf and Lausanne, he entered, in 1880, the Federal Polytechnic School, where he paid particular attention to mathematics, mechanics, and physics. After receiving his diploma, he remained in the school to pursue special studies in electricity in the electro-technical labora-

tory of Professor H. F. Weber. In 1885 he received the degree of Doctor of Science from the University of Zurich.

In the same year, M. Palaz entered the Central Telephonic Service of the Swiss Confederation at Berne; but in 1886 accepted a position in the *Bureau Internationale des Poids et Mesures* at Sèvres, and later became one of the editors of *La Lumière Électrique.* In 1889, he was called to a professorship at the University of Lausanne, where he now occupies the position of Professor of Industrial Electricity.

Dr. Palaz was one of the Swiss members of the chamber of delegates at the Chicago Electrical Congress of 1893.

PREFACE.

SINCE the introduction of the electric light, the public has been attracted by the advantages of an abundant illumination, and its requirements have become greater and greater, while the stimulus given the lighting industry has become the more active. The development of electric lighting has directed a great deal of research to the present conditions of the production of light, and to the best methods of measurement and distribution. The result has been a complete transformation in photometric methods, which now constitute an important whole. But as the reports of this research are scattered in special journals, both French and foreign, to make use of them is very difficult, if not impossible.

This it is which has led me to co-ordinate the acquired results and to mold them into a homogeneous whole, in order to furnish the engineer charged with the installation and operation of lighting apparatus, the varied information of which he has need. My task has been made easy by the fact that since 1887 I have followed the progress of photometric methods in the various articles which I have published in the journal *La Lumière Électrique.*

There does not exist, so far as I know, any French work on industrial photometry; we must be content with the scanty information contained in special chapters of treatises devoted to lighting by gas or by electricity. In Germany, a small manual by Krüss, *Die Electrotechnische Photometrie,* has had a certain success which led me to form the project of making a French translation of it. But since the date of its publication (1885) this work has, to a certain extent, become antiquated in consequence of the

great progress resulting from the numerous researches of these last years; further, some parts are somewhat incomplete. Accordingly, I gave up this first plan, without, however, abandoning the idea of publishing a work on the subject.

It is this work which I now present to the public, in the hope that it may render some service to all those, electrical engineers or gas engineers, who are concerned with the complex questions of lighting. I am fully conscious of the imperfections of my work, the first of this extent. I shall be particularly grateful to all who will kindly communicate to me their observations or their criticisms.

I have dwelt with great care on the numerous photometric apparatus invented especially during these last years, in order to indicate what a variety of methods and apparatus are at present at our disposal in photometry. This variety of apparatus has not prevented me from studying in detail the practical apparatus in current use, among others the photometers of Foucault, Bunsen, and Lummer and Brodhun.

The chapter devoted to photometric standards is very full; we cannot insist too strongly on the importance of an easy and exact reproduction of the light-standard. It has appeared to me indispensable to place before the eyes of the reader the most accurate results furnished by the numerous photometric standards which have been used up to the present.

The various apparatus auxiliary to practical photometry are considered in the fourth chapter; in the fifth, I have treated the photometric properties of incandescent and arc-lamps, and have completed these data by remarks on common light-sources, and on the progress to be realized in the production of artificial light.

A chapter on the distribution of lighting completes the work. This chapter is necessarily somewhat incomplete, for both from the physical and constructive points of view, artificial lighting, public and private, would demand study for itself; but it has been necessary to limit myself in order not to exceed, beyond measure, the bounds set for my work.

I have indicated the principal bibliographical sources of the memoirs which I have mentioned, in order to place the reader desirous of completing the study of a special problem, in a position to have recourse to the originals. I have set myself a limit, in order not to overload the work with bibliographical notices, and it has seemed to me useless to reproduce, at the end of the volume, a complete bibliography of the problems of photometry, and the list of the numerous memoirs consulted.

A. PALAZ.

LAUSANNE, December, 1891.

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PHOTOMETRY.

CHAPTER I.

PRINCIPLES OF PHOTOMETRY.

The Fundamental Photometric Law.

1. Common sources of light, *e.g.* flames, incandescent bodies, the voltaic arc, etc., have finite dimensions; but to establish the fundamental law of photometry which Bouguer and Lambert were the first to discover, it is necessary to consider first a theoretical source of light formed by a luminous point. It is then possible to generalize the results obtained by applying them to luminous sources of finite dimensions.

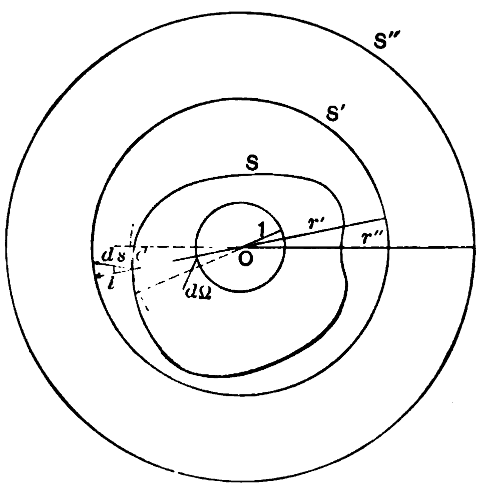

FIG. 1.

Let O be a source of light so small that it may be considered as concentrated at a point. If this is placed at the center of a spherical shell S' (Fig. 1), having a radius r' and a homogeneous and

uniform surface (covered, for example, with a layer of white-lead), all parts of the inner surface will appear equally illuminated.

The source emits a determined *quantity of light* Q. The whole of this quantity of light is received by the surrounding shell whose surface is $4\pi r'^2$. Since all parts of the surface appear equally illuminated, a unit area situated anywhere on the surface will receive a quantity of light

$$q' = \frac{Q}{4\pi r'^2}.$$

A second concentric spherical shell of radius r'' will also receive the same quantity of light Q, but a unit surface of this sphere will receive only the quantity

$$q'' = \frac{Q}{4\pi r''^2}.$$

From this it follows that

$$\frac{q'}{q''} = \frac{r''^2}{r'^2}. \tag{1}$$

That is to say, *the quantities of light received per unit area on concentric spheres of radii* r′ *and* r″ *are inversely proportional to the squares of the radii* r′ *and* r″. (*The Law of Distances.*)

2. Further let us consider a closed surface of any form situated within the spherical shell S'. This surface evidently receives the same quantity of light Q as the surrounding surface. Let us cut off any portion of this surface dS by a cone having its vertex at the center O of the sphere; let $d\Omega$ be the solid angle corresponding to dS; that is to say, the corresponding portion of the surface of a sphere having a radius of unity. The quantity dq of light received on dS is then the same as that received by the element $d\Omega$; now this receives the quantity of light

$$\frac{Q}{4\pi} d\Omega;$$

then

$$dq = \frac{Q}{4\pi} d\Omega.$$

But $\dfrac{Q}{4\pi}$ is a constant which measures the emission of light from the source O; this is called the *total intensity*, and is designated by the letter I; from this it follows that

$$I = \frac{Q}{4\pi}. \tag{2}$$

The preceding equation then becomes

$$dq = Id\Omega.$$

If $d\Omega = 1$, then $dq = I$; that is to say, *the total intensity is equal to the quantity of light emitted in a solid angle equal to unity.*

Let us call ds the projection of the element dS on the plane perpendicular to the radius vector r of the element dS, i being the angle between the normal to dS and the radius vector, and we have

$$ds = dS \cos i,$$

and
$$\frac{d\Omega}{ds} = \frac{1}{r^2};$$

then
$$d\Omega = \frac{ds}{r^2} = \frac{dS \cos i}{r^2},$$

consequently
$$dq = Id\Omega = \frac{IdS \cos i}{r^2}; \tag{3}$$

that is, *the quantity of light received by an element of surface* dS, *whose normal makes an angle* i *with the direction of the luminous ray, is proportional to the cosine of the angle of incidence* i.

Let the *intensity of illumination* of a surface at a given point be the ratio of the quantity of light received by the element dS of the surface at this point to the area of this element; intensity of illumination is generally designated by e. Then

$$e = \frac{dq}{dS} = \frac{I \cos i}{r^2}. \tag{4}$$

If the surface dS is equal to unity, $e = dq$; the intensity of illumination may then also be considered as being the quantity of light received by a unit surface.

3. Instead of considering the luminous source concentrated at a point, let us assume that it has the form of a sphere of radius R. Each element dS of the surface furnishes part of the luminous emission, and the quantity of light emitted by this surface dS is

$$dq = Q\frac{dS}{4\pi R^2} = \frac{Q}{4\pi R^2}dS = \frac{I}{R^2}dS.$$

In these formulæ, Q represents the total quantity of light emitted by the source, and I its total intensity; if $dS = 1$, it follows that

$$dq = \frac{Q}{4\pi R^2} = \frac{I}{R^2}.$$

The expression $\frac{Q}{4\pi R^2} = \frac{I}{R^2}$, which represents the quantity of light emitted normally by a unit of surface of the luminous source, is called the *intrinsic intensity*, or the *brilliancy*, of the source; this is designated by the letter i*; then

$$i = \frac{Q}{4\pi R^2} = \frac{I}{R^2}; \tag{5}$$

it follows that $$Q = 4\pi R^2 i.$$

The total quantity of light Q emitted by a luminous source is therefore equal to the product of its brilliancy by its surface. The total intensity I may then be deduced by the aid of equation (2).

Let us consider the source of light limited by a plane surface AB; let us suppose the intrinsic intensity i constant at all points of the source; the quantity of light emitted by the surface S of the plane AB, determined by the opening of the screen E, is equal to iS, and the emission of light takes place normally to the plane AB.

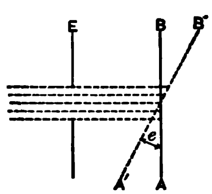

FIG. 2.

Now let the surface AB be inclined and placed in the position $A'B'$; a portion of the surface S' of the plane AB corresponds to the opening S of the screen. It is shown by experiment that the quantity of light traversing the opening of the screen does not change. So if e designates the angle between the planes AB and $A'B'$, we have

$$S' = \frac{S}{\cos e};$$

and since the light emitted has not varied, and since the surface of emission has increased in the ratio of 1 to $\cos e$, the intrinsic intensity i must have diminished in the same ratio; it then follows that

$$i' = i \cos e.$$

* The same letter, i, is used to designate the angle of incidence and the intrinsic intensity, because there is no possibility of confusion.

This empirical law, given for the first time by Lambert*, is known under the name of the *law of the cosine.*

Another proof of the accuracy of the law of the cosine follows from the fact that an incandescent metallic sphere appears in the dark as a uniformly luminous disc. Any element of the surface of the visible hemisphere then sends to the eye the same quantity of light as that which would be emitted normally by the projection of the element on the base of the hemisphere.

It should be remarked, however, that the law of the cosine is exact only in so far as phenomena of diffraction and reflection may be neglected, and where the angle e is very small. This should be considered as an approximate law, and not one rigorously true. Certain recent experiments of Seeliger†, in particular, have shown that the deviations of this law from the results of direct observation frequently exceed one per cent, even for very small angles of incidence.

4. *The fundamental law of photometry.* — We may summarize the preceding laws in a single formula. Let dS be an infinitesimal element of a luminous surface at a point where the intrinsic intensity is i; let dS' be an infinitesimal element of a second surface S' illuminated by the first; let us call d the distance between the two elements, e the angle that the line joining them makes with the normal to dS, and e' the angle which the same line makes with the normal to dS'. The quantity of light emitted by dS and received by dS' is given by the formula

$$q = i\frac{dSdS'}{d^2}\cos e\cos e'.$$

Assuming that the dimensions of a quantity of light are the same as those of a quantity of heat, and taking account of the relations

$$Q = 4\pi I,$$

$$i = \frac{[I]}{[S]},$$

$$e = \frac{[I]}{[d]^2},$$

* Lambert, *Photometria sive de mensura et gradibus luminis*, 1760.

† *Berichte der bay. Acad. der Wiss.*, 1888, p. 201, and *Lum. Él.*, Vol. XXV. p. 615.

we have the following dimensions,

$$\text{dim. } Q = \text{dim. } I = [ML^2T^{-2}],$$

$$\text{dim. } i = \text{dim. } e = [MT^{-2}].$$

The choice of the unit for the intensity I determines also the choice of the unit of brilliancy and the unit of illumination, as the preceding formulæ show.

We shall have occasion to revert to this special point when we take up the study of the usual sources of light and the problems of the distribution of light.

The Intensity of Light according to the Undulatory Theory.

5. According to the undulatory theory, light is the result of the vibratory movement of the ether propagated along a straight line. The equation of this movement is

$$v = \frac{v_1}{d} \sin \frac{2\pi}{T}\left(t - \frac{d}{c}\right).$$

In this equation v is the velocity of a molecule of ether of mass μ at the point considered and at the time t; this point is situated at a distance d from the luminous source where the maximum amplitude is v_1; the period of the vibratory movement is T, and the relation of the wave-length λ to the velocity of propagation c of the vibratory movement is

$$\lambda = cT.$$

It is known that the color of light depends upon the value of the wave-length. A light is said to be monochromatic when it emits vibrations of ether of the same wave-length only. Ordinary luminous bodies emit complex light; that is, they produce a complex vibratory movement.

This complex movement is formed by the superposition of simple undulatory movements, each corresponding to a single wave-length; the undulatory movements of the ether possess the property of having the velocity of propagation $c\left(=\frac{\lambda}{T}\right)$ independent of the wave-length; at least, the most careful experiments indicate this.

The quality of the light from a given source depends on the wave-length of the radiations emitted, and on the proportion in which the vibrations of the different wave-lengths unite in the formation of the complex light emitted.

The intensity of vibratory movement at a given point where the superficial element dS is situated is equal to the *vis viva* of the molecules of ether μ which are on this element at the time t; but this *vis viva* varying continually, we consider the mean *vis viva* of these molecules during the period of one oscillation to represent this intensity; we have

$$I=\frac{1}{T}\int_0^T\frac{\mu}{2}\left(\frac{dv}{dt}\right)^2dt,$$

and, replacing v by its value, we obtain

$$I=\mu\frac{\pi^2}{T^2}\frac{v_1^2}{d^2}.$$

The intensity of a monochromatic luminous vibration is then a quantity mathematically defined; we see that it is proportional to the square of the maximum amplitude v_1. But we cannot determine this intensity directly, for it is not possible to measure the quantities μ and v_1; we may ignore the mass μ, since this factor disappears in the comparison of two luminous intensities.

The comparison of two monochromatic luminous intensities then reduces to the comparison of the amplitudes of their vibratory movement. Unfortunately it is not possible to measure this element as, for example, we measure wave-lengths, and so it is necessary, in order to measure the intensity of a luminous radiation, to have recourse to the actions that it exerts on various substances and phenomena.

The Various Actions of Light.

6. Without taking into account the action of light on the electric and magnetic properties of certain bodies and on certain electrical phenomena, we may distinguish three different kinds of actions of light, — calorific, chemical, and illuminating.

For a long time there have been distinguished in the light from any given source, in sunlight for instance, chemical, calorific, and illuminating rays. This distinction is entirely arbitrary, and the curves which represent the variations of intensity relative to the

chemical, calorific, and luminous rays of a given light are only in reality the graphic representation of absorption-spectra of substances which have been used in their study; for example, salts of silver, lampblack, and the retina.

The presence of waves of ether of a determined length is proved by the aid of one of the phenomena mentioned above. We know, for example, that it is only vibrations of the ether whose wave-length is included between 0.360 μ* and 0.810 μ that act on the eye and produce an impression of light.

Other vibrations, whose wave-lengths are greater or smaller, have no luminous action on the retina, and can only be proved to exist by the action which they exert on other substances,—an action which is calorific or chemical; however, the fact that it is impossible, by any given process, to observe the presence of vibrations of a determined wave-length in a luminous pencil does not permit us to conclude that they do not exist, for they frequently require exceedingly sensitive methods and apparatus to prove their existence.

Radiations of small wave-length (ultra-violet) are generally shown to exist by their chemical action, in the same way that radiations of considerable wave-length (infra-red) are shown to exist by their calorific action.

However, we should not conclude that the calorific action of the first and the chemical action of the second are null; for a ray of determined wave-length may exert at the same time the three actions,—chemical, calorific, and luminous.

Below there will be found certain data relative to the principal radiations which may be shown to exist by means now used.

Quality of Radiations and Means of recognizing them.	Wave-Lengths in Microns.	Character of the Vibratory Movement.
Ultra-violet rays (photography).	0.185	Extreme ray of the aluminium spectrum obtained by a spark from induction coil. (Cornu.)
	0.295	Extreme limit of the solar spectrum at sea-level. (Cornu.)
Visible radiations (the eye).	0.360	Limit of lavender light, visible for normal eyes.
	0.810	Extreme limit of dark red light.
Beginning of the ultra-red (phosphorescence).	1	Extreme possible limit of wave-length in the ultra-red. (Draper.)

* 1 micron = 1 μ = 0.001 mm.

Quality of Radiations and Means of recognizing them.	Wave-Lengths in Microns.	Character of the Vibratory Movement.
Thermal action (bolometer).	2.7	Sensible limit of solar rays which penetrate the atmosphere at Allegheny. (Langley.)
Radiations from terrestrial sources (bolometer).	5.3	Limit with prism of rock salt.
	7.5	Approximate position of the maximum of a black surface at 100° C.
	11	Black surface at 0° C.
	30	Approximate estimation of the minimum value of the longest heat-wave with a prism of rock salt.
Sound vibrations (ear).	14,000	The shortest perceptible wave-length (Savart, 24,000 vibrations per second).

This table shows that the extent of the normal spectrum perceptible to the eye is very narrow; this extent does not exceed fifteen thousandths of the spectrum perceptible by photographic and calorimetric methods.

The Photometric Action of Light.

7. From a photometric point of view, the only radiations which are to be taken into account are those which are perceptible to the eye. Now all the vibrations with wave-lengths above $0.36\,\mu$ (about), and principally those above $0.81\,\mu$, contribute to the calorific action of a luminous pencil, while it is only those whose vibrations are included between the above two limits which act upon the eye.

It is the same with the chemical action of a luminous pencil; in this case it is especially those radiations whose wave-lengths are below $0.36\,\mu$ which produce the greatest part of the total chemical action of the pencil.

We see then that chemical or calorific phenomena of light cannot be used to measure its photometric intensity, especially when the nature of the work executed by the luminous pencil on a body — work which is shown by elevation of temperature or by chemical decomposition — depends on work already executed on other bodies by the same pencil.

To illustrate, let us take a luminous pencil of well-determined intensity, and pass it through an alum solution; the photometric action of the luminous pencil will not vary noticeably, while its calorific action will be considerably diminished.

If, then, we should compare the photometric action of a luminous pencil, after its passage through an alum solution, with that of another luminous pencil which has not passed through any solution of this kind, by comparing their calorific actions, we would make a great error.

The same remark applies to the other physical phenomena on which light has an influence (variation of the electrical resistance of selenium under the influence of light, variation of the magnetic moment of a bar-magnet, actino-electric discharges, etc.).

The luminous intensity of a pencil of light differs essentially from the intensity of vibratory movement defined by the undulatory theory. From the photometric point of view, light is manifested by a sensation, by a simple physiological phenomenon. The luminous intensity of a pencil of light is not the energy of the vibratory movement of the ether, but only the action of this energy on our visual organ.

Sensibility of the Eye for Photometric Observations.

8. The eye is then the standard piece of apparatus for all photometric operations; it is the photoscope required for every comparison of intensity of two luminous bodies; the eye here plays the same *rôle* as the galvanometer or the electrometer in electrical measurements by zero methods. We can therefore only tolerate for photometric measurements a healthy and well-formed visual organ. Furthermore, the obligatory employment of the eye imposes, in photometric measurements, a limit of precision determined by its sensibility.

Since it is the eye alone which can appreciate the photometric qualities of a luminous pencil or of a given illumination, we do not measure, in reality, the luminous intensity of a source of light or the intensity of illumination of a surface, but the excitation produced on the eye and the optic nerve. Now any one may prove that the visual organ is incapable of distinguishing whether an illumination is m or n times as great as another illumination; the eye can only judge that one is greater than the other, but without being able to estimate their ratio. This fact, verified experimentally every day, is one of the consequences of a general law which governs the greater part of the sensations. This is the *psycho-physical law* of Fechner, according to which the intensity of the sensation is proportional to the logarithm of the excitation*.

* Helmholtz, *Optique Physiologique*, p. 415.

It is known that in every sensation we denote by *stimulus threshold* the inferior limit, below which the stimulus is too slight to produce a perceptible sensation; the *maximum stimulus* is the superior limit, above which an increase in the intensity of the stimulus produces no increase in the intensity of the sensation.

The value of the sensation which corresponds to the stimulus threshold is called the *minimum sensation;* that which corresponds to the maximum stimulus is called the *maximum sensation;* the name *sensible value* or *physiological unit* is also given to the stimulus threshold; for this quantity is employed as unity in measurements of the stimulus.

In the case where the equality of two illuminations is established, the physiological unit is the limiting value which the difference of the two illuminations should attain in order that this difference may be perceptible to the retina.

To determine this quantity, we may proceed in the following manner invented by Bouguer: a white screen is illuminated by two equal sources of light (two equal candles), and a rod is placed in front of the screen, on which it projects two shadows (Rumford's photometer). One of the candles is moved away until the corresponding shadow ceases to be visible.

Let a be the distance of the screen from the nearer light, b its distance from the farther light; the intensities of illumination produced on the screen by these two lights are in the inverse ratio of a^2 to b^2. Bouguer found $\frac{a}{b} = \frac{1}{8}$, while Fechner obtained $\frac{a}{b} = \frac{1}{10}$; it then follows that Bouguer could distinguish $\frac{1}{64}$ of the luminous intensity, and Fechner $\frac{1}{100}$. Arago observed that, by moving the candle, still smaller differences may be noticed, and thus found $\frac{1}{131}$. Lastly, Helmholtz was able to distinguish differences of illumination of $\frac{1}{188}$ between the concentric circles of a disc, and at times differences of $\frac{1}{150}$ and even $\frac{1}{167}$.

The disc was then illuminated by diffuse daylight. In illuminating the disc by direct sunlight, perception of the differences of illumination became more difficult. The sensible value in the comparison of two illuminations depends on the intensity of the illuminations or of the lights that are considered; it is maximum for a mean value, and smaller when the illuminations are too intense or too weak.

Thus Masson found that the sensible value (stimulus threshold) was maximum when the luminous intensities to be compared were

of the order of diffuse daylight; he obtained, under these conditions, $\frac{1}{186}$ for the threshold value of excitation; that is, he was able to distinguish differences of illumination of even $\frac{1}{186}$.

The practical conclusion from this is that in photometric comparisons we should so manage that we have to compare only illuminations whose intensity approximates the value of the illumination produced by diffuse daylight.

The preceding shows, moreover, that the eye is incapable of appreciating the inequality of illumination of two contiguous surfaces within about 0.01, even when the colors that are compared are identical, as in the measurements indicated above. This fact limits the precision of photometric measurements.

From the moment when the difference of intensity of two lights, or the difference of illumination of two surfaces, has passed the sensible value corresponding to the observer and to the conditions of the experiment, the intensity of the sensation varies, according to the psycho-physical law of Fechner, verified by the numerous measurements of E. H. Weber. According to these measurements the increment of sensation dS is proportional to the ratio between the increment of the excitation dI and the primitive excitation I; that is,

$$dS = k\frac{dI}{I},$$

or, integrating,

$$S = k \log I - C.$$

The sensation S is null for an intensity of excitation equal to the sensible value I_0; we have then

$$C = k \log I_0,$$

and consequently

$$S = k \log \frac{I}{I_0}.$$

This formula, which expresses the psycho-physical law of Fechner, shows then that when the intensity of luminous excitation (illumination) passes from a given value to a value m times as great, the sensation increases in the ratio of a to $(a + \log m)$.

These facts clearly explain why the eye is unable to appreciate with precision the ratio of the two luminous sensations produced by two different sources, but can appreciate only their equality.

Variations of the Sensibility of the Eye with the Color of the Light.

9. The sensibility of the eye varies with the nature of the luminous rays; all the preceding figures relative to the visual

sensibility of several observers, have reference to white light, but these values are no longer the same if the nature of the light changes.

There is very little exact knowledge concerning the relative values of the stimulus threshold for the different regions of the spectrum. We give, however, the values obtained by Ebert (E) and his assistant (S) by the aid of a method giving sufficiently accurate results *.

The following table gives the relative values of the stimulus threshold for five regions of the spectrum. They are given in the first double column.

Color.	Length of Wave in μ.	Relative Stimulus Threshold.		Stimulus Threshold Referred to the Same Energy of Vibratory Motion.	
		E	S	E	S
Red	0.675	0.8	0.6	34	25
Yellow	0.590	2.3	2.0	17	15
Green	0.530	0.5	0.5	1	1
Bluish green . .	0.500	1.2	0.8	2	1.3
Blue	0.470	7.3	6.8	[illegible]	3

These values were obtained using an Argand gas-burner. To reduce them in terms of the normal solar spectrum, it will suffice to take into account the measurements of the relative energy in the different regions of the solar spectrum made, for instance, by Langley, and the measurements of the energy of the spectrum of a gas flame made by O. E. Meyer. The numbers in the third double column are thus obtained. The sensibility of the excitation is then given by the reciprocals of the threshold values.

The table shows that the sensibility of the luminous excitation is maximum for green and minimum for red. Consequently the energy of the vibratory movement, which contributes to the production of a luminous sensation, is maximum when the length of the wave is the same as that of radiations in the green region of the spectrum. That is to say, when λ is about 0.530 μ.

These conclusions are confirmed by the very careful observations made by Langley † on the visual energy in the different regions of

* *Wiedemanns Ann.*, Vol. XXXIII. p. 136. *Lum. Él.*, Vol. XXVII. p. 139.
† *Lum. Él.*, Vol. XXXI. p. 144.

the normal solar spectrum. He also found that the same quantity of vibratory movement produces in the green an impression 100,000 times as great as that which is produced in the dark red (0.750 μ).

Below are the figures which represent the luminous sensation produced by the same quantity of energy in the various parts of the spectrum. The luminous sensation produced in the dark red (0.750 μ) is taken as unity.

	Violet.	Blue.	Green.	Yellow.	Orange.	Red.	Dark Red.
Length of wave in μ.	0.400	0.470	0.530	0.580	0.600	0.650	0.750
Luminous sensation.	1600;	62,000;	100,000	28,000;	14,000	1,200	1.

Composition of the Light emitted by Various Luminous Sources.

10. A body when heated sends forth rays, that is to say, it causes a vibratory movement of the ether, the nature of which depends on the temperature. With Draper it was assumed, until within a few years, that all bodies begin to emit rays perceptible to the eye when their temperature reaches 525° C., and that these rays belong to the extreme red of the normal spectrum where the wave-lengths are greatest.

Some years ago H. F. Weber* discovered that a solid body whose temperature is being raised, commences to emit light before it becomes incandescent. The first sensible trace of light in the spectroscope is a hazy, gray band which appears in that part of the spectrum which corresponds to the yellow and yellowish green. If the temperature continues to rise, the spectrum of the rays emitted by the heated body increases on both sides of this gray band. Weber found the first trace of light at 417° C. with gold, at 390° C. with platinum, and at 377° C. with iron.

This phenomenon is easily explained by what has been shown above concerning the eye's sensibility, which is maximum for green rays; consequently the first luminous ray emitted by a body just beginning to be luminous should appear in this region of the spectrum.

Having passed the temperature at which the first rays are perceived, the brightness of the incandescent body increases very rapidly with the temperature. Rays of longer and shorter wave-

* *Sitzungsber. der Berliner Acad.*, 1887, p. 491; *Lum. Él.*, Vol. XXX. p. 31.

length are added to the yellow and yellowish green, and as complete incandescence is attained, the increase of rays of great refrangibility becomes very rapid.

The spectrum of the light emitted by a body raised, for example, to a temperature of 720° C. includes all the colors up to a reddish orange (from the A to the C line); at 780° C. it extends to the bright orange (G), and at 1165° C. it includes the whole spectrum between the lines A and H. Above 1165° C. the ultra-violet rays, which are not perceptible to the eye, are to be added to the preceding.

Below are the values obtained by Violle* for the luminous intensity of four different regions of the spectrum of a disc of platinum raised to high temperatures; the unit adopted for the rays of each color is the intensity of the corresponding rays in the spectrum of melting platinum.

Temperature. C.	$\lambda = 0.656\,\mu$. (C)	$\lambda = 0.5892\,\mu$. (D)	$\lambda = 0.586\,\mu$. ($E = 0.527\,\mu$.)	$\lambda = 0.482\,\mu$. ($F = 0.416\,\mu$.)
775°	0.00038	0.00007	0.00003	
954°	0.00197	0.00124	0.00073	
1045°	0.00645	0.00450	0.00271	0.00133
1500°	0.303	0.271	0.225	0.155
1775°	1.000	1.00000	1.000	1.000

If the intensity of the corresponding rays in the spectrum of the carcel lamp is taken as a unit, the following table is obtained: —

Temperature.	$\lambda = 0.656$.	$\lambda = 0.5892$.	$\lambda = 0.586$.	$\lambda = 0.482$.
775°	0.00300	0.00060	0.00030	
954°	0.01541	0.01105	0.00715	
1045°	0.0505	0.0402	0.0265	0.0162
1500°	2.371	2.417	2.198	1.894
1775°	7.829	8.932	9.759	12.16

These two tables show the rapidity with which the luminous intensity increases as the temperature is raised. For instance, the intensity of the rays having a wave-length of $0.589\,\mu$ is more than

* *Comptes Rendus*, Vol. LXXXVIII. p. 171, and Vol. XCII. p. 866.

eight hundred times as great at the temperature of melting platinum (1775° C.) as it is at the temperature of melting silver (954° C.).

It can be seen from the second table how much more intense are the highly refrangible rays emitted by incandescent platinum than those of the carcel lamp.

The higher the temperature rises, the more intense become the rays of short wave-length. If the temperature is low, rays of great wave-length preponderate, and the light appears red. In order that light may be white like sunlight, it is necessary that the rays of the various wave-lengths should be combined in the same ratio as the corresponding rays of the normal solar spectrum. If the red rays are more intense than in the normal spectrum, the light appears red; and in the same way it appears violet if the violet rays are more intense.

In the following table are the values obtained by Crova* for the luminous intensity of rays of the same wave-length emitted by different sources, the luminous intensity of the wave whose length is 0.676 μ being taken as unity.

Wave-length	0.676 μ	0.605 μ	0.560 μ	0.523 μ	0.486 μ	0.459 μ
Voltaic arc	1.000	0.707	0.597	0.506	0.307	0.228
Drummond light	1.000	0.573	0.490	0.299	0.168	0.073
Carcel lamp	1.000	0.442	0.296	0.166	0.080	0.017

These values show that in the light of the carcel lamp the red rays are nearly sixty times as intense as the violet, while in the light of the voltaic arc they are only four times as intense, that is, one-fifteenth as much. Moreover, it is well known that the flame of the carcel lamp appears red beside the voltaic arc.

Taking as a standard the rays of the carcel lamp, W. Pickering† found the following values for certain rays of different luminous sources.

Rays of the Spectrum.	C.	D.	b′	Between F and G.
Candle	73	100	104	134
Gas-light	74	100	103	125
Voltaic arc	61	100	121	735

* *Comptes Rendus*, Vol. LXXXVII. p. 322.
† *Nature*, Vol. XXV. p. 340.

Finally, O. E. Meyer * obtained the following values, taking the rays emitted by a gas flame as a standard, and making the intensities corresponding to the *D* line unity.

Rays of the Spectrum.	B.	D.	E.	G.
Petroleum lamp . . .	0.66	1.00	1.40	1.00
Sunlight, direct . . .	4.07	1.00	0.43	0.15
Sunlight, diffused . .	1.25	1.00	0.50	0.41
Voltaic arc	1.10	1.00	0.40	0.10
Incandescent lamp . .	0.30	1.00	1.40	1.10

The above results are sufficient to allow the classification of sources of light according to the nature of the light which they emit. The units not being the same in the preceding tables, it is not possible to compare directly the values which they contain. We may, however, state that the usual sources of light can be classed as follows, with reference to the kind of light emitted: carcel lamp, candle, gas-light, petroleum lamp, incandescent platinum, Drummond light, voltaic arc, sunlight.

The Photometric Elements of Luminous Sources.

11. *Definitions.*—The intensity of a source of light varies in general with the direction of the luminous rays. For a long time observers have limited themselves to considering only the luminous intensity in a horizontal plane passing through the center of the luminous body, that is, the *horizontal luminous intensity;* the development of illumination by intensive burners and by the voltaic arc has required more attention to be paid to the problem, and new factors to be introduced into the study of a light-center. Thus there has been introduced the notion of *mean spherical intensity*, which plays an important part in the comparison of the photometric qualities of light-centers.

If we lay off in various directions lines passing through the luminous source, whose lengths measure the intensity of the rays emitted in those directions, the locus of the points thus obtained forms a surface called the *photometric surface* of the source.

In arc-lights, for instance, the photometric surface may be con-

* *Monatsber. der Berliner Acad.*, 1880. [See article by H. G. Vogel, p. 801.]

sidered in general as one of revolution about the common axis of the two carbons, although, in many cases, this condition is not perfectly fulfilled, on account of defects in the homogeneity of the carbons and in their centering.

The meridian of the photometric surface has a well-determined form. In order to construct it, we measure the luminous intensity for inclinations not differing too much in the different azimuths. We choose, in general, inclinations varying by 10 degrees, and four azimuths differing by 90 degrees. We take, then, as the luminous intensity for a given inclination the mean of the values obtained for this inclination in the four azimuths. To construct the curve, we take as the unit of luminous intensity the maximum luminous intensity. The graphical construction is simplified by employing a sheet prepared in advance, formed of concentric circumferences described around the point A as center, having respectively for radii 0.1, 0.2, 0.3, etc., the line AP being taken as unity; and of straight lines passing through the point A and making angles of 10 degrees, 20 degrees . . . , above and below the horizontal AH. Along these straight lines the corresponding luminous intensity is laid off in terms of the maximum intensity AE taken as unity. The line $ABCDEF$ passing through the points thus determined is the meridian curve of the photometric surface of the center; it shows at a glance what is the relative intensity in any given direction, AD for example.

12. *Mean horizontal intensity.* — The mean horizontal intensity of a light-source is the mean of the values of the intensity, measured in all directions in the horizontal plane passing through the source. Practically, it is sufficient to make these measurements in a certain number of symmetrical directions and to calculate the mean.

If the photometric surface of a light-source is cut by a horizontal plane passing through the source, the curve of intersection represents the variations of the horizontal intensity. The mean horizontal intensity is then represented by the mean value of the radius vector of this curve. To determine practically the mean horizontal intensity of a light-center, it is sufficient to make measurements in a small number of different directions, four or eight for instance, symmetrically arranged.

13. *Mean spherical intensity.* — The mean spherical intensity may be defined as the sum of the illuminations received by a sphere

of radius 1, concentric with the light-source, divided by the surface of the sphere. In other words, it is the intensity of the light-source rendered uniform, *i.e.* emitting luminous radiations of constant intensity in all directions.

In the calculation of mean spherical intensity it is generally assumed that the photometric surface is a surface of revolution; consequently the luminous intensity I is independent of the azimuth, and only varies with the inclination of the ray. This intensity is then a function of the inclination θ to the horizontal; consequently

$$I = f(\theta).$$

The total intensity then will be the mean of the partial intensities relative to each direction. To obtain this mean, let us consider the part of the photometric surface comprised between the two parallels defined by the angles θ and $\theta + d\theta$. These two angles differing very little, we may suppose the intensity I constant. The quantity of light which falls on this zone of the photometric surface is equal to that which the corresponding zone of the unit sphere receives. Now the height of this zone being $\cos\theta \, d\theta$, its surface is equal to $2\pi \cos\theta \, d\theta$, and the quantity of light which it receives is given by the expression $2\pi I \cos\theta \, d\theta$. Consequently the total quantity of light received by a zone of the photometric surface and the corresponding zone of the sphere is equal to $\int_{\theta_1}^{\theta_2} 2\pi I \cos\theta \, d\theta$, θ_1 and θ_2 being the relative inclinations of the parallels which limit the zone. The mean intensity of illumination of the zone will then equal the total quantity of light received divided by the surface; that is,

$$\frac{\int_{\theta_1}^{\theta_2} 2\pi I \cos\theta \, d\theta}{2\pi(\sin\theta_2 - \sin\theta_1)}.$$

The mean spherical intensity is equal to the intensity of the mean illumination of the unit sphere, that is, to the total quantity of light received by the unit sphere divided by its surface; which gives

$$\frac{\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} 2\pi I \cos\theta \, d\theta}{4\pi}.$$

Practically, the law according to which the intensity varies with the inclination is not simple enough to allow the integration to be

effected directly. We are obliged to effect it by approximation. The calculation is simplified by the employment of a curve which is deduced immediately from the meridian curve of the photometric surface in the following manner.

14. *Calculation of mean spherical intensity.* — To obtain the different points of this curve (Fig. 3) it is sufficient to draw, through the points where the prolongations of the radii AB, AC, AD, etc., meet the circumference, horizontal lines and to lay off on these,

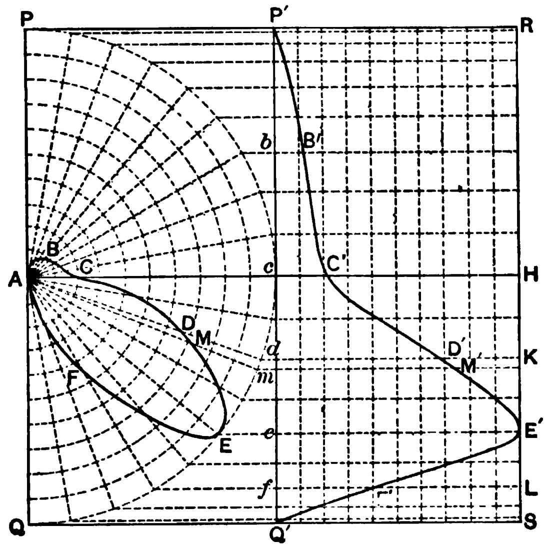

Fig. 3.

from the point where they meet the perpendicular $P'Q'$ drawn at a distance 1 from the point A, lengths equal to the radii vectores which measure the relative intensities; we have then $bB' = AB$, $cC' = AC$, etc. We may facilitate the construction by employing a sheet prepared in advance *.

The length dm on the line $P'Q'$ corresponding to the inclinations θ and $\theta + d\theta$ is equal to $\cos\theta\, d\theta$, and the length dD' is equal to the intensity I; consequently the surface $dmM'D'$ is equal to $I\cos\theta\, d\theta$. Then the product of this surface by 2π represents the quantity

* *Comtes rendus des essais Photométriques à l'exposition d'Anvers, en* 1885, par M. Rousseau.

of light received by the zone corresponding to $d\theta$. It follows that the entire surface $P'C'E'Q'$ is equal to $\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} I\cos\theta\, d\theta$, and multiplied by 2π represents the total quantity of light received by the unit sphere. The mean spherical intensity being equal to $2\pi\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} I\cos\theta\, d\theta$ divided by the surface of the sphere (4π), is thus given by the expression $\frac{\text{surface } P'C'E'Q'}{2}$. Now the surface of the rectangle $P'RSQ'$ is equal to 2, hence the mean spherical intensity is the ratio of the surface $P'RSQ'$ to the surface $P'C'E'Q'$. The determination of the mean spherical intensity may then be made graphically; it is sufficient to evaluate the surface $P'C'E'Q'$ by the aid of a planimeter.

We may, however, dispense with a graphic determination by effecting the calculation in the following manner: we replace the meridian curve of the photometric surface by a polygonal line having its angles at the points determined directly by observation, which is the same thing as regarding the mean luminous intensity of each zone as being equal to the arithmetical mean of the intensities I' and I'' corresponding to the inclinations θ' and θ'' of the circles at the bases of the zone; the quantity of light received by this last is then

$$2\pi h\frac{I'+I''}{2} = 2\pi(\sin\theta'' - \sin\theta')\frac{I'+I''}{2}.$$

Making the measurements at the angles $\theta_1, \theta_2, \theta_3, \ldots \theta_n$, the quantity of light received by the total zone is equal to

$$2\pi\left[(\sin\theta_2 - \sin\theta_1)\frac{I_1+I_2}{2} + (\sin\theta_3 - \sin\theta_2)\frac{I_2+I_3}{2}\cdots \right.$$
$$\left. + (\sin\theta_n - \sin\theta_{n-1})\frac{I_{n-1}+I_n}{2}\right].$$

The surface of this zone being equal to $2\pi(\sin\theta_n - \sin\theta_1)$, the corresponding mean intensity is equal to

$$\frac{(\sin\theta_2 - \sin\theta_1)\frac{I_1+I_2}{2} + \cdots + (\sin\theta_n - \sin\theta_{n-1})\frac{I_{n-1}+I_n}{2}}{\sin\theta_n - \sin\theta_1}.$$

Supposing that θ_1 equals $-\frac{\pi}{2}$, and θ_n equals $+\frac{\pi}{2}$, we obtain the mean spherical intensity; the denominator is then equal to 2.

A knowledge of the exact distribution of light in the upper hemisphere has only a limited importance with respect to the illumination; this is especially the case with the arc-lamp. For this reason it is sufficient to determine the luminous intensity at intervals of 30 degrees in the upper hemisphere. In this case the formula which gives the mean hemispherical intensity (for the upper hemisphere) becomes, the area of its surface being 2π,

$$\frac{\sin 30° - \sin 0°}{2}(I'_1 + I'_2) + \frac{\sin 60° - \sin 30°}{2}(I'_2 + I'_3) + \frac{\sin 90° - \sin 60°}{2}(I'_3 + I'_4).$$

In the arc-lamp I_4 (the intensity in the vertical direction) is null, and the formula may be written

$$0.250(I'_1 + I'_2) + 0.183(I'_2 + I'_3) + 0.067\, I'_3.$$

Taking a difference of 10 degrees for the measurement in the lower hemisphere, the formula for mean hemispherical intensity becomes

$$\frac{\sin 10° - \sin 0°}{2}(I_1 + I_2) + \frac{\sin 20° - \sin 10°}{2}(I_2 + I_3) + \cdots$$

or $0.0868(I_1+I_2) + 0.0842\,(I_2 + I_3) + 0.0790(I_3+I_4) + 0.0714(I_4+I_5)$
$+ 0.0616\,(I_5 + I_6) + 0.05\,(I_6 + I_7) + 0.0368(I_7 + I_8)$
$+ 0.0226\,(I_8 + I_9) + 0.0076\,I_9.$

If we wish to reduce the number of measurements from 9 to 6 (the intensity in the vertical direction being null), it is sufficient to make measurements at intervals of 15 degrees. We then have for the mean hemispherical intensity

$$\frac{\sin 15° - \sin 0°}{2}(I_1 + I_2) + \frac{\sin 30° - \sin 15°}{2}(I_2 + I_3) + \cdots$$

or $0.1294(I_1+I_2) + 0.1206(I_2+I_3) + 0.1036(I_3+I_4) + 0.0794\,(I_4+I_5)$
$+ 0.050(I_5 + I_6) + 0.017\,I_6.$

The preceding method of calculation was employed by Rousseau for the photometric measurements at the Antwerp Exposition.

It has great advantages. Some experimenters, however, prefer the following method.

The quantity of light received by the zone $\theta_1\theta_2$ is equal to

$$2\pi\int_{\theta_1}^{\theta_2} I\cos\theta\, d\theta = 2\pi\int_{\theta_1}^{\theta_2} y d\theta,$$

in which $y = I\cos\theta$. We then replace, with sufficient approximation, the above integral by the expression $\frac{y_1 + y_2}{2}$, supposing $y_1 = I_1\cos\theta_1$ and $y_2 = I_2\cos\theta_2$, with the condition that θ_1 and θ_2 be sufficiently near one another.

Assuming that the angles $\theta_1, \theta_2, \theta_3, \dots \theta_n$, are equally spaced, that is, that $\theta_1 - \theta_2 = \theta_2 - \theta_3$, etc., $= \Delta\theta$, the quantity of light received by the total zone $\theta_1\theta_n$ is given by the expression

$$\pi(y_1 + 2y_2 + 2y_3 + \cdots + 2y_{n-1} + y_n)\Delta\theta.$$

The mean intensity corresponding to this zone is

$$\frac{y_1 + 2y_2 + 2y_3 + \cdots + 2y_{n-1} + y_n}{2(\sin\theta_n - \sin\theta_1)}\Delta\theta,$$

and the mean spherical intensity becomes

$$(y_1 + 2y_2 + 2y_3 + \cdots + 2y_{n-1} + y_n)\frac{\Delta\theta}{4}.$$

Finally a third method, employed at the Electrical Exposition at Paris in 1881, consists in dividing the unit sphere into horizontal zones sufficiently narrow, and multiplying the surface of each of these zones by the luminous intensity of the ray which corresponds to its mean parallel. The sum of the products thus obtained, divided by the surface of the sphere (4π), is equal to the mean spherical intensity of the light-source.

[A modification of this method, as used in the Physical Laboratory of the University of Michigan, consists in measuring the intensity in some determined number of azimuths, 12 for example at intervals of 30°, horizontally, and at inclinations of 30°, 60°, and 90° above and below the horizontal. The mean of each set of 12 is multiplied by the area of the zone whose bases (or base, in the case of the polar zones) lie at a distance of 15° from the set. The area of the whole sphere is taken as unity. The sum of these products is equal to the mean spherical intensity. That is:

Mean Spherical Intensity $= 0.0170\, I_{+90°} + 0.1294\, I_{+60°} + 0.2242\, I_{+30°}$
$+ 0.2588\, I_{0°} + 0.2242\, I_{-30°} + 0.1294\, I_{-60°} + 0.0170\, I_{-90°}$.

In the case of incandescent lamps $I_{-90°}$ is zero.]

INCANDESCENT LAMP TEST.

PHYSICAL LABORATORY, University of Michigan.

Photometer Readings, L = ...

Longitudes	0	30	60	90	120	150	180	210	240	270	300	330	360	Volts.	Amp.	Watts
North Pole																
60° N																
30° N																
Horizontal																
30° S																
60° S																
South Pole																

Candle Power, Methven Screen = ...C. P.

Longitudes	0.360	30	60	90	120	150	180	210	240	270	300	330	A Mean.	B Area of Zone.	A x B
North Pole														0.0170	
60° N														0.1294	
30° N														0.2242	
Horizontal														0.2588	
30° S														0.2242	
60° S														0.1294	
South Pole														0.0170	

Mean Horizontal C. P. =
Watts per Mean H. C. P. =
Watts per Mean S. C. P. =

Mean Spherical C. P.

Test of....................Lamp No.............made by

....

...........................189....

CHAPTER II.

PHOTOMETERS.

15. Photometers are apparatus by which we may compare the luminous intensities of two given sources of light; they depend on the following principle: making the illuminations, produced on a given surface by the two lights, vary in a continuous and determinate manner until these illuminations are equal.

This fundamental principle which is at the base of every photometer is an immediate consequence of the fact (§ 8) that the eye appreciates with maximum precision the equality of the illuminations of two surfaces, while the precision with which this organ can determine the ratio of two illuminations is absolutely illusory.

The equality of the illuminations which are compared may be obtained in many different ways:

A. By the application of the fundamental photometric law, that is by varying the distance or inclination of the surfaces whose illuminations are to be made equal;

B. By methods of diaphragmation and of dispersion;

C. By methods based on the properties of polarized light and by methods of mixture of the lights of the sources that are being compared;

D. By methods based on visual acuteness.

The majority of photometers may be included in one of these four categories, although the disposition and construction of the parts, as well as the manner in which the equality of the two illuminations is determined, vary considerably in different apparatus.

E. Besides these photometers those should be mentioned which depend on the various actions of light and which cannot be included in one of the preceding categories.

F. There should also be added to the usual photometers intended for the comparison of the total intensity of light-sources, those which are combined with spectrometers in such a manner as to permit photometric comparisons of different regions of the spectra of the two sources to be studied.

A. PHOTOMETERS BASED ON THE FUNDAMENTAL PHOTOMETRIC LAW.

16. We know that the intensity of illumination produced on an element of surface is given by the formula

$$e = \frac{I \cos i}{d^2},$$

I being the intensity of the luminous source, d the distance from it to the element, i the angle of incidence of the luminous ray on the element.

The illumination may be modified by varying the value of d or that of i. If the distance alone is modified, supposing that the luminous rays meet the screen at the same angle of incidence, photometers are obtained based on the law of the distance. If the distance remains constant and the angle of incidence i alone varies, photometers are obtained based on the law of the inclination.

17. In all photometers based on the law of the distance, the inclination at which the luminous rays of the two sources compared fall on the surface whose illumination is sought, is constant; it is the distance alone of this surface from the two lights which varies.

Consequently the part which is essential and common to all these photometers is one or two divided scales, along which the two lights or the screen may be moved. In the majority of cases the arrangement is such that the two lights and screen are along a straight line; the whole is then mounted on an optical bench.

Lambert (Rumford) Photometer.

18. This photometer, invented in 1760 and first used by Lambert, bears generally the name of Rumford, because this English physicist used it to such a great extent at the beginning of this century.

Let L_1 and L_2 be the two lights (Fig. 4) whose luminous intensities I_1 and I_2 are to be compared, T an opaque pencil, and AB a vertical white screen. The light L_1 projects at L'_1 a shadow of the pencil T, which is illuminated by the light L_2 only, while the shadow L'_2, projected by the latter, is illuminated by L_1 only. By suitably moving the lights L_1 and L_2, we succeed in obtaining the same illumination at L'_1 and L'_2; the eye judges with niceness the moment when this condition is fulfilled. The distances d_1 and d_2

from L_1 and L_2 to the screen are then measured, and we have the relation

$$\frac{I_1}{d_1^2}=\frac{I_2}{d_2^2}.$$

Generally the two lights are moved along a divided scale perpendicular to the screen, but sometimes in any manner whatever; we then neglect the law of inclinations which requires the inclination

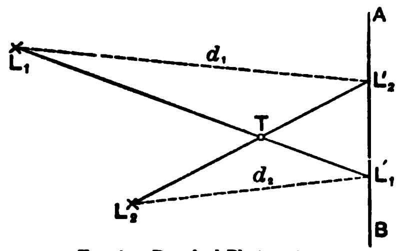

FIG. 4. — Rumford Photometer.

to be constant, in order that the law of distances may be exact; we may take account of this last, and we will find that if L_1 remains fixed, the light L_2 may be moved along a curve whose form might be studied would space permit. But the want of exactness of Rumford's photometer renders this correction deceptive.

In practice if we wish to measure rapidly the intensity of a luminous source, say within 10 or 15 per cent, the Rumford photometer is very valuable in that it is easy to set up, but it makes no pretension to giving results which are rigorously exact.

Bouguer Photometer.

19. This is the oldest photometer. The screen AB is divided into two equal parts by a partition CC, normal to the illuminated

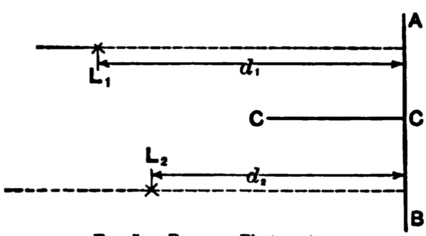

FIG. 5. — Bouguer Photometer.

surface. One of the lights L_1 illuminates the half CA of the screen, while the other L_2 illuminates the other half CB; in order to

establish equality of illumination of the two halves of the screen, the two lights may be moved along divided scales which are perpendicular to the screen.

Most frequently one of the lights, L_1 for instance, is fixed; the other alone is movable.

The opaque screen is generally replaced by one semi-transparent, say either ground-glass or a sheet of paper; then the equality of illumination of the two halves of the screen is observed by placing oneself on the side opposite the light.

Foucault Photometer.

20. The Foucault photometer is only a simple modification of the Bouguer photometer. The partition C does not extend to the screen, but may be moved at will by a special contrivance in such a manner as to reduce to a simple line the shadow which separates the two halves of the screen, one illuminated by L_1, and the other by L_2.

To make a reading with the photometer, one of the lights is moved until the illumination of the opalescent screen is as uniform

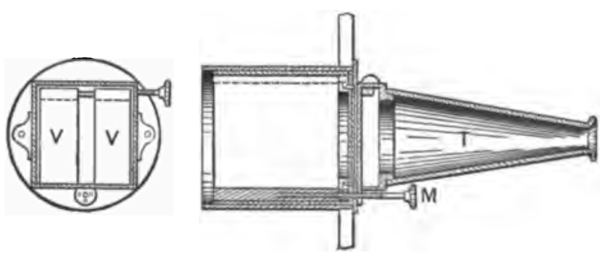

FIG. 6.—Foucault Screen.

as possible, which is determined by standing behind the screen and looking through the tube T either directly or by the aid of a telescope.

The rays from the two lights do not always fall normally on the screen; but care should be taken that they fall invariably at the same angle in order that the factor $\cos i$ may be constant.

Figure 6 gives a cut of the Foucault screen such as is generally used; the adjusting screw M serves to move the partition.

Figure 7 represents the complete installation of a photometric bench provided with a Foucault screen.

The standard (a carcel lamp) is placed at the right at the distance of one meter; the light which is being studied is mounted on a piece which is movable along a divided scale; this movable piece is governed by a crank placed within reach of the observer.

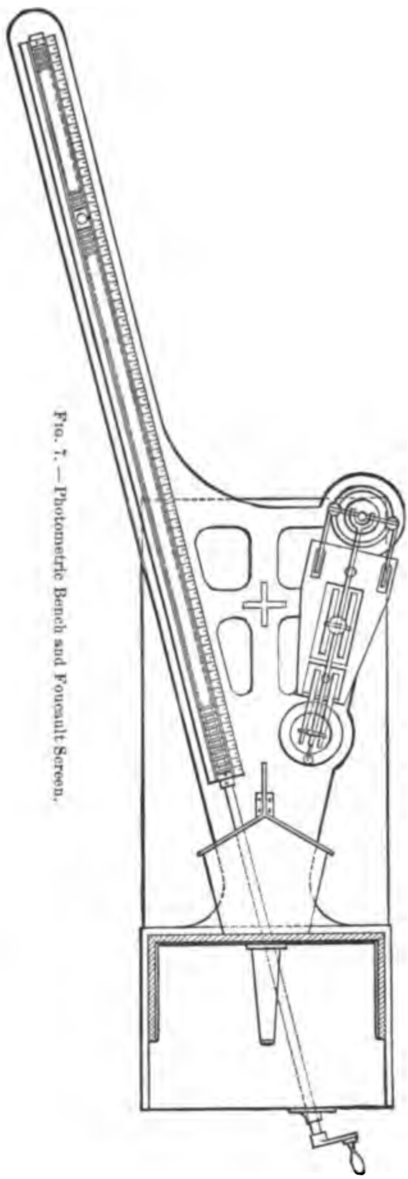

Fig. 7.—Photometric Bench and Foucault Screen.

Figure 8 represents a simplified form of the Foucault photometer, such as the Continental Gas-meter Company of Paris makes. The screen is placed in a box fixed on a support at the front of the apparatus.

This form of the Foucault photometer has a disadvantage which is relatively considerable: it is necessary that the two lights should be placed on the same side of the screen. We may easily overcome this inconvenience by the aid of an arrangement invented by Ritchie (Fig. 9), in which two mirrors m_1, m_2 making an angle with one another throw upon the screen AB the luminous rays coming from the two sources. These mirrors being at right angles, the light which comes from L_1 and L_2 is reflected normally upon the surface of the screen.

Figure 10 shows the details of the photometer employed by Violle in his researches on the absolute standard of light*.

The shutters of the photometric box are open at the side to show the interior.

* *Lum. Él.*, Vol. XXXIV. p. 52.

The side shutters have circular openings for the passage of the rays of light. Two front shutters, of which only one is shown in the figure, keep the light from the two sources from reaching the observer.

The screen E is placed at the end of a telescope which allows the observer to verify the equality of illumination of the two divisions of the screen. The two mirrors M and M' are fastened by two plates of metal to the large toothed wheel R, governed by the crank m, which acts on the rod a and the toothed wheel r. This mechanism serves to turn the mirrors, that is, to substitute the mirror M for the mirror M' and *vice versa*, in order to eliminate

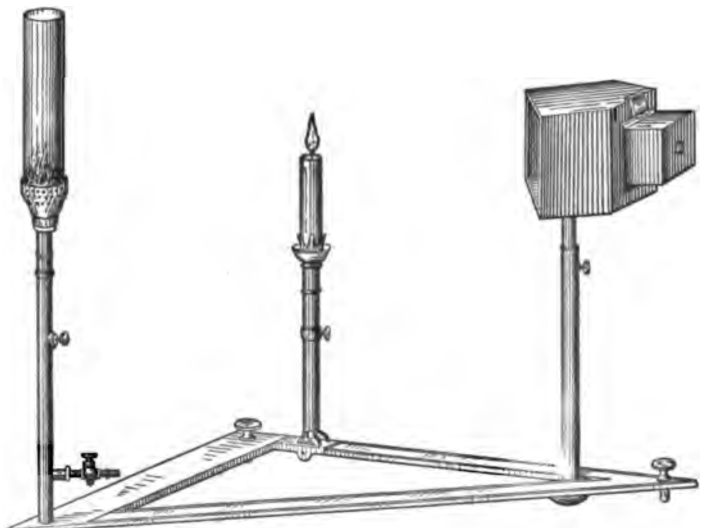

FIG. 8. — Foucault Photometer (Simplified Form).

the error resulting from differences in the coefficients of absorption and reflection of the two mirrors.

In order to effect this movement, it is sufficient to turn the crank m until the wheel R has turned 180 degrees, which is indicated by two stops, one at each end.

The figure shows, besides, the construction of the photometric bench, on which the screen is moved by hand by means of a small carriage running along two rails.

As the observer determines, by means of the equality of the brightness of the two divisions of the screen, the equality of the

illuminations produced by the two sources of light which are being compared, it is necessary that the two tints should have the same proportion as the illuminations which produce them.

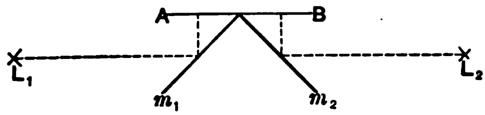

FIG. 9. — Ritchie's Arrangement.

Now the brightness of the screen depends on the coefficients of absorption, of reflection, and of transparency of the opalescent plate. In order that the equality of the brightness of the two halves of the screen may correspond to the equality of the illuminations produced by the two lights, it is necessary that these three coefficients have respectively the same values for the two divisions of the screen; that is, that the screen be perfectly homogeneous.

If this condition is not exactly fulfilled, the resulting error may, however, be eliminated by repeating the measurement after having turned the screen so that the right division becomes the left, and *vice versa*.

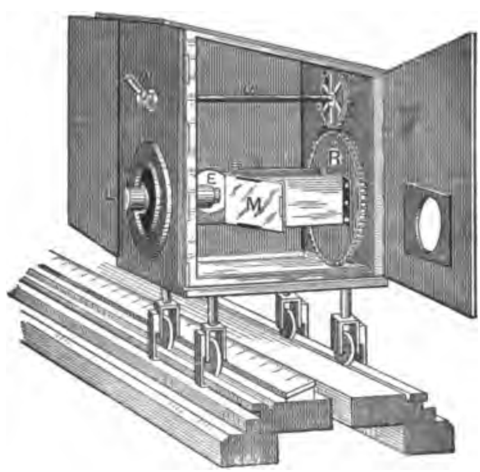

FIG. 10. — Foucault Photometer (Violle's Pattern).

21. *Construction of the Foucault screen.* — The nature of the opalescent screen of the Foucault photometer being of very great importance for exactness of measurements, we give some details

of its construction, taken in great part from a memoir by Crova* on diffusing screens. The information which we give here may be applied also to the construction of diffusers, whose use [as a secondary source of light] plays a quite important *rôle* in certain photometric apparatus, and to which we shall return later.

The screen should be somewhat transparent, but not so much so that the light-source may be distinguished through it; also the surface should be as uniform as possible. Foucault used a screen made of a sheet of plate-glass on which a thin layer of starch or dried milk had been deposited very uniformly; when the layer was dry, it was protected by another sheet of glass fastened at its edges on the first, yet avoiding all contact with it: this result was attained by having previously glued on the second glass a frame formed of narrow strips of paper.

The following is the manner in which Deleuil prepares Foucault screens. Wheat-starch is mixed with distilled water, the liquid is passed through a very fine gauze, and after having been allowed to settle for a moment, the milky liquid is decanted and afterwards briskly agitated and poured by means of a pipette upon a sheet of glass laid absolutely horizontal. The glass should have been previously cleaned with the most scrupulous care.

When the milky liquid has spread to the edges, it is allowed to stand, then the glass is given a slight inclination by means of one of the leveling-screws of the tripod on which it lies, and the water is drained off by means of a strip of filter-paper acting as a siphon; finally it is allowed to dry as it lies. The milky liquid should be used immediately, and the temperature should not exceed 18° C.; experience shows what degree of opaqueness to give the liquid.

As to the opaqueness of the layer, it is sufficient to make it of such a thickness that in looking at the sun through the screen we can neither distinguish its outline nor its position. In certain cases screens constructed in this manner are a little too opaque. Crova has succeeded in obtaining them more translucent and of a remarkable homogeneity by employing beet-root starch, whose grains are spherical, of great limpidity, and very small.

To obtain this starch we put the grain to soak for several days in water frequently renewed, then each grain is cut in two by means of a very fine scapula; with a sharp needle, and making use of a

* *Ann. de Chim. et de Phys.* (6), Vol. VI. p. 342.

magnifying-glass, we detach the little particles of starch contained in the grain, which appear as small white points.

This starch is ground in water in a glass mortar, and the milky liquid passed through very fine muslin. This method of preparation is very long, but it gives screens of remarkable fineness and uniformity of grain.

Ground-glass may also be used, but it is very difficult to regulate its degree of opaqueness and to give it the necessary uniformity. This surface is also extremely changeable and of remarkable instability; the least friction, or even slight contact with an organic surface, is sufficient to produce a local change in opaqueness which it is impossible to remedy; also when a ground-glass satisfying the desired conditions has been obtained, it is necessary, as for the Foucault screen, to protect it in a positive manner by a transparent sheet of glass fastened at its edges without touching it. Some have succeeded in obtaining opal glasses, very homogeneous, of a milky appearance, without an appreciable grain, that have parallel faces; but these screens modify by a phenomenon of diffraction the tint of the incident light, and that which they diffuse appears slightly reddish.

This alteration of the tint by opal glasses offers no inconvenience if the screen is employed simply to determine the equality of illuminations; but it becomes an obstacle when the screen serves to weaken the light.

Relief-Photometers.

22. In the preceding photometers the two divisions of the screen are placed in the same plane.

In order that the two sources of light that are compared may be placed along the axis of the same photometric bench, on the two sides of the screen, it is necessary to have recourse to the system of mirrors invented by Ritchie.

It is possible, however, to obtain the same result in the following manner, proposed at first by Villarceau and afterwards adopted with a slight modification by Sylvanus P. Thompson and Starling.

In the relief-photometer of Villarceau, the screen is formed by two opaque plates, p_1 and p_2, placed vertically on the optical bench and making between them a right angle (Fig. 11). The plate p_1 is illuminated by the rays of light coming from L_1 only, while the plate p_2 receives the rays from L_2 only. The equality of illumination of the two faces of the screen may be determined with

the greatest facility; for the whole screen then appears as a single, plane, illuminated surface, in which the edge of the diedral angle of the screen is no longer perceptible. The luminous intensities of

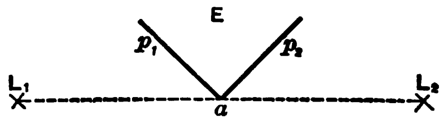

FIG. 11. — Relief-Photometer.

the two lights are then in the direct ratio of the squares of their distances from the edge of the screen.

Conroy* has modified the arrangement of the screens of the relief-photometer of Villarceau, so as to increase the precision of the measurements by removing the difficulty that there was in making the edges of the two divisions disappear. The following is the arrangement invented by Conroy, the details of which are given in Fig. 12.

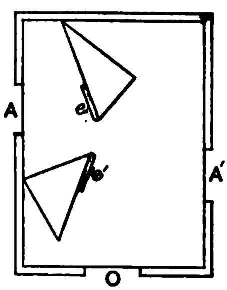

FIG. 12. — Conroy Screen.

The box inclosing the two screens is placed on the photometric bench in such a manner that the light coming from the light-centers to be compared enters by the circular openings A and A'. The screens e and e' are fixed at the extremities of the hypothenusal faces of the two triangular prisms shown; they are simply cut out of a sheet of paper which is white and slightly glazed.

The observer looks through the opening O and determines the moment when the two screens e and e', being equally illuminated, appear as a single surface. Conroy found that it was advantageous in the measurements to observe the screens at an angle of incidence of about 60 degrees, they being illuminated at an angle of 30 degrees; these conditions are realized in the figure.

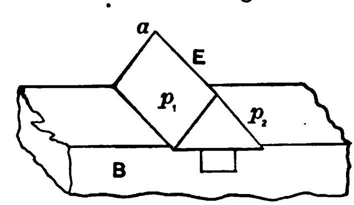

FIG. 13. — Thompson-Starling Screen.

23. In the apparatus of Thompson and Starling (Fig. 13) the cuneiform screen is arranged so that the edge a is horizontal; this arrangement requires the observer to verify the equality of the illuminations by placing himself above the

* *Phil. Mag.* (5), 1883, Vol. XV. p. 425.

screen, that is, between the two lights. If it is desired to avoid this condition, it is possible to have recourse to a mirror inclined at 45 degrees with the horizontal, and throwing forward the image of the screen.

Bunsen Photometer.

24. Of all the industrial photometers, that of Bunsen is certainly the one that is employed most frequently, particularly in Germany. This is because its manipulation is quite rapid and its indications relatively very precise.

Bunsen's photometer is based on the following property: a spot of oil or grease on a sheet of paper appears bright against a dark

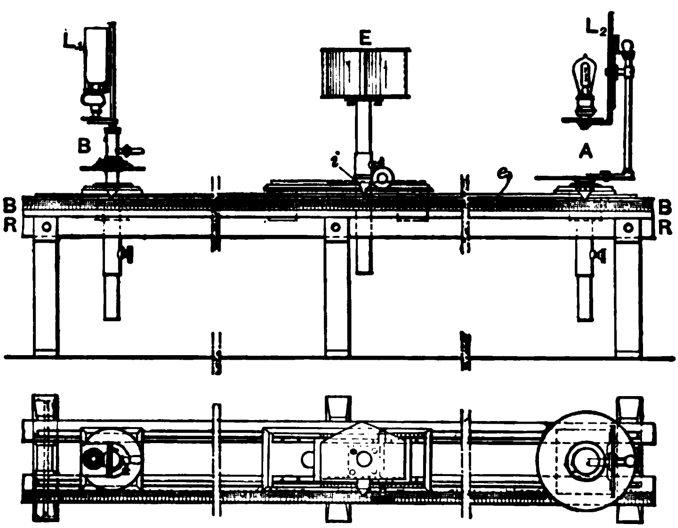

FIG. 14. — Bunsen Photometer.

background, when we see it by transmitted light, and dark against a light background, when we see it by reflected light. Consequently if the paper is equally illuminated on both sides, the spot should neither be bright on a dark background nor dark on a bright background; it should then disappear completely.

The construction of Bunsen's photometer is very easily understood (Fig. 14). The screen E and its accessories are mounted on a carriage movable on a divided bench (an optical bench), on which

are placed at A and B the two sources of light L_1 and L_2 whose intensities are to be compared. The screen is so arranged that the rays coming from the two lights are perpendicular to it.

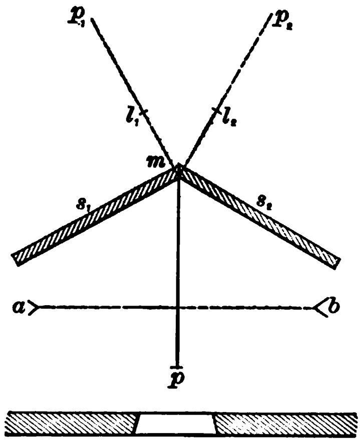

FIG. 15. — Rudorff's Mirror.

25. In order that we may observe simultaneously the two sides of the screen, a system of mirrors, proposed by Rudorff*, is frequently employed (Fig. 15).

In this arrangement the screen p constitutes the bisecting plane of the angle, 140°, formed by the two mirrors S_1 and S_2. The screen is illuminated by the rays a and b coming from the lights that are being compared. The observer places his eye at o, and sees, through the opening which is left in the side of the photometer, the two faces of the screen reflected at p_1l_1 and p_2l_2 by the mirrors S_1 and S_2.

The gravest defect of this arrangement comes from the fact that the two images of the spot, p_1l_1 and p_2l_2, are too far from one another and are separated by the shadows ml_1 and ml_2; it is impossible to avoid this inconvenience, since it is necessary that the spot should be at a sufficiently great distance from the edge of the mirror to be always outside the obscure zone ml.

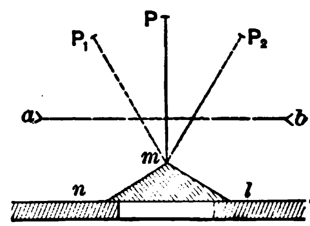

FIG. 16. — Hefner Prism.

Von Hefner Alteneck † has modified Rudorff's arrangement and replaced it by that of Fig. 16, in which the mirrors are replaced by a prism nml placed before the screen mp; the images are contiguous and no zone of shadows interferes with the observations.

Krüss ‡ has modified this arrangement so as to avoid the deformation of the images produced by reflection and refraction in the prism.

* *Journal für Gasbeleuchtung*, 1869, p. 567.
† *Elektr. Zeitschr.*, 1883; *Lum. Él.*, Vol. X. p. 500.
‡ *Centralblatt für Elektr.*, Vol. VI. p. 781.

Figure 17 gives the details of his modification. The screen P is placed in the median plane of the two prisms I and II. The angle formed by the faces of these prisms is so chosen that the rays which fall perpendicularly on the face A_1 of the prism I, and which come from the points ab of the screen, are reflected at B_1, C_1, and A_1, and emerge from the prism perpendicularly to the face D_1. The rays follow an analogous trajectory in the prism II. A tube of variable length to suit the observer may be placed before the faces D_1 and D_2; this tube is terminated by a diaphragm of small opening which fixes the position of the eye in the plane of the screen.

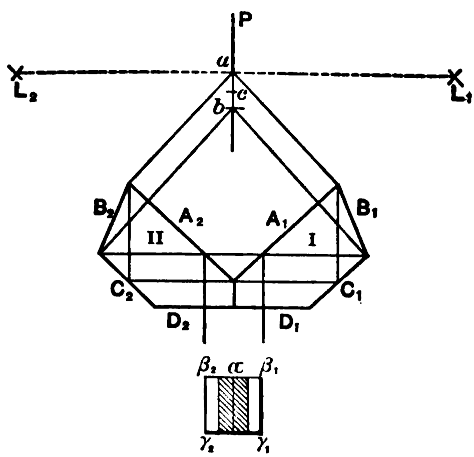

FIG. 17. — Krüss Prism.

The eye then sees the visual field divided into halves by the line of separation a of the two faces D_1 and D_2; at the right is found the image of the right side of the screen illuminated by one of the lights L_1. The image of the left side of the screen illuminated by the other light L_2 is found at the left. The image of a thus falls in the zone α, and that of b in the lateral parts of the visual field β_1 and β_2.

If the part ac of the screen represents the spot, γ_1 and γ_2 represent the images of the edges of the spot, and the comparisons are exact.

A Bunsen screen may be transformed immediately into a Foucault screen by suppressing the spot or removing it outside of the field of vision.

The nature of the paper which constitutes the screen and that of the spot have a considerable influence on the precision of the measurements. In order to take account of them in an exact manner, a complete theoretical study of the apparatus should be made.

Theory of the Bunsen Screen*.

26. Let us consider the most general case in which the two faces of the screen are not identical, that is, they have different coefficients of transparency and reflection. We may afterwards simplify the formula by assuming the identity of the two faces.

Let us designate by a_1 the coefficient of absorption, by t_1 the coefficient of transparency, by r_1 the coefficient of reflection of the opaque part of the left face of the screen, and by a_1', t_1', r_1', the same coefficients relative to the left face of the spot. Let us designate in the same way by a_2, t_2, r_2, and a_2', t_2', r_2', the same coefficients relative to the right face of the screen.

The coefficient r_1, for instance, determines the illumination of the opaque part of the left face of the screen due to the light L_1 at the left, while t_1 determines the illumination of the same part of the screen due to the light L_2 at the right; the other coefficients determine in the same way the illumination of the other parts of the screen.

These coefficients vary with the direction from which the observer looks at the screen. Assuming that the screen satisfies the law of Lambert, we should have

$$a+t+r=1.$$

It may practically be assumed that this law is satisfied, on condition that the screen is always observed from the same direction.

Let e_1 be the intensity of illumination of the left side of the opaque part of the screen, and let e_1' be the intensity of illumination of the left side of the spot; designate by e_2 and e_2' the corresponding quantities relative to the right side of the screen. Further, let I_1 and I_2 be the intensities of the lights at the left and the right, d_1 and d_2 their distances from the screen; we obtain immediately, neglecting a factor in the final result,

* Léonard Weber, Théorie du Photomètre de Bunsen; *Wiedemanns Ann.*, Vol. XXXI. p. 676; *Lum. Él.*, Vol. XXXI. p. 267; Boulouch, Sur le photomètre de Bunsen, *Comptes Rendus*, Vol. CV.

$$e_1 = \frac{I_1 r_1}{d_1^2} + \frac{I_2 t_1}{d_2^2}, \qquad e_1' = \frac{I_1 r_1'}{d_1^2} + \frac{I_2 t_1'}{d_2^2},$$

$$e_2 = \frac{I_2 r_2}{d_2^2} + \frac{I_1 t_2}{d_1^2}, \qquad e_2' = \frac{I_2 r_2'}{d_2^2} + \frac{I_1 t_2'}{d_1^2}.$$

There are, then, on each face of the screen two intensities of illumination, viz. e_1 and e_1' on the left face, and e_2 and e_2' on the right face.

Observations may then be made in three different ways, viz.:

1°. The two illuminations e_1 and e_1' are made equal, that is, the spot disappears on the left face of the screen;

2°. The two illuminations e_2 and e_2' are made equal, which corresponds to the disappearance of the spot on the right side of the screen;

3°. The ratios $\frac{e_1'}{e_1}$ and $\frac{e_2'}{e_2}$ are made equal, that is, the spot stands out from the rest of the sheet of paper with the same intensity on both sides of the screen.

Suppose the two lights fixed and the screen movable; call L the position of the screen corresponding to the first method of observation, R the position corresponding to the second method, M the position corresponding to the third method of measurement.

The point M is generally situated between L and R. As to the relative positions of L and R, two cases are distinguished according to the nature of the screen.

In the first case, the point L is situated at the right of M, and the point R at the left; these three points then succeed one another in the following order: R, M, L going from left to right; the screen is then called a *negative screen;* if the three points succeed one another in the inverse order, it is called a *positive screen.*

The sign of the screen depends upon the coefficients of transparency and reflection of the spot and of the opaque paper adjacent.

With a positive screen a case may be presented where the three points L, M, R coincide. The spot appears bright on a dark background in the position M; it is dark on a bright background with a negative screen. [See Appendix A.]

These two distinctive characteristics obtain only when the spot is more transparent than the rest of the screen; they should be inverted when it is less transparent, which takes place, for instance, when it is formed by an opaque varnish.

If the screen is managed in such a manner as to make the spot on the left face of the screen disappear (position L), we have $e_1 = e_1'$; that is,

$$\frac{I_1 r_1}{d_1^2} + \frac{I_2 t_1}{d_2^2} = \frac{I_1 r_1'}{d_1^2} + \frac{I_2 t_1'}{d_2^2}.$$

Whence,
$$I_1 = \frac{(t_1' - t_1)\, d_1^2}{(r_1 - r_1')\, d_2^2} I_2 .$$

Designate by p the ratio $\frac{d_1^2}{d_2^2}$, and we have

$$I_1 = \frac{(t_1' - t_1)}{(r_1 - r_1')} p_1 I_2. \tag{1}$$

Observing the disappearance of the spot on the right side in the same manner (position R), we have

$$I_1 = \frac{(r_2 - r_2')}{(t_2' - t_2)} p_2 I_2. \tag{2}$$

If we assume that the coefficients are in equal pairs, that is,

$$r_1 = r_2, \quad t_1 = t_2, \quad r_1' = r_2', \quad t_1' = t_2',$$

we obtain, taking the product of the two preceding relations,

$$I_1 = \sqrt{p_1 p_2}\, I_2. \tag{3}$$

This formula supposes that the two faces of the screen are identical; we may free ourselves from this restriction by making two new settings p_3 and p_4, having turned the screen around. We have then

$$I_1 = \sqrt[4]{p_1 p_2 p_3 p_4}\, I_2. \tag{4}$$

An analogous result is obtained by employing the third method, in which the observations are made in the position M: such a point is chosen as to obtain equal contrasts of illumination, that is, we make

$$\frac{e_1'}{e_1} = \frac{e_2'}{e_2}.$$

The ratio of the illuminations, and not their difference, is considered since, from the psycho-physical law of E. H. Weber (§ 8), the perception of the differences of two sensations is proportional to their ratio, and not to their difference.

We have then

$$\frac{e_1'}{e_1}=\frac{I_1r_1'+I_2t_1'p_1}{I_1r_1+I_2t_1p_1},$$

$$\frac{e_2'}{e_2}=\frac{I_2r_2'p_1+I_1t_2'}{I_2r_2p_1+I_1t_2}.$$

The condition

$$\frac{e_1'}{e_1}=\frac{e_2'}{e_2}$$

gives, then, for I_1 the quadratic equation

$$K_1I_1^2+p_1(K_3-K_4)I_2I_1+K_2p_1^2I_2^2=0,$$

in which

$$K_1=r_1't_2-r_1t_2', \qquad K_2=r_2t_1'-r_2't_1,$$

$$K_3=t_2t_1'+r_2r_1', \qquad K_4=t_1t_2'+r_1r_2'.$$

Assuming the identity of the two sides of the screen, we have $K_3=K_4$ and $K_1=K_2$; the ordinary formula is then obtained,

$$I_1=P_1I_2. \tag{5}$$

In the general case the coefficients r and t may be eliminated by taking a second setting p_2 at M_1 after having turned the screen. The factors K_1 and K_2, and K_3 and K_4 are then inverted, and we obtain

$$I_1=\sqrt{p_1p_2}I_2. \tag{6}$$

27. To complete this brief theory of the Bunsen photometer, it is necessary to deduce in addition the formulæ for calculating errors of measurement.

The error ΔI_1 of the result depends on the error Δp of the setting p; the following considerations give the algebraic expression of this dependence.

In the case where one is limited to a single measurement (formula 2),

$$\frac{\Delta I_1}{I_1}=\frac{\Delta p_1}{p_1}. \tag{2'}$$

In the case where two settings p_1 and p_2 are made, it may be assumed that the errors Δp_1 and Δp_2 are equal. We then have (formula 3 or 6)

$$\frac{\Delta I_1}{I_1}=\frac{1}{2}\sqrt{2}\frac{\Delta p}{p}. \tag{3' and 6'}$$

Finally, if four settings have been made, we have

$$\frac{\Delta I}{I_1}=\frac{1}{4}\sqrt{4}\frac{\Delta p}{p}. \tag{4'}$$

It is necessary then to evaluate first of all the ratio $\frac{\Delta p}{p}$.

This ratio depends on two factors which vary according to the nature of the observation (L, R, M). The first is a function of the coefficients r and t, that is, a function of the nature of the screen; the second factor depends on the psycho-physiological qualities of the eye, and in particular on the faculty, more or less great, of the eye to perceive the equality of illumination of two surfaces (at R or L), or to perceive the equality of the contrasts of two illuminations.

Designate by

$$\Delta q_1=\frac{\Delta e_1'}{\Delta e_1},$$

$$\Delta q_2=\frac{\Delta e_2'}{\Delta e_2},$$

$$\Delta Q=\frac{\Delta\left(\frac{e_1'}{e_1}\right)}{\Delta\left(\frac{e_2'}{e_2}\right)},$$

the ratios of the illuminations or of the contrasts still perceptible to the eye at the limit; there will be found after many reductions the following values for the ratio $\frac{\Delta p}{p}$, in the three principal positions of the screen:

$$\frac{\Delta p}{p}=f_1\Delta q_1 \quad (\text{at } L),$$

$$\frac{\Delta p}{p}=f_2\Delta q_2 \quad (\text{at } R),$$

$$\frac{\Delta p}{p}=F\Delta Q \quad (\text{at } M).$$

The constants f_1, f_2, F, have the following values:

$$f_1=\frac{r_1}{r_1-r_1'}+\frac{t_1}{t_1'-t_1},$$

$$f_2=\left[\frac{r_2}{r_2-r_2'}+\frac{t_2}{t_2'-t_2}\right],$$

$$F=\frac{1}{\frac{r_1-t_1}{r_1+t_1}+\frac{t_1'-r_1'}{t_1'+r_1'}}$$

For simplicity it has been supposed, in the calculation of F only, that the two sides of the screen are identical.

The factors f_1, f_2, and F are the coefficients of sensibility of the screen for the positions L, R, and M. The minimum value of f_1 or f_2 is equal to 1, while that of F is equal to 0.5. However, these coefficients have values in reality much higher.

Construction of the Bunsen Screen.

28. In the construction of the Bunsen screen the coefficients of sensibility f_1, f_2, and F should be made as small as possible. For this it is necessary that the coefficients r_1' and t_1 should be very small, r_1 and t_1' being very great. The best screen is therefore made from a dull, white, opaque disc, while the spot should be as transparent as possible, and affected by an insensible coefficient of reflection.

The error of the measurements depends also on the sensible values of the perception, Δq_1, Δq_2, and ΔQ. Now these sensible values become less in proportion as the delimitations of the spot and of the opaque part of the screen become more perfect. With certain screens we may have favorable values for the coefficients of sensibility f and F, but unfavorable values for the sensible values Δq and ΔQ.

With regard to this, the following numbers obtained by Leonard Weber of Breslau, with eight different screens, allow an exact idea of the size of these coefficients to be formed. These eight screens, designated by the figures 1 to 8, have considerably different coefficients, which is furthermore a consequence of the differences in their construction, which the following description takes into account:

1. Toepler Screen: A sheet of white paper pierced with a circular hole, covered on each side with a sheet of tracing-paper and put together without glue.

2. Two thin pieces of white cardboard pierced with a circular hole, and between them a sheet of tracing-paper.

3. and 4. Krüss Screens: Sheets of white school-paper with a paraffine spot.

5. White cardboard pierced with a hole, covered with a sheet of tracing-paper blackened with plumbago so as to have unequal faces.

6. Two sheets of white paper exactly alike, pierced with a hole, and covered on each side with a sheet of tracing-paper.

7. Two sheets of white paper, between which is a sheet of tracing-paper.

8. Oiled paper, the spot being formed on each side by a band of white varnish.

The following table includes values obtained for the ratios of the coefficients r and t of the left face of the screen; with the exception of number 5, the differences between the two faces are insensible. The sixth column includes the mean of the values of f_1 and f_2, which, moreover, differ slightly; the seventh includes the mean of the corresponding values Δq_1 and Δq_2; that is,

$$\Delta q = \frac{\Delta q_1 + \Delta q_2}{2}.$$

The last two columns include the values of F and ΔQ.

Number of the screen.	$\frac{r_1}{r_1'}$	$\frac{t_1'}{t_1}$	$\frac{r_1}{t_1}$	$\frac{t_1'}{r_1'}$	f	Δq	F	ΔQ
1	1.71	3.48	4.80	1.24	2.98	0.89 0.96	1.34	1.49 1.49
2	4.54	0.25	....	2.33	1.30	1.88 2.10	0.71	2.26 1.56
3	1.94	1.71	2.74	1.21	3.41	1.11 0.76	1.77	0.79 0.90
4	1.93	3.51	6.18	1.10	2.55	1.18 1.24	1.32	1.97 1.52
5	3.23	9.83	26.34	1.21	....	....	....	....
6	1.75	10.76	12.38	1.52	2.53	0.86 0.71	0.95	0.84 2.53
7	3.76	6.47	13.26	1.83	1.58	1.42 1.67	0.87	1.16 0.93
8	1.38	2.66	4.94	0.84	4.06	0.47 0.36	1.92	0.73 0.99

The values of Δq and ΔQ included in the first horizontal line are relative to the visual power of Weber, those in the second to that of his assistant. Knowing $f \cdot \Delta q$, and $F \cdot \Delta Q$, it is easy to calculate the error of the setting in the positions L, R, or M of the screen; it is seen then that the precision attained with the different screens

is always less than the precision limit which results from the conditions of sensibility of the eye; it varies, furthermore, with the observer.

Thus the best screen is, not that for which the coefficients of sensibility or the coefficients Δ have a minimum value, but that for which the product $f \cdot \Delta q$ is a minimum.

The following is a very simple method of constructing a screen whose properties are very satisfactory: a sheet of white paper is stretched on a board; a disc of brass twenty millimeters in diameter fitted with a handle is heated, then plunged into a bath of paraffine; after being allowed to drip a little, it is carefully placed on the sheet of paper; a certain number of spots are thus made successively, and the best is chosen; finally, the superfluous paraffine is removed by the aid of a sheet of blotting-paper, on which a hot iron is moved about (moderately hot, so as not to injure the edges of the spot).

In order to facilitate its reversal, the screen is mounted in a movable frame in a box; this frame should be fitted with lamellar springs to keep the sheet perfectly rigid.

The reversal of the screen, in some form or other, is necessary in all measurements of precision, because the nature of the two faces of the screen may change with great rapidity.

Likewise, it is not sufficient to have determined, at a given time, the equality of the two faces of the screen in order that this result may be used rightly in later measurements, since the physical conditions of the two faces of the screen may have varied..

If, for example, one of the faces of the screen has been exposed more than the other to diffused light, very appreciable differences in the value of the coefficients *a*, *t*, and *r* may be immediately noticed. Screens should therefore be kept in the dark in the interval between measurements.

The list given above indicates other methods of constructing the screen; the advantages of one over another are not such that one may be indicated as preferable to the others.

Screens of Elster and Joly.

29. To the screens mentioned above should be added the quite peculiar screen invented by Elster *, of Breslau. This screen con-

* *Lum. Él.*, Vol. II. p. 540.

sists of two rectangular blocks separated by a sheet of opaque metal (Fig. 18); the whole is compressed so as to form a single block only. The translucent substance should be so chosen as to diffuse light equally in all directions.

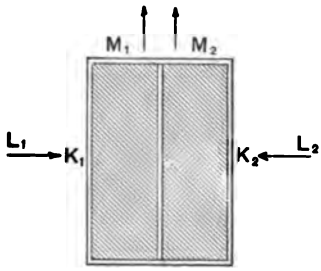

Fig. 18.

The rays of light coming from the sources compared L_1 and L_2 fall normally on the faces K_1 and K_2 of the screen; the observer determines when there is equality of illumination of the two halves of one of the vertical faces M_1, M_2, parallel to the rays of light. The illuminations of the faces K_1, K_2 are then equal also. Stearine or paraffine may be employed for the translucent block; moreover, opaque glasses may also be used with advantage.

30. The screen invented at about the same time by Joly* of Dublin is analogous to that of Elster; it depends on the following principle:

When a translucent body is slit, the light which crosses it is reflected in part by the faces of the slit, so that the part of the body on one side appears darker than that on the other. If this slit is turned so that the light falls equally on the two faces, or indeed, if two sources of light illuminate the two faces equally, the slit is no longer visible.

Joly employed two rectangular blocks of paraffine put together face to face; the screen is then placed so that the slit is vertical, that is, perpendicular to the rays from the two sources.

In place of paraffine, Joly has also employed opal glass, which has the advantage of lasting longer and of permitting more exact adjustment. The two rectangular blocks are, in this case, cemented with Canada balsam; if a sheet of silver is introduced between, an Elster screen is obtained. The face of the block is carefully adjusted normal to the slit; the observer viewing this face determines by the disappearance of the slit the moment when the two halves of the screen are equally illuminated. The blocks which gave the best results were $20 \times 50 \times 11$ mm. in size.

Differences of transparency in the two halves of the translucent

* *Lum. Él.*, Vol. XXIX. p. 238.

blocks in these two screens are compensated by reversing the screens; the mean of the values obtained in the two cases is free from this cause of error.

Optical Screen of Lummer and Brodhun.

31. In this carefully elaborated screen, the inconvenience resulting from the transparency of the spot and the paper is completely overcome, so that each of the two fields, whose illuminations are being compared, receives light from a single source only; furthermore, the line of separation of the two fields is very sharply defined, and disappears completely at the moment of equality of illumination.

The screen of Lummer and Brodhun* is based on the following arrangement:

Let l and λ be two surfaces (Fig. 19) illuminated and emitting diffuse light; A and B two right-angled triangular prisms, put together by their hypothenusal faces, so that the light coming from λ incident on the part hi of the face of B is brought to 0, while the part of the light coming from l, falling on the part qh, is transmitted directly to 0. The eye placed at 0, being accommodated to the face pi, sees a uniformly illuminated surface when the illuminations of the surfaces l and λ are in a determined ratio.

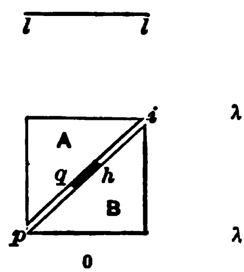

FIG. 19.

These conditions may be realized by the aid of many different arrangements of the prisms, among which the following (Fig. 20) has given the best results:

The hypothenusal face of the prism A which rests on the hypothenusal face of the prism B has the form of a spherical surface cut in a small circle by a plane. The rays which come from l traverse this surface of contact without any reflection or refraction, and there is obtained an elliptical field in the uniformly illuminated hypothenusal face of the prism B. This elliptical field is very sharply outlined, and the line of demarcation disappears completely at the moment of equality.

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* *Zeitschrift für Instrumentenkunde*, 1889, p. 41; *Lum. Él.*, Vol. XXXIII. p. 410.

The opaque screen P (Figs. 20 and 20 *bis*), whose two faces correspond to the surfaces l and λ of the preceding figure, is placed perpendicular to the photometric bench. The diffuse light emitted by the faces of the screen falls on the mirrors e and f, which reflect it normally upon the faces of the prisms A and B. (In the figure we see the mirror f only.) The observer determines when there is equality of illumination of the two fields by the aid of a telescope, r. The photometric box is carried by the horizontal trunnion a, which rests with its extremities between the screws m_1 and m_2 fixed at the ends of vertical supports S_1 and S_2. The screen is placed in the frame n, which permits it to be reversed at will. The mirrors

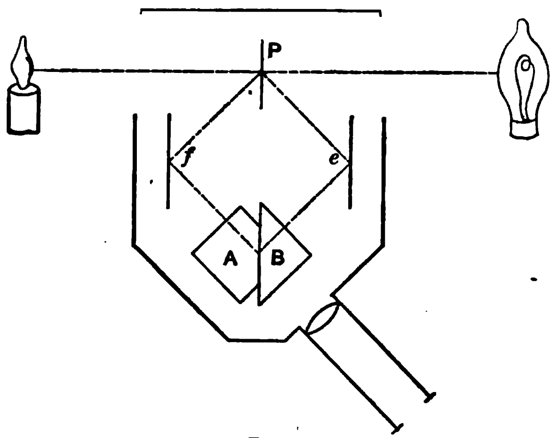


FIG. 20.

and prisms are provided with regulating screws. The photometer box is provided, furthermore, with a cover, having an opening to admit the screen, to whose faces the light from the sources to be compared comes through lateral circular openings.

The supports S_1 and S_2 have stops which serve to fix the position of the photometric box after its reversal; this last operation is necessary in order to eliminate errors of regulation and differences in the two faces of the screen, which is formed by two sheets of paper separated by a sheet of tinned paper.

The optical screen of Lummer and Brodhun has a coefficient of sensibility equal to unity, while that of the screens of Krüss or Elster varies between 2.5 and 3.5. This screen has, then, much greater sensibility than ordinary screens, and this sensibility is still further increased from the fact that the line of separation of the

two fields is sharply defined, which gives to the coefficient Δq a very small value. The mean error of a determination has been found equal to about 0.5 per cent.

The photometric determinations obtained with this screen are effected by equalizing the illuminations of the two parts of the

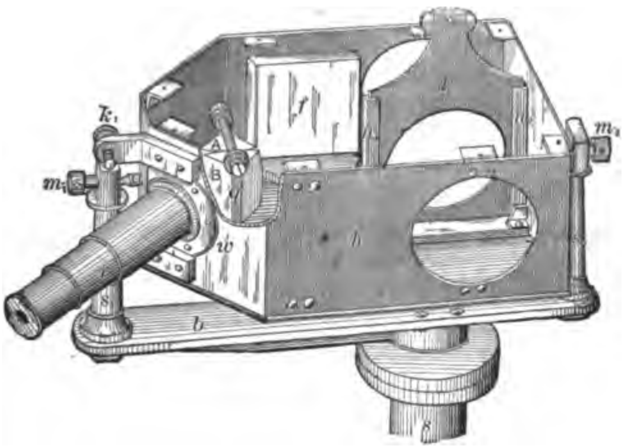


FIG. 20 *bis*. — Lummer-Brodhun Optical Screen.

field. They are then analogous to the settings L and R of the Bunsen screen. Brodhun and Lummer* have recently modified their apparatus in such a manner as to be able to effect also the setting M of the Bunsen screen, in which the two ratios or the contrasts of illumination of the two parts of each face of the screen are equalized; the photometer then becomes a contrast photometer.

This result is obtained in the following manner: The hypothenusal face of the two prisms in contact is divided into four rectangular fields (Fig. 21), 1, 2, 3, 4; the opposing surface is carefully removed in fields 1 and 3, and left intact in the others; the two prisms are afterwards pressed against each other, and put in the place of the original prism. We then obtain in fields 1 and 3 reflected light coming from the right, and in fields 2 and 4 transmitted light coming from the left. The line ab divides the visual field into halves, each one of which corresponds to one of the faces of the Bunsen screen, and includes

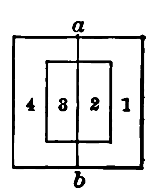


FIG. 21.

* *Zeitschrift fur Instrumentenkunde*, 1890, p. 461.

an interior and an exterior region; the first corresponds to the spot of the Bunsen screen, and the second to the opaque part. But, at the moment of the setting, we should have a uniform brightness and not equal contrasts; to obtain the last result, plates of glass of equal thickness are placed on one of the faces of the right angle of each prism, so as to diminish the brightness of parts 2 and 3 of the field. The contrasts may be modified at will by varying the thickness of these plates.

The precise measurements of Lummer and Brodhun have shown that the sensibility of the apparatus attains its maximum value for a contrast of 4 per cent only. The precision is then more than double that obtained with equal illuminations (settings *R* or *L*). The mean error of a setting has been found equal to 0.24 per cent for a contrast of 3 per cent, and 0.81 per cent for a contrast of 18 per cent. With the Bunsen screen this error varies generally from 1.5 to 4 per cent.

From what precedes the conclusion may be drawn that the optical screen of Lummer and Brodhun is at present that which presents the greatest advantages. [For use with arc-lamps, see Appendix B.]

Arnoux Photometer.

32. The number of photometers based on the second fundamental law of photometry is very limited. The apparatus of R. Arnoux is almost the only one belonging to this category. At first thought we may say that photometers based on the law of the cosine cannot give results rigorously exact; for this law is only approximate, and is not at all exact for large angles of incidence*.

The apparatus of Arnoux (Figs. 22 and 23) is composed of two plates of ground-glass *a* and *a'*, making with one another an arbitrary angle of 90°. A very thin partition *b* of blackened copper, situated in the bisecting plane, prevents all exchange of light between the two plates. This arrangement is fixed at the end of a copper tube *c* in which there slides a second tube *d* fitted with a lens *f* with cross-wires whose line of sight is in the bisecting plane of the two plates and perpendicular to their common edge. The apparatus is supported by an optical stand *h*; it may also turn about the line of intersection of the two plates.

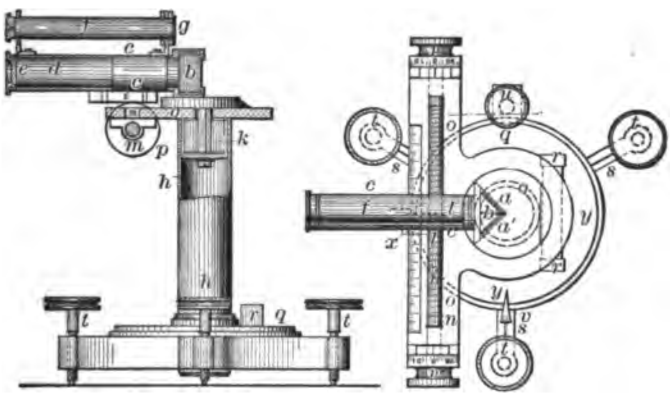
Below the tube *c* is fixed a band of metal *i*, one of whose vertical faces coincides with the bisecting plane of the two plates; this

* *Lum. Él.*, Vol. XXIII. p. 555.

face is constantly pressed against the edge of a nut l, under the influence of a spiral spring k, placed in the stand h. This nut, governed by the micrometer screw m, moves longitudinally in a slot n. The upper plate o in which this slot is cut is fitted on the edge parallel to the slot with a divided scale before which moves the zero mark x cut on one of the faces of the movable nut l. The zero of the scale is so arranged as to be in the bisecting plane of the two plates, while the slot and the axis of the micrometer screw are perpendicular to this plane.

We measure, then, the deviation of the telescope from its initial position by the trigonometrical tangent. The pitch of the screw being one millimeter, and the head being divided into 10 parts, displacements of 0.1 millimeter may be measured.

To make measurements the lights may be arranged in two different ways.



FIGS. 22 and 23. — Arnoux Cosine-Photometer.

In the first, the two lights L_1 and L_2 and the middle point of the edge of the two divisions are on the same horizontal straight line.

Let β be the angle made by the rays from the two lights with the normals to the two plates of the photometer in their initial position, and, further, let α be the common angle through which it is necessary to turn them to obtain equality of illumination. The illuminations of the two divisions of the photometer will be equal when

$$\frac{I_2 \cos(\beta + \alpha)}{d_2^2} = \frac{I_1 \cos(\beta - \alpha)}{d_1^2},$$

I_1 and I_2 designating the intensities of L_1 and L_2, and d_1 and d_2 their distances to the edge of the photometer.

We may then write this relation

$$\frac{I_2}{I_1}=\frac{d_2^2}{d_1^2}\cdot\frac{1+\tan\alpha\tan\beta}{1-\tan\alpha\tan\beta}.$$

If the angle between the faces is 90°, we have $\beta=45°$, $\tan\beta=1$; and if r expresses in millimeters the distance from the axis of the micrometer screw to the axis of rotation, and n the number of turns made by this screw, whose pitch is one millimeter, we have

$$\tan\alpha=\frac{n}{r},$$

whence

$$\frac{I_2}{I_1}=\frac{d_2^2}{d_1^2}\cdot\frac{r+n}{r-n}.$$

In the second method, the lights L_1 and L_2 are placed at right angles with reference to the edge of the photometer; but this arrangement has certain inconveniences.

However, we shall not dwell longer on this apparatus, which is very carefully planned, but which does not give results sufficiently exact in all cases.

B. PHOTOMETERS BASED ON THE EMPLOYMENT OF DIAPHRAGMS AND DIVERGING LENSES.

33. In photometers of the first category, the equality of brightness of the two halves of the screen is obtained by varying the distances of the lights from the screen, or the inclination of the luminous rays on it.

In photometers of the second category, the equality of brightness is obtained by various methods, which diminish the luminous intensity in a well-determined ratio. Among these, special attention should be paid to methods of diaphragmation, dispersion, and those which are based on the phenomena of absorption. Before entering on the special study of these methods, let us first make a brief exposition of their theory.

Theory and Properties of Diaphragms.

34. Let us consider the illumination produced on the wall of a dark room by light coming from outside, and passing through [a translucent diaphragm placed behind] an aperture in the opposite

wall; if the rays of the luminous pencil are parallel, the wall will be uniformly illuminated. But if the aperture is diminished one-half, the quantity of light received on the opposite side will also be diminished in the same ratio; in general, the quantity of light received by the wall which serves as a screen will be directly proportional to the size of the aperture.

Let I_1 be the intensity of a source of light L_1 placed normally to, and at a distance D_1 from the diaphragm; the intensity of illumination of this last is then $\frac{I_1}{D_1^2}$, and the quantity of light received by the surface of the diaphragm, being proportional to its area S_1, is then equal to $\frac{I_1 S_1}{D_1^2}$. This surface acts in turn as a source of light; and at a distance d_1 from the diaphragm, the illumination produced by the illuminating surface S_1 on a surface S normal to the rays of light is equal to

$$e_1 = \alpha_1 \frac{I_1 S_1}{D_1^2 d_1^2}.$$

α_1 is a factor of proportionality, which depends on phenomena of reflection, refraction, and diffusion, of which the screen is the seat. This intensity of illumination may then be varied by varying the surface S_1 of the diaphragm.

The second source of light L_2 being in the same way placed at the distance D_2 from a second diaphragm S_2, the intensity of illumination e_2 produced by this last on the surface S, normal to the rays of light coming from L_2 and situated at a distance d_2 from the diaphragm, is equal to

$$e_2 = \alpha_2 \frac{I_2 S_2}{D_2^2 d_2^2}.$$

If the factors α_1 and α_2 are known, the ratio of I_1 to I_2 may be immediately determined by measuring the surface S_1 and S_2 of the two diaphragms at the moment when the above equation is satisfied.

Instead of measuring the coefficients α_1 and α_2, which are moreover hardly constants, it is simpler to eliminate them by combining the measurements in a special manner. This result is easily reached by employing an auxiliary light L_0, on whose constancy we may rely.

The intensity of illumination produced on the screen S by the rays of light from L_0 is given by the relation

$$e_0 = \alpha_0 \frac{I_0 S_0}{D_0^2 d_0^2} = K_0 I_0 S_0,$$

in which I_0 represents the luminous intensity of L_0, S_0 the area of the diaphragm, D_0 its distance from L_0, and d_0 its distance from the screen.

Illuminating the screen simultaneously by the lights L_0 and L_1, we succeed in equalizing the illuminations e_0 $(=K_0I_0S_0)$ and e_1 $(=K_1I_1S_1)$; we have then

$$K_0I_0S_0=K_1I_1S_1. \tag{1}$$

The coefficients K_0 and K_1 have the following values:

$$K_0=\frac{\alpha_0}{D_0^2d_0^2},$$

and $$K_1=\frac{\alpha_1}{D_1^2d_1^2}.$$

Next the light L_2 is substituted for the light L_1, and equality of the two corresponding illuminations e_0' $(=K_0I_0S_0')$ and e_2 $(=K_1I_2S_2)$ is established by varying the opening of the diaphragms,

that is, $$K_0I_0S_0'=K_1I_2S_2. \tag{2}$$

The factors K are the same as in the preceding observation since the arrangement of the apparatus has undergone no modification.

Dividing these two equations member by member, we obtain

$$\frac{S_0}{S_0'}=\frac{I_1S_1}{I_2S_2},$$

whence $$\frac{I_1}{I_2}=\frac{S_0S_2}{S_0'S_1}. \tag{3}$$

In the majority of cases, the diaphragm interposed in the path of the rays of light from L_1 and from L_2 may be suppressed; in this case equation (3) becomes simply

$$\frac{I_1}{I_2}=\frac{S_0}{S_0'}.$$

The Properties of Dispersion Lenses.

35. Let us suppose that a double-concave lens be placed at a distance p (Fig. 24) from the source of light L and that a screen be placed at a further distance δ. The distance of the screen from

the source of light is then $p+\delta$. The divergence of the luminous pencil proceeding from L is increased, on passing through the lens, in such a manner that the rays of light seem to come from the virtual focus L', situated at a distance p' from the optical center of the lens.

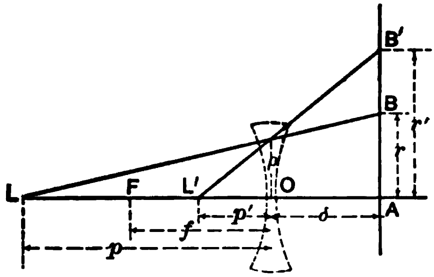


FIG. 24.

The photometric screen being placed perpendicular to the optical axis of the lens, the refracted pencil of light then illuminates a circle of radius $r'(=AB')$, while the original pencil would only have been distributed on a circle of radius $r(=AB)$. Neglecting the correction arising from the fact that the screen is plain and not spherical, a correction furthermore negligible if observations are made near the axis, the intensities of illumination obtained in the two cases are inversely proportional to r^2 and r'^2; now L and L' are conjugate foci; and calling f the focal distance of the lens, we have, applying the fundamental equation of diverging lenses,

$$\frac{1}{p'}-\frac{1}{p}=\frac{1}{f},$$

and

$$p'=\frac{pf}{p+f}.$$

The radius of the lens being equal to ρ, we have

$$\frac{r'}{\rho}=\frac{p'+\delta}{p'},$$

and

$$\frac{r}{\rho}=\frac{p+\delta}{p};$$

whence

$$\frac{r'}{r}=\frac{p(p'+\delta)}{p'(p+\delta)};$$

and we obtain, after making certain reductions,

$$\frac{r'}{r}=1+\frac{\delta p}{f(p+\delta)}.$$

Consequently the intensities of the illuminations produced with and without the interposition of the lens in the path of the rays of light are to one another in the ratio N:

$$N=\left(\frac{r'}{r}\right)^2=\left(1+\frac{\delta p}{f(p+\delta)}\right)^2.$$

In other words, the illumination produced on a screen by the source of light is the same as if its distance had been increased in the ratio $\frac{r'}{r}$; we must then introduce into the calculation, in place of the distance $d\ (=p+\delta)$, the distance d' expressed by the relation

$$d'=\frac{r'}{r}(p+\delta)=(p+\delta)+\frac{\delta p}{f},$$

or

$$d'=d+\frac{\delta p}{f}.$$

The above formula may also be put in another form; now $P=d-\delta$; hence

$$d'=d+\frac{\delta(d-\delta)}{f},$$

or

$$d'=d\left(1+\frac{\delta}{f}\right)-\frac{\delta^2}{f}.$$

In this equation the modified distance is expressed as a function of the real distance, of the constant f of the lens, and of the distance δ of the lens from the screen.

Putting $$a_1=1+\frac{\delta}{f},$$

and $$a_2=\frac{\delta^2}{f},$$

we have $$d'=a_1d-a_2.$$

36. We may then calculate in advance the various values of a_1 and a_2 for a given lens and for various values of δ.

The weakening effect of the lens is null when the bisecting plane of the lens coincides with the screen, that is, when $\delta = 0$; it is also null if $p = 0$, that is, if the source of light is at the optical center of the lens. The weakening effect is maximum for an intermediate position determined by equating to 0 the derivative of d' with respect to δ.

We obtain then $\delta = \frac{d}{2}$, whence we conclude that the maximum dispersive effect is produced when the lens is placed at equal distances from the light and the screen.

In the above calculation, no account has been taken of the weakening of the rays of light due to absorption and reflection produced by the lens. At first sight we might assume that these causes of weakening are insensible, considering the slight thickness of a diverging lens, especially near its center.

Aryton and Perry have done this in making formulæ for their dispersion-photometer. However, precise measurements made since allow the conclusion that the weakening action due to reflection and absorption by the lens frequently attains a value of from five to eight per cent.

It may be determined, furthermore, that the weakening of the pencil of light, caused by the lens, comes entirely from phenomena of reflection, and not at all from phenomena of absorption; the weakening is, in fact, the same with plates of glass of different thickness as with lenses.

Below will be found the ratios of the illuminations observed on the screen by Voller*, and obtained with and without the lens, for various lights; the results obtained with plates of glass of various thickness complete the table.

	Lens. $f = 18$ cm.	Lens. $f = 50$ cm.	Plates of Glass of a Thickness of		
			1.0 mm.	2.9 mm.	4.8 mm.
Candle	.....	.....	0.923	0.916	0.923
Petroleum lamp . . .	.....	.....	0.914	.....	0.910
Gas-burner	0.966	0.933	0.902	0.918	0.910
Incandescent lamp . .	0.937	0.900	.....	.....	.....

The result of these measurements shows, then, that we should determine experimentally the weakening factor of a given lens

* *Abhandl. des Naturwiss. ver. zu Hamburg* (7), Vol. II. p. 40.

before employing it in photometric comparisons; we may, however, eliminate this reduction by compensating the weakening action of the lens by inserting in the path of the rays of light from the photometric standard plates of equivalent thickness.

Cornu's Method.

37. This method* is based on the following property of converging lenses, discovered and already used by Bouguer: if with a converging lens a real image of a luminous source is formed, and the aperture of the objective is modified by inserting a diaphragm of greater or less opening, the size and position of the image are not modified; on the contrary, the intensity of illumination of the image is proportional to the opening of the diaphragm, provided that this opening is always small with respect to its distance from the light. This last property is evident, since the quantity of light which contributes to the formation of the image is proportional to the surface of the lens, met by the incident rays.

For a diaphragm, Cornu made use of the following arrangement known as a *cat's eye.*

It is formed by two metallic plates each with a square opening, AB and $A'B'$, made to glide on one another by a pinion working in two racks C, C' (Fig. 25).

In one of their extreme positions the two squares are in coincidence, and a maximum square opening allows the passage of the light; in the other extreme position, the opening of one of the plates is covered by the solid part of the other so that no light may pass; in the intermediate positions, the free opening has the form of a square, whatever its dimensions. Further, since the pinion is fixed, and its rotation makes one of the plates advance as much as the other recedes, the center of the variable square remains fixed in front of the optical center of the lens. Consequently, the opening is always proportional to the square of the displacement of the movable plates — the diagonal of the opening — measured on the graduated scale of the apparatus.

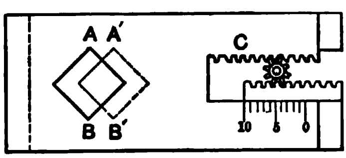


FIG. 25.

The most simple means of utilizing this method consists in

* *Journal de Physique*, Vol. X., 1881, p. 189; *Lum. El.*, Vol. III. p. 221.

employing two identical objectives, fitted with these diaphragms and so placed that their optical axes cross at about twice their common focal lengths. Each of them produces, on a white screen, the image of a small opening in another diaphragm in front of that part of each of the lights which it is desired to compare.

The diaphragm of the lens in front of the smaller light being open to its fullest extent, the opening of the other is regulated until equality of illumination of the two images is obtained.

To better determine this moment, the apparatus is so managed that the images of the openings of the small diaphragms are in contact along one of their edges; this edge disappears then at the moment of equality.

This apparatus permits an easy measurement of the intrinsic intensity of a light at various points, by employing an auxiliary light.

Cornu has given his apparatus another form which is more practical and more general, and which does away with the screen.

The untinned mirror* is replaced by a mirror of black glass AA' (Fig. 26), ending in a rectilinear edge A normal to the plane of the principal axes of the objective. The focal planes are so regulated as to pass exactly through this edge. A microscope of small magnifying power allows one to see the images of the two sources simultaneously, on each side of the rectilinear edge. By suitably regulating the position of the lights, the two regions to be compared are brought into contact at the edge. To render the comparison still more exact, the two regions are isolated by the aid of a circular diaphragm CC' introduced in the focal plane of the eye-piece of the microscope. The visible field consists then of a small circle divided into halves by the almost invisible line formed by the edge; one of the halves has a fixed intensity, the other an intensity which is rendered variable by the aid of the photometric screen: these are the best conditions for obtaining equality of the two intensities. Under these circumstances, and above all if

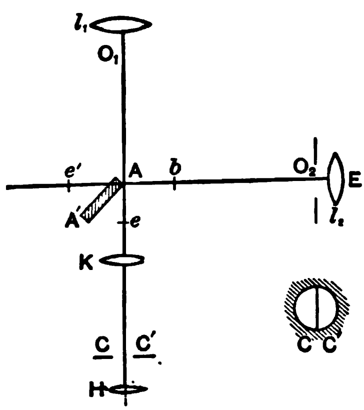


FIG. 26.—Cornu's Arrangement.

* *Journal de Physique*, Vol. X., 1881, p. 192.

care is taken to diminish the intensities to a certain limit, the eye acquires so great sensitiveness that the least difference in the composition of the lights betrays itself by a difference of tint which becomes troublesome in appreciating their equality; it is only lights which are strictly identical or monochromatic which give an absolutely satisfactory impression of equality.

The surfaces to be compared may be extremely small; if the focal images are quite pure and obtained by the aid of achromatic objectives, the microscope which serves as an eye-piece may greatly magnify them; the apparatus then is able to measure the brightness of extremely small images.

The above apparatus gives only the intrinsic brilliancy of the different regions of the lights which are studied.

To compare the total intensities, we should use diffusing screens on which the rays of the lights to be studied fall, and which are placed immediately before the diaphragms of the two lenses.

Napoli's Photometer.

38. This photometer* puts into practice in a very ingenious manner the principles of diaphragmation.

Suppose a disc pierced with a hole at any distance from the center, and a light placed before this hole; if the disc is turned, the image is displaced circularly on the disc, and forms, by the persistence of the image on the retina, a uniformly illuminated ring. The brightness of this ring is independent of the velocity of the disc, and depends only on the surface of the opening, that is, on the surface of the diaphragm. The ring will then be more or less bright according as the surface of the opening is increased or decreased, and the intensity of brightness will be proportional to the size of the opening.

Figure 27 represents the principal part of the photometer, namely, two peripherally notched discs *D* and *D'*, of the same diameter, in juxtaposition; they move on one another in such a way as to present spaces either open or more or less filled at the will of the observer.

One of these discs *D* is fixed to an axis set rotating by a crank, a fly-wheel, and an endless cord.

The second disc *D'* is movable on the axis of the disc *D*, and carries a cylinder with a helicoidal slot, in which a pin held by the

* *Séances de la Soc. de Phys.*, 1880, p. 53; *Lum. Él.*, Vol. II. p. 133.

sleeve *C* engages; this sleeve, by means of a key, may move longitudinally in a second slot made in the axis of the disc *D*, and following a generatrix. It ends in circular teeth and is operated by the button *B* and the pinion *P*.

Turning the button *B*, the teeth advance or recede, and the disc *D* moves on *D'*; the pitch of the slot is so calculated that, for the whole course of the sleeve, the two discs may pass from a position completely intercepting the light to one where all the slots are open. A pointer fixed at *B* on the axis of the pinion indicates on a dial the opening of the slots.

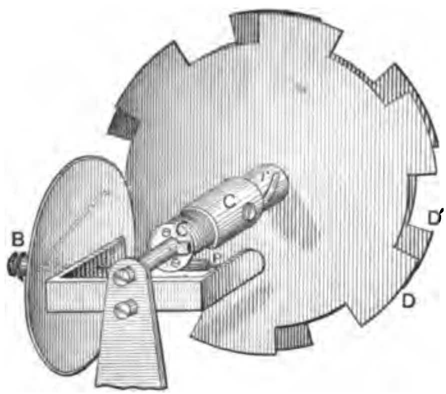


Fig. 27.

The apparatus is mounted in connection with a Foucault screen; one-half is illuminated by a standard light, the other by the light to be studied, reduced in the ratio indicated by the pointer of the apparatus. It is sufficient to move the button *B* until the two divisions of the screen are equally bright.

It may be mentioned that the same principle was applied to the construction of photometers, at about the same time, by Guthrie * in England and Hammerl † in Germany.

Ayrton and Perry's Dispersion Photometer.

39. This photometer ‡ is based on the properties of diverging lenses. Equality of illumination is determined by Rumford's method.

* *Chem. News*, Vol. XLIX. p. 262.
† *Elektr. Zeitschrift*, Vol. IV. p. 262.
‡ *Phil. Mag.* (5), Vol. IX. p. 117.

The opaque body, the equality of whose shadows is determined, is a rod placed in front of a sheet of white paper; the photometric standard is movable along a graduated scale. The light to be studied (electric lamp) throws its rays upon a plane mirror, which reflects them upon a concave lens; this disperses them and reduces the illumination produced in an easily calculated ratio, so as to obtain equality of the two illuminations without moving the light.

The mirror makes an angle of 45° with its axis of rotation which is perpendicular to the disc and parallel to the axis of the lens; hence all the rays reflected by the mirror and passing through the center of the lens have the same angle of incidence, 45°, and thus undergo the same absorption, whatever be the position of the light with respect to the mirror.

Further, the particular value of this angle, 45°, is such that the angle through which the disc must be turned to reflect a pencil of light first horizontal, then inclined, gives immediately this inclination. The whole apparatus may turn about an axis so that the pencil reflected by the mirror and refracted by the lens is projected upon the middle of the screen.

As to the luminous intensity I of the light studied, adopting as a unit the light employed, and applying the formulæ for dispersion lenses, we obtain

$$I = \frac{1}{\delta'^2}\left[x + \frac{\delta(x-\delta)}{f}\right]^2$$

in which

f is the focal length of the lens;

r the horizontal distance from the light to the mirror;

r' the distance from the mirror to the screen;

θ the angle of elevation of the light, or the inclination of its pencil of rays directed toward the mirror;

x the quantity $x = r' + r \sec \theta$;

δ the distance from the lens to the screen;

δ' the distance from the photometric standard at the time when the shadows are equal.

To the results thus obtained, we should apply the correction which takes account of the loss due to reflection on the surfaces of the lens.

Crova's Method.

40. Crova's method* is a method of diaphragmation combined with the employment of a diffuser on which the light from the sources to be compared falls.

On a sheet of ground glass, opal glass, or a Foucault screen, are let fall normally, the rays emitted either by the light to be measured L_1, or by the standard light L_2.

Every point of the back of the diffuser may be considered a source which emits light whose intensity depends on the nature of the diffuser; but whatever be this law, the rays diffused in a nearly normal direction have a uniform intensity if the screen is homogeneous, whatever be the point from which they emanate.

We place, then, behind the diffuser, an opaque screen having a slit whose size may be varied at will. The intensity of illumination at a point of the screen is then proportional to the surface of the diffuser. Varying the surface of the latter by means of the variable slot in the diaphragm, we may make this illumination equal to that of another part of the same screen, illuminated by a constant auxiliary light.

When lights of very different intensities are compared, it is necessary to place them at different distances from the diffuser; for if the distance were the same for the two lights, it would be necessary, in the case of the intense light, to give to the opening of the diaphragm a very small surface, which might bring about phenomena of diffraction. This necessity of placing the two lights at different distances D_1 and D_2 from the diffuser does not complicate the method; S_1 and S_2 representing the area of the opening of the diaphragm in the two cases, we have

$$\frac{I_1}{I_2} = \frac{S_2 D_1^2}{S_1 D_2^2}.$$

This method has given good results, in the comparison of arc-lights for instance. The apparatus used by Crova is composed of a vertical Foucault screen, one-half of which is lighted by a Carcel lamp, which serves as an intermediate standard. This lamp is placed in a blackened box, fitted with a large horizontal tube 50 cm. in length, also blackened inside, which allows the light to fall on the screen at an angle of 45°.

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* *Comtes Rendus*, Vol. XCIX. p. 1067.

The other half of the Foucault screen receives the light from the diffuser, which is placed at the end of a horizontal tube of the same length as the preceding to which it is perpendicular; in this way the two parts of the screen receive light at the same angle, 45°.

As has been said, immediately behind the diffuser is found the diaphragm, whose aperture may be varied by means of a micrometer screw. Finally the board on which the photometer is fixed may turn about a vertical axis, so as to allow the tube carrying the diffuser to be placed in all azimuths.

The apparatus invented by Crova also permits the measurement of the luminous intensities in any direction. To accomplish this, the tube at the end of which the diffuser is placed may be moved in a vertical plane, perpendicular to the axis of the tube containing the lamp. A divided circle fixed on this last tube allows the angle between the normal to the diffuser and the horizontal to be read; the value of this angle permits the calculation of the ratio of the intensities I_1 and I_2.

Let us suppose, to measure I_1, that the diffuser is vertical, while in measuring I_2 its normal makes the angle i with the horizontal; for a uniform illumination of the screen we shall have

$$\frac{I_1}{D_1^2} S_1 \cos 45^\circ = \frac{I_2}{D_2^2} S_2 \cos 45^\circ \cos i,$$

whence

$$\frac{I_1}{I_2} = \frac{S_2 D_1^2}{S_1 D_2^2} \cos i.$$

It should be remarked that the diffusion photometer of Crova supposes a perfect diffuser; for the construction of the latter, let us refer to the details that we have given concerning the construction of the Foucault screen (§ 21). An essential condition of Crova's photometer is, furthermore, the necessity of choosing diffusers whose opaqueness varies with the intensity of the lights to be compared.

Mascart's Photometer.

41. This apparatus* uses at the same time diffusers and lenses provided with diaphragms; it allows the comparison of lights of very different intensities, and the measurement of the intensities of rays of light in any direction without measuring the angles.

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* *Bull. de la Soc. int. des Électriciens*, 1888, p. 103; *Bull. d. sc. Phys.*, Vol. I. p. 250.

The incident light strikes the Foucault screen *D*, passes through it, and is reflected in a mirror *M* along the axis of the apparatus (Fig. 28).

A lens *C* placed against the movable diaphragm, and distant from the screen twice its focal length, produces the image of this screen *D* on the Foucault screen *E*.

From the lamp *L*, the height of whose flame is regulated by means of its image thrown on the ground glass *G*, proceeds a pencil of light which is concentrated by a lens on the Foucault screen *D*, of the same dimensions as the first.

A lens *C* placed against the second movable diaphragm, and distant from the screen double its focal length, gives the image of this

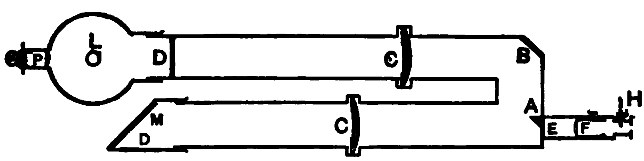

Fig. 28. — Mascart Photometer.

screen on *E*, after having been reflected by the mirror *B* and the prism *A*.

An adjustable lens at the point *F* enables us to see clearly the Foucault screen *E*, on which are projected the two pencils, each of them occupying half of the disc.

The effect of the difference of coloring is corrected by means of the diaphragm *H*, composed of glass of various colors, which permits the equalization of the tints of the lights. Practically, the apparatus is used as follows:

After having lighted the small lamp, the height of whose flame is regulated by means of its image in the ground glass, we arrange the apparatus so that the screen *D* receives normally the light of the standard lamp L_1, placed at a meter's distance from this screen.

Next we vary the opening of the diaphragms until the illuminations of the two halves of the screen *E* are equal; these openings being rectangular, their surface is proportional to their breadth *n*, read directly on a divided scale. The intensity of the auxiliary lamp being *I*, we have then

$$\alpha I n_1' = \beta I_1 n_1.$$

We next repeat the same operation with the light L_2, whose intensity I_2 we wish to measure; we then have

$$\alpha I n_2' = \frac{\beta I_2}{D^2} n_2,$$

D being the distance of the light L_2 from the diffuser. We then have

$$\frac{I_2}{I_1} = \frac{n_1 n_2'}{n_1' n_2} D^2.$$

In Mascart's photometer, as in that of Crova, we have assumed that the quantity of light emitted normally by the diffuser is proportional to its surface and to the quantity of light which it receives. This hypothesis has been confirmed by direct measurements by Mascart, and its precision has been established with errors less than three per cent. It follows from this that the ratio $\frac{n_1}{n_1'}$ should vary inversely as the square of the distance, since the quantity of light received by the diffuser varies in the same manner. We may then determine this ratio once for all; that is, we may take the intensity of the lamp of the apparatus as an intermediate standard.

Beside the preceding apparatus, Mascart has had constructed by Pellin a small apparatus which may be held in the hand, in which the auxiliary light is a small petroleum lamp. Further, the slitted diaphragms are replaced by discs pierced with holes of unequal size; the illumination of the two parts of the screen E then varies by sudden leaps, but without any resulting inconvenience.

Employment of Absorbing Media.

42. We may also equalize the illuminations of the two halves of the photometric screen by placing in the path of the rays of light from one of the two sources that are being compared, media more or less opaque which absorb a part of the light. We may employ with success smoked glass of varying thickness and tint. Before using this glass its absorbing power is carefully determined. To make the illuminations of the two halves of the screen exactly equal, it is necessary to insert suitably chosen glasses. But in proceeding in this way the illuminations do not vary continuously.

To eliminate this inconvenience, Sabine * has proposed employing a single smoked glass cut on a bevel, whose absorbing power varies with its thickness. By introducing little by little this wedge in the path of the pencil of light coming from the source to be compared, the illumination of the screen is varied in a continuous manner. The amount which the wedge enters the slot is measured on a divided scale; this gives immediately the corresponding weakening of the pencil of light, determined in advance once for all. Sabine's photometer includes also a diaphragm whose arrangement has nothing novel.

The employment of the absorbing wedge has a serious inconvenience. The luminous intensity is not weakened equally in all parts of the section of the luminous pencil, because of the unequal thickness of the wedge. The photometric screen is consequently not uniformly illuminated. Spitta † has overcome this inconvenience by replacing the wedge by two bevelled sheets, with their hypotenusal faces put together. Moving the two wedges on one another, the total thickness varies, but it is constant for a certain length, greater than the size of the luminous pencil. The absorption of this double wedge is determined in advance for the different positions of the two superposed halves. It then suffices to read their position on a divided scale in order to obtain the absorption.

This arrangement has not yet been employed in industrial apparatus, but it is ingenious and merits notice, because of the applications to which it is susceptible.

The weakening of the rays of light by the absorbing media may give excellent results when the lights compared are of the same tint. If this is not the case, the absorption is not equal for the two lights; there are then produced secondary actions of which it is difficult to take account, and which greatly complicate the measurements.

C. POLARIZATION AND COMPENSATION PHOTOMETERS.

43. In the photometers already described, the illumination of the two parts of the screen has been equalized, either by varying the distance or the inclination of the rays, or by interposing in the path of the latter, dispersion lenses, diaphragms, or absorbing media. In addition to these various ways of diminishing the intensity of

* *Phil. Mag.* (5), Vol. XV. p. 22.

† *Proceedings of the Royal Society, London,* Vol. XLVII. p. 15, 1889.

a pencil of light, there still exists one, based on the properties of polarized light.

Let us recall, in a few words, the properties of polarized light. We know that light is the result of transverse vibrations of the ether. In the case of ordinary light, the vibrations take place in all directions, in a plane perpendicular to the direction of propagation.

If a ray of sunlight is passed through a rhombohedron of quartz, it is decomposed into two distinct rays, the ordinary ray and the extraordinary ray. These rays are said to be polarized. The undulations of the ether are produced always normally to the direction of propagation of the wave, but they take place for each ray in a single direction, instead of in all directions.

In the ordinary ray the undulations take place in a plane perpendicular to the plane determined by the incident ray and the normal to the surface of the crystal. In the extraordinary ray the undulations take place in the plane of the incident ray and the normal to the surface of the crystal.

A ray of natural light may then be considered as constituted of two independent rays, whose intensity is equal to one-half of that of the natural ray, and which are polarized in planes perpendicular and parallel to the plane of incidence.

We know that the analyzer of a Nicol prism is formed of a rhombohedron of Iceland spar cut in two along a plane perpendicular to the plane of the principal diagonals of the bases and passing through the obtuse angles which are nearest one another; the two halves are afterward cemented together with Canada balsam, whose index of refraction is smaller than the extraordinary index of Iceland spar, but greater than the ordinary index. Consequently the ordinary ray undergoes total reflection at the surface of separation, so that the prism allows only the extraordinary ray to pass.

If this polarized ray is received on a second Nicol prism, whose principal section makes with that of the first an angle α, the luminous intensity I' of the ray which emerges from the second Nicol prism is related to that of the ray which emerges from the first by the law of Malus:

$$I' = I \cos^2 \alpha.$$

Varying the angle α from 90° to 0, we may vary I' from 0 to I in a well-determined manner. We have thus a precise process for varying the luminous intensity of a given pencil of light previously polarized.

Duboscq's Photometer.

44. Polarization photometers are based on this principle. The first in date is that of Arago *, based on a property of polarized light which he had just discovered; viz. when a pencil of natural light falls on a pile of plates of glass, the quantity of light polarized by reflection is equal to that polarized by refraction.

Duboscq † has constructed a piece of apparatus based on the same principle and which has been wrongly attributed to Babinet. Below is the description of the apparatus (Fig. 29). The lights L_1 and L_2 are placed behind sheets of ground glass; the rays which they emit meet a pile of plates of glass P, which reflects the light coming from L_1 and refracts that coming from L_2 in the direction of the eye. These lights are partially polarized in rectangular azimuths; if their intensities are equal, the quantities of polarized light which they contain will produce natural light. We determine the absence of polarization by the aid of Savart's polariscope, composed of the double rotation plate Q and the analyzer N.

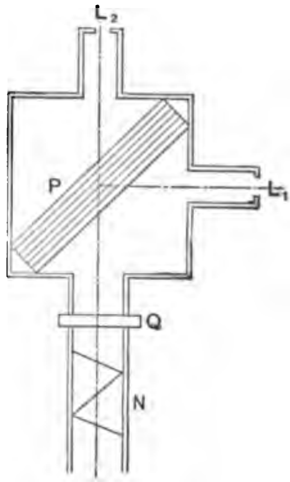

FIG. 29. — Duboscq's Photometer.

The distance of the unknown light L_2 is varied until this result is attained. We then have, designating by d_1 and d_2 the distances of the lights from the diaphragms, and by I_1 and I_2 their intensities,

$$I_2 = \frac{d_2^2}{d_1^2} I_1.$$

Wild's Polarization Photometer.

45. Wild's polarization photometer ‡ is the only one which has been especially contrived for industrial measurements. The first form of this photometer dates from 1859, but Wild later on worked out an industrial form which was constructed by Pfister of Berne §.

* *Comptes Rendus*, Vol. XIII. pp. 840, 967.

† *Cours de Physique*, par Jamin et Bouty, Vol. III. fasc. 3, p. 579.

‡ *Poggendorf's Annalen*, Vol. XCIX. p. 235; Vol. CXVIII. p. 193.

§ *Mélanges physiques de Saint Pétersbourg*, Vol. XII.; *Bull. des sc. phys.*, Vol. I. p. 578.

The lights to be compared, S and S', are placed in the direction of the axes of the tubes A and B (Fig. 30), which make between them an angle of 70° 50′, twice the complement of Brewster's angle (54° 35′). The pencils of light emitted by each of them pass through the diffusers of opal glass D, D, then through the Nicol polarizers P, P, fixed on the divided circles C, C, which may be turned before the fixed verniers by means of the rods b, b. They next meet a pile of plates of glass G, G, directed along the bisector of the angle formed by the axes of the tubes A and B, with the faces of which they make, consequently, an angle of 35° 25′. The reflected portions of the pencils are then polarized in the plane of incidence, and the transmitted portions in the perpendicular plane.

The reflected portion of the pencil A and the transmitted portion of B fall on the telescope L held by a tube F, diametrically oppo-

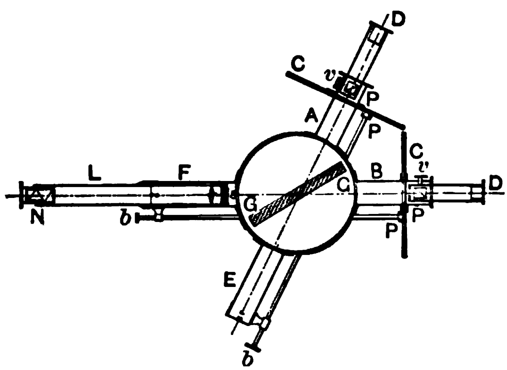

FIG. 30.—Wild's Photometer.

site to B. This telescope includes, beside the objective and the eye-piece, a double plate of quartz S and a Nicol prism N, which together constitute a Savart polariscope; it may easily be removed from the tube F and placed in the tube E, diametrically opposite A, in which fall the transmitted portion of the pencil A and the reflected portion of the pencil B.

To use the apparatus we commence by putting the principal section of one of the plates of the polariscope in the plane of incidence of the rays which fall on the pile of plates of glass. This is accomplished by closing the opening A and turning the telescope until the fringes disappear. At this instant the principal section of one of the plates coincides with the plane of polarization of the portion of the pencil B which is refracted through the pile

of glass plates; consequently the principal section of the other plate is in the plane of incidence.

We then turn the circle C of the tube B until the zero of the graduation comes to the zero of the vernier; then after having unclamped the screw V, we turn the polarizer P until complete extinction takes place in the telescope; in this position the plane of polarization of the pencil which traverses P is in the plane of incidence. The polarizer of the tube A is regulated in the same way by placing the telescope at F.

By a glance at the graduated circles we may know the angles which are formed with the plane of incidence by the planes of polarization of the pencils of light which emerge from the polarizers.

This done, we proceed to compare the intensities of the lights L_1 and L_2. We may remark that for any position of the polarizers, the pencil of light transmitted by the pile and coming from B, does not have the same intensity as the reflected pencil coming from A. These two pencils polarized at right angles act on the polariscope like partially polarized natural light, and as they are slightly divergent, fringes appear. They disappear when the two pencils have the same intensity. If, then, α_1 and α_2 are the angles which the planes of polarization of the polarizers A and B make with the planes of incidence, and d_1 and d_2 the distances from the sources of intensity I_1 and I_2 to the diffusers D, the intensity of the reflected pencil is $\frac{I_1}{d_1^2} A \cos^2\alpha_1$, and that of the transmitted pencil $\frac{I_2}{d_2^2} B \sin^2\alpha_2$, A and B being the coefficients of diminution of the light passing through the diffusers and the polarizers; accordingly, we have

$$\frac{I_1}{d_1^2} A \cos^2\alpha_1 = \frac{I_2}{d_2^2} B \sin^2\alpha_2,$$

whence

$$\frac{I_1}{I_2} = \frac{d_1^2 B \sin^2\alpha_2}{d_2^2 A \cos^2\alpha_1},$$

an equation which gives the ratio of the intensities when that of the coefficients A and B is known. This last ratio, which, theoretically, should be equal to unity when the different parts of the tubes A and B are identical, is determined experimentally, once for all, by interchanging the position of the lights with respect to the tubes, and seeking new values of α_1 and α_2 which make the fringes disappear. Assuming that the intensities of the pencils of light falling on the telescope are equal, we obtain a relation which, united with the pre-

ceding, permits the elimination of the ratio of the intensities and the calculation of $\frac{A}{B}$.

The method of Wild's photometer, then, depends on the phenomenon of the disappearance of the fringes when two pencils of light of equal intensity and polarized at right angles are superimposed. This method allows, according to Wild, the comparison of the intensities of two lights within from $\frac{1}{500}$ to $\frac{1}{1000}$. Its sensibility is then considerably greater than that of the Bunsen screen. It has, on the other hand, the inconvenience of fatiguing the retina, because of the persistence of the luminous impressions, and requiring a great deal of attention, for the eye frequently sees the fringes after they have disappeared, and it is only after a moment of repose that we can really be sure of their disappearance. For this reason, the exactness of this method is in reality less than the preceding figures would imply.

Wybauw's Compensation Method.

46. This method* differs from the preceding methods; it consists essentially in this: one of the two faces of the photometer whose illuminations are to be compared receives in the ordinary way the rays from the light to be studied, an electric light for instance. The other face receives only a known or easily calculated fraction of this same light, a fraction to which is added as much of the light emitted by the photometric standard as is necessary to make the illuminations of the two faces of the photometer become equal.

Wybauw has made a practical application of his idea in the construction of a photometer of the Foucault genus. In this apparatus, due to an ingenious arrangement of the mirrors, the path pursued by the rays of light projected on one of the surfaces to be illuminated is increased in a suitable ratio, relatively to the path of the rays of light projected on the other surface; a carcel lamp then serves to equalize the difference thus obtained in the illuminations of the two surfaces.

The Compensation Photometer of Krüss.

Krüss has realized Wybauw's idea in quite a practical manner by employing the Bunsen photometer †.

Let I_1 and I_2 be the lights to be compared, E the photometric screen, BD a mirror whose plane makes an angle ϵ with the line I_1I_2.

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* *Bull. de la Soc. belge des électriciens*, Vol. II. No. 5.

† *Lum. Él.*, Vol. XIX. p. 118.

The photometric screen E receives, then, on one side the light which comes directly from I_1, on the other side the light from the same source which has just met the mirror BD and been reflected along the path I_1AE, and finally the direct rays from the light I_2.

It is easy to determine the exact formula for the apparatus. The formula which Krüss gave at first has been advantageously modified by Strecker*. We will confine ourselves to giving the results of the latter without entering into the details of their calculation; for, since the invention of Grosse's mixture photometer, which solves the problem of compensation in a most perfect manner, Krüss's compensation photometer has lost its importance.

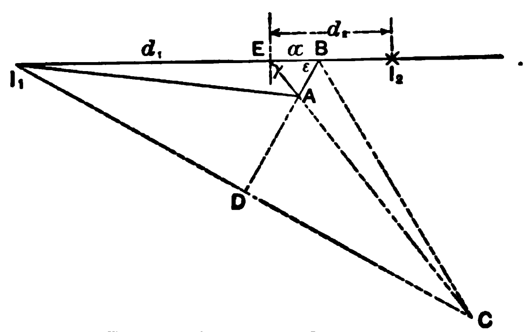

FIG. 31.—Compensation Photometer.

Below are Strecker's conclusions. Designating by a the horizontal distance from the axis of rotation of the mirror BD to the screen E, the angle ϵ must be comprised between 60° and 70°, and the distance from the unknown light I_1 to the screen must be comprised between $10a$ and $15a$. Under these conditions, the formula to be employed is

$$I_1 = I_2\left(\frac{d_1}{d_2}\right)^2 k \cdot \frac{1}{1 + \frac{ak\phi}{d_1}},$$

in which $$k = \frac{1}{1 + \sigma \cos 2\epsilon}.$$

The coefficient of absorption of the mirror (about one per cent) is represented by σ, and ϕ represents a constant whose values are, for

* *Elektr. Zeitschrift*, 1887.

$\epsilon = 60°$	65°	70°	75°
$\phi = 8.8$	17	25	31

The coefficient k should not exceed 3.

Grosse's Mixture Photometer.

47. This photometer * depends at the same time on the employment of Wybauw's compensation method and on phenomena of polarization.

It includes a prism P of spar (Fig. 32), formed of two rectangular isosceles prisms abd, bcd, separated by a thin layer of air, and cut with reference to the optical axis of the crystal so that the ordinary ray coming from an incident ray normal to one of the faces is totally reflected at the diagonal plane bd. Consequently the incident pencil V will give, in the direction UR, a pencil polarized

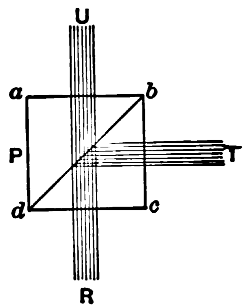

FIG. 32.

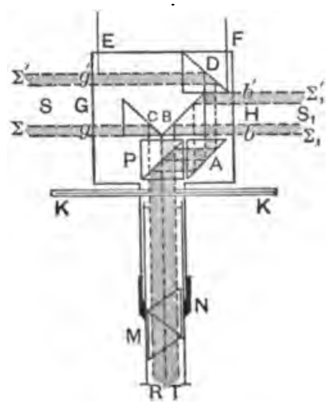

FIG. 33.

perpendicularly to the plane of incidence, and the incident pencil T will give in this same direction a pencil polarized in the plane of incidence. In Grosse's photometer the pencils which emerge from the prism P fall on a nicol N (Fig. 33), whose principal section makes with the plane of the figure an angle measured by the movement along the divided circle KK of an index on the tube M. Totally reflecting prisms A, B, C, D reflect in suitable directions the rays which traverse the plates of ground glass G and H cut from the same sheet and illuminated by the lights to be compared.

* *Zeitschrift für Instrumentenkunde*, 1888, pp. 95, 129, 347; *Lum Él.*, Vol. XXXI. p. 221; and *Bull des sc. phys.*, Vol. I. p. 583.

Finally, screens E and F, moving in two slots, permit us to limit the pencils of light which enter the instrument.

If the screens are in the position indicated in the figure, the left half R of the field of the instrument is illuminated by the pencils Σ and Σ_1', the first coming from the light S, and the second from the light S_1. The other half T of the field is illuminated by the pencils Σ' and Σ_1. These pencils have the same section, and consequently cut on the plates of ground glass G and H equal surfaces g, g', b, b', whose area may be taken as unity. The pencils Σ' and Σ_1', undergoing the same transformations, have the same coefficient of diminution A; for a like reason, the pencils Σ and Σ_1 have the same coefficient of diminution B. If, now, we designate by I and I_1 the intensities of the sources, by d and d_1 their distances to the plates G and H, and by α the angle of the principal section of the analyzer N with the plane of the figure, we have for the intensities of the rays emerging from the analyzer and coming

from the	pencil		Σ	$B\dfrac{I}{d^2}\cos^2\alpha,$
"	"		Σ'	$A\dfrac{I}{d^2}\sin^2\alpha,$
"	"		Σ_1	$B\dfrac{I_1}{d_1^2}\cos^2\alpha,$
"	"		Σ_1	$A\dfrac{I_1}{d_1^2}\sin^2\alpha.$

The screen F being so inserted as to intercept the pencil Σ_1', when the two portions of the field of the instrument are equally illuminated, we have

$$B\frac{I}{d^2}\cos^2\alpha = A\frac{I}{d^2}\sin^2\alpha + B\frac{I_1}{d_1^2}\cos^2\alpha.$$

If the screen E is inserted so as to intercept the pencil Σ', the screen F being withdrawn, we have, when the field is uniformly illuminated,

$$B\frac{I}{d^2}\cos^2\alpha' + A\frac{I}{d_1^2}\sin^2\alpha' = B\frac{I_1}{d_1^2}\cos^2\alpha',$$

α' corresponding to the new position of the analyzer. Eliminating A and B in the two preceding equations, we have an equation which gives the ratio of the intensities.

According to the preceding, the comparison of the intensities of the two sources necessitates two operations; but the ratio $\frac{A}{B}(=k)$ is a constant of the instrument; it may be determined once for all. When it is known, the ratio of the intensities is deduced by a single operation from the formula

$$\frac{I}{I_1}=\frac{d^2}{d_1^2}\frac{1}{1-k\tan^2\alpha}.$$

Krüss of Hamburg, the maker, has given the apparatus, a practical form which allows it to be fitted directly to the photometric bench in place of the Bunsen arrangement (Fig. 34).

The split rectangular prism *P*, the triangular prism *A*, both of calcarious spar, and the three totally reflecting prisms *B, C, D* are placed in a closed box *C*; the lateral walls of this box are made of

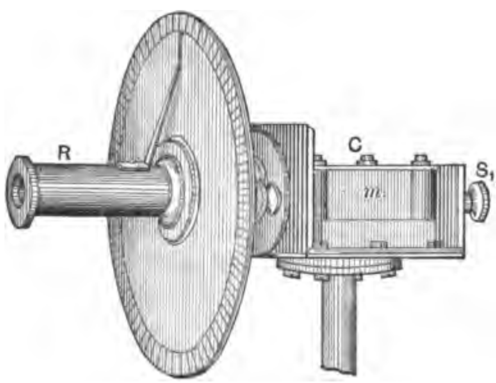

FIG. 34.—Grosse's Photometer.

two plates of ground glass G_1 and H_2 (at m_1 and m_2). On the back of the box *C* are found two buttons S_1 and S_2 which govern the screens *E* and *F*. The front has a circular opening through which the rays of light pass, to fall afterwards on the Nicol prism *N* placed in the tube *R*. The Nicol prism is movable about the axis of the tube, and an index indicates the rotations on the divided circle *K*. When the index of the Nicol prism is on the zero of the scale, either the ordinary or extraordinary rays must be extinguished; the regulating screw allows this result to be exactly obtained.

This photometer may be employed in different ways, either as an ordinary photometer without compensation, by pushing the two screens *E, F* entirely in, or as a unilateral compensation photome-

ter, by withdrawing only one of the screens, or finally as a bilateral compensation photometer, by withdrawing both screens *.

Grosse has perfected his apparatus † by adding, between the spar prism and the analyzer *N*, a Soleil double-rotation plate of quartz, which introduces phenomena of coloring by polarization. The measurement consists then in obtaining equality of the tints of the two regions by moving the lights to be compared. We cannot enter into the details of this modification.

D. PHOTOMETERS BASED ON VISUAL ACUTENESS. (HETEROCHROMATIC PHOTOMETRY.)

General Methods of Heterochromatic Photometry.

48. In all that precedes, we have supposed implicitly that the two lights compared have the same tint; that is, that the rays emitted by the two lights have the same composition. However, this is not the case in reality. The light chosen as a photometric standard generally emits rays whose composition differs from that of the rays emitted by the light which is studied. As a result the two divisions of the photometer whose illuminations are to be equalized are differently colored, which renders the measurements very uncertain, if not impossible. It is in fact difficult to judge exactly of the equality of illumination of two surfaces of different tints; this is only accomplished in practice in an approximate and wholly conventional manner, for the impressions are of different natures, and the same observer, with an interval of only a few minutes, does not come to the same conclusions; *a fortiori* two observers may obtain different, or even contradictory results. It should be added furthermore that the optical illusion, on account of which the line of separation of the two illuminated divisions of certain photometers disappears at the instant of equality, no longer exists, which deprives these photometers (Foucault, Joly, Elster, Lummer, etc.) of a considerable part of their exactness.

For this reason von Helmholtz has been able to say that "of all the comparisons effected by the aid of the eye between the intensities of different sorts of composite light, there is not one which possesses an objective value independent of the nature of the eye."

Fortunately this assertion is not rigorously true, for measure-

* *Lum. Él.*, 1889, Vol. XXXI. p. 224.

† *Chemiker Zeitung*, 1887, No. 94.

ments of relative intensity would then be about impossible in the great majority of cases, since it is rarely that we find two lights of exactly the same tint.

The obstacles to an exact comparison of two lights of different tints are the result of a singular property of the eye discovered by Purkinje*, which von Helmholtz † has thus enunciated: "Intensity of sensation is a function of the luminous intensity which differs with the kind of light." Intensity of sensation increases and decreases more slowly for the blue than for the red, for the same variation of objective luminous intensity.

To be definite, let us consider two sources of colored light, one yellow and the other blue, and let us put them before a Rumford photometer (§ 18) so that the two shadows of the opaque pencil, placed in front of the white screen, appear equally bright. If we simultaneously double the quantity of light thrown upon the screen by the two sources, we shall find that the yellow shadow appears brighter than the blue shadow, while if we reduce to one-half of their original values the quantities of light thrown by the two sources, the blue shadow will appear brighter than the yellow. Consequently if we wish to express the intensity of the blue light as a function of the intensity of the yellow taken as unity, we shall find a number which varies according to the conditions of the comparison.

The number which should represent the intensity of the blue light will be found less when we employ a greater quantity of yellow light to effect the measurement, and, on the contrary, greater when the chosen quantity of yellow light is less.

It follows from what precedes that if a reddish light, as that of the carcel lamp, is compared with a bluish light, as that of the arc lamp, the ratio of the intensities varies with the illumination of the photometric field. In the single case when the intensities of brightness of the two halves approach zero, does the photometer give ratios which are independent of the absolute value of the intensities.

This, as Crova has shown, explains why Allard has been able to determine with sufficient precision the ratio of the luminous intensities of radiants of very different tints, by taking the precaution to half close the eye while looking at the photometric screen; by

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* *Zur Physiologie der Sinne*, Vol. II. p. 109.

† *Optique Physiologique*, p. 421 (French translation).

so doing the approach of the eyelids cuts off, like a diaphragm, the pupillary opening more and more, and the intensity of the retinal field tends toward zero, the limit near which all sensation of color disappears, the luminous impression being still sufficient to enable one to judge of the equality of the two regions, which then appear a uniform gray.

The preceding facts show that the comparison of luminous intensities of differently colored radiants should depend on other methods than those which are in use in the photometric comparisons of lights of the same color. Before explaining these methods, let us recall that, in general, two quantities of light are equal to one another, when, received by the eye of the same observer, they produce on him the same effect; but this effect should be independent of the coloring of the light, and only dependent on its intensity.

These two conditions may be satisfied in two ways, as Macé de Lépinay and Nicati * have shown, taking as a starting-point two quite different functions of the eye which correspond with sufficient exactness to the two expressions *to distinguish* and *to see.*

49. The first method is based on the following phenomenon:

If light of any color and of an intensity which grows less and less, falls on a printed page, we experience an increased difficulty in reading, and the observer, to distinguish the characters, must approach nearer and nearer the object. It is this which is expressed when we say that *visual acuteness diminishes with diminution of intensity of illumination.* Let us recall that visual acuteness is measured by the reciprocal of the angle under which a definite object (ordinarily printed characters) must be seen in order to recognize its form.

This fact is intimately connected with intensity of illumination, or, more exactly, intensity of the light perceived by the eye, and moreover is completely independent of the nature of the color impressions produced on the eye by the light which produces the illumination.

We may then form a basis for a photometric method by considering two quantities of light equal when, *falling on the same uncolored object* (black on a white background) *always placed at the same distance from the observer, they enable him to perceive the details with the same nicety,* or in other words, *when they reduce the visual acuteness to the same value.*

This method was invented by Celsius and employed afterwards by W. Herschell.

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* *Ann. de Chim. et de Phys.*, 5e série, Vol. XXIV. p. 289; Vol. XXX. p. 145.

We know that visual acuteness is measured by presenting to the eye letters of different sizes, placed at an invariable distance (five meters, for example), and noting the dimensions of the smallest letter which is clearly seen. Another method which is more exact consists in employing signs of definite sizes and having the observer approach until he begins to distinguish them. The visual acuteness is, in this case, directly proportional to the distance from the observer to the object at the moment when he perceives it distinctly.

The letters which are used are made of heavy black lines whose width is equal to the interval between them. We take as the unit of visual acuteness ($v = 1$), an acuteness such that for the observer the distance between the consecutive lines subtends an angle of 1′. If the width of the lines is 1 mm., the visual acuteness is 1 when the distance from the observer is 3.44 m. At a distance of 1 m., the visual acuteness is $v_1 = \frac{1}{3.44} = 0.29$, and at n m. it is $v_n = 0.29\,n$.

Macé de Lépinay and Nicati have found that it is preferable, for photometric measurements, to replace the letters by three black horizontal lines on a white background, 5 mm. long, 1 mm. wide and 1 mm. apart. To obtain lines perfectly black, the most simple method is to cut them in a sheet of white waxed paper, and to place the sheet before a cavity lined with black velvet. In this way the black lines reflect only a negligible part of the light which strikes them.

In order to compare the intensities of two radiants by this method, we may proceed as follows:

We illuminate the conventional signs adopted to determine the visual acuteness by the light from the photometric standard, placed at a definite distance d_1; then the observer approaches until the eye, at the distance d, clearly perceives them. Next we illuminate them by the radiant to be studied, which is moved until the observer perceives them anew with the same facility as before. Let d_2 be the distance of the radiant corresponding to the limit of perception.

At this instant the visual acuteness is the same as before, and consequently the illuminations are equal. I_1 being the intensity of the photometric standard, I_2 that of the radiant which is studied, we have,

$$\frac{I_1}{d_1^2} = \frac{I_2}{d_2^2},$$

whence

$$I_2 = I_1 \frac{d_2^2}{d_1^2}.$$

In practice we may employ with advantage as conventional signs the logarithms of any table; the detached page of the table plays the part of the screen on which the numbers appear with more or less clearness, according to the intensity of illumination.

This method should only be employed when the two radiants to be compared are very differently colored, for its precision does not exceed about 10 per cent; this degree of precision may be considered satisfactory for differently colored radiants, but it would not be sufficient for lights of the same tint.

50. Beside the preceding method based on visual acuteness, that in which equality of illumination of two continuous divisions is determined directly may also be used for the comparison of radiants of different tints, but with an important restriction. When two neighboring divisions are illuminated, the one exclusively by one of the radiants, the other exclusively by the other, experience shows that however different the coloring of these two contiguous divisions may be, provided they are small enough, the eye may appreciate with a certain exactness the moment when these two divisions appear equally illuminated.

The restriction mentioned above is relative to the dimensions of the divisions whose equality of brightness is to be established. The eye appreciates in fact with correspondingly greater difficulty the coloring of a surface, as this surface becomes smaller. It follows from the very complete measurements of Macé de Lépinay and Nicati that the two regions illuminated by the radiants compared should subtend an angle less than 45′. Consequently the size of the divisions should not be greater than 6.5 mm. if the observer is at a distance of 50 cm. Below are the limit values of the size of the divisions for various distances of the observer.

Distance.	Size.
0.1 m.	1.3 mm.
0.3	3.9
0.5	6.5
1.0	14.1
2.0	26.2
3.0	39.2
5.0	65.5

The limit value which the size of the illuminated division should not exceed for a given distance of the observer, is so deter-

mined that the ratio of the intensities of the two radiants of different tints may be independent of their absolute intensity or, at least, not depend on it more than on Purkinje's phenomenon. For this the retinal images should be smaller than 0.002 mm. which corresponds to a visual angle of 45′: this value is exactly 0.002 mm. for the blue, and 0.004 mm. for the red. In the majority of lights, the most refrangible radiations are the least intense; we may almost always double the values of the limit sizes, which results in giving to the visual angle a value of 1° 30′.

This method consists, then, in employing, for the comparison of differently colored radiants, the usual photometers, taking care that the size of the divisions whose illuminations are to be made equal do not exceed the above limits.

The preceding is sufficient to give an idea of the two principal methods which may be used in the comparison of lights of different colors. It remains for us to enter into the details of the methods and the practical apparatus.

Macé de Lépinay's Method *.

51. This method depends on the following fact demonstrated by experiment, which has enabled Crova to establish his method for the optical measurement of high temperatures.

When bodies of the same temperature and of different emissive powers are placed in obscure surroundings, they emit light of very different intensity, but of the same composition.

This law is directly applicable to the usual radiants which are all constituted of particles of carbon rendered incandescent by the high temperature to which they are brought.

Let I be the intensity of a radiant, deduced by direct comparison with the photometric standard, *e.g.* the carcel lamp. Let us designate by R the intensity of one of its red rays, of determined wave-length, measured by the spectro-photometer with reference to the intensity of the same kind of ray of the carcel lamp; and by G the intensity of one of its green rays, defined in a similar way. If for the first radiant another at the same temperature is substituted, the three quantities I, G, and R remain proportional, and the two ratios $\frac{I}{R}$ and $\frac{G}{R}$ retain the same values.

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* *Comptes Rendus*, Vol. XCVII. p. 1428.

If the temperature of the radiant studied varies in a continuous manner, it will be the same with the composition of the light which it emits, and the two ratios $\frac{I}{R}$ and $\frac{G}{R}$ will vary also in a continuous manner. We shall then be correct in writing

$$\frac{I}{R}=f\left(\frac{G}{R}\right).$$

The intensity of any ordinary radiant could then be determined by simply measuring the intensities G and R, if the nature of the function $f\left(\frac{G}{R}\right)$ were known in advance.

This solution of the problem would not be practical, for it would necessitate the employment, always delicate, of a spectro-photometer. But the exactness of the entire reasoning remains if we substitute for the spectro-photometric measurements made with rays of determined wave-length, in the red and the green, measurements made by means of the Foucault photometer. The divisions should then be observed through two solutions, one red, and the other green; these solutions should always be employed in the same state of concentration and of the same thickness; they should in addition furnish rays sensibly simple, in order that the two divisions of the screen may have the same color.

The solutions which best fulfil the above conditions are: a solution of pure perchloride of iron in water at 38° B. (Baumé), and a solution of pure chloride of nickel in water at 18° B. The two solutions should have a thickness of 3 mm. The first transmits red rays only, and the second green rays only.

To determine by experiment the function $f\left(\frac{G}{R}\right)$, Macé de Lépinay made 52 measurements, comparing with the carcel standard successively, a regulating lamp with a straight chimney, one with a bulging chimney, a petroleum lamp, the Drummond light, and finally sunlight diffused by a white screen of sulphate of barium.

The following formula is deduced from these measurements:

$$\frac{R}{I}=1+0.208\left(1-\frac{G}{R}\right).$$

By means of this formula the following numerical table, which is sufficient in practice, was calculated:

$\frac{G}{R}$	$\frac{I}{R}$	$\frac{G}{R}$	$\frac{I}{R}$
0.8	0.96	2.0	1.26
1.0	1.00	2.2	1.33
1.2	1.04	2.4	1.41
1.4	1.09	2.6	1.50
1.6	1.14	2.8	1.60
1.8	1.20	...	....

The degree of exactness which this method gives is shown by the following verifications:

With a Swan lamp (at 12 volts and 0.95 amp.) Macé de Lépinay found:

$$G = 0.167,$$

$$R = 0.184;$$

whence $$\frac{G}{R} = 0.908,$$

$$\frac{I}{R} = 0.98;$$

then $$I = 0.180.$$

Direct experiment gave $$I = 0.182.$$

With the Drummond light, the following results were obtained:

$$G = 6.59,$$

$$R = 5.04;$$

whence $$\frac{G}{R} = 1.31,$$

$$\frac{I}{R} = 1.07;$$

that is, $$I = 5.39.$$

Direct measurement gave $$I = 5.43.$$

L. Weber's Photometer.

52. The preceding formula of Macé de Lépinay may be put in the form

$$I = kR,$$

representing by I the intensity of the radiant obtained by the method of equal illumination, by R the intensity of the red light obtained by means of a solution of perchloride of iron in water at 38° B., and by k a coefficient which takes account of the physiological elements of the problem, and which is, according to the measurements of Macé de Lépinay,

$$k = \frac{1}{1 + 0.208\left(1 - \frac{G}{R}\right)}.$$

In this equation G represents the intensity of the green light obtained by passing the rays of light through a solution of pure chloride of nickel at 18° B.

Weber's photometer permits the easy measurement of the intensities G and R, by the aid of which the intensity I is deduced from the preceding formula*.

It is composed of a horizontal tube A (Fig. 35), in which are placed the photometric standard H, and a ground glass s, whose movements, parallel to the axis of A, are governed by the screw f, and may be measured with very great exactness. At one of the extremities of A, and movable about the axis of this tube, is found a second tube B, provided in the same way with a ground glass s_1; it has at g an opening before which the eye of the observer is placed.

When the tube B is directed at any radiant, the vertical edge on the left of the prism of total reflection P (Fig. 36) divides the visual field into two portions; the half at the left is found to be illuminated by the observed radiant, and the half on the right by the standard of comparison H. Moving the glass s from right to left, and, if necessary, making use of the glass s_1, we succeed in obtaining perfect equality in the illumination of the two halves of the visual field. Under these circumstances it is sufficient to place before the eye a plate of glass which has been made red by means of a deposit of suboxide of copper, in order that the two visual fields may appear

* *Elektrotechnische Zeitschrift*, Vol. V. p. 166; *Lum. Él.*, Vol. XII. p. 468.

absolutely monochromatic, even when the tints of the radiants compared differ notably from one another.

Weber was not satisfied with the values of the coefficient k deduced from the formula of Macé de Lépinay; he determined this coefficient directly by means of his apparatus and two plates of ground glass on which particular designs were photographed. These two plates are fitted to each piece of apparatus, which enables one

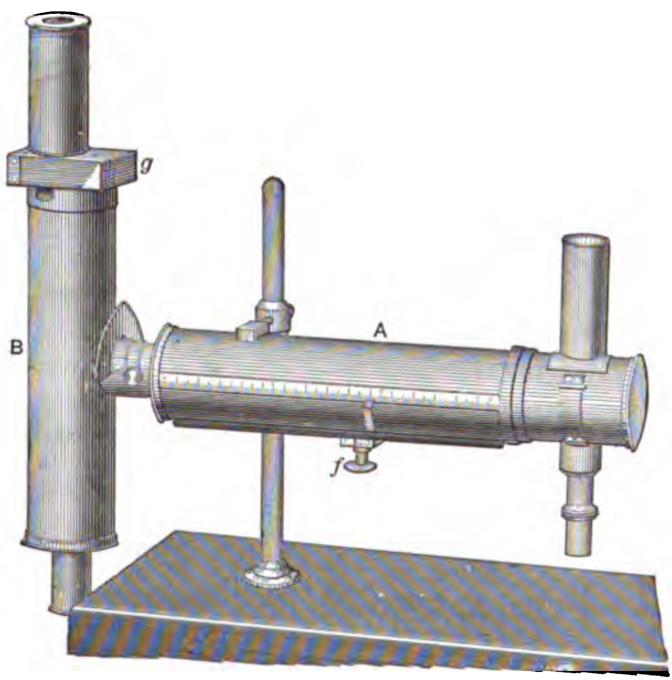

FIG. 85. — Weber Photometer.

to verify, by the method of equal visual acuteness, the results obtained by the method of equal illumination.

These plates are obtained in the following way: first there are drawn, on a rectangle of paper 40 cm. by 20 cm., eight squares in which are traced concentric circles in which the width of the white of each circle is equal to the width of the black. A photographic reduction to a twentieth gives very clear plates, 2 cm. by 1 cm., in which the width of the eight systems of circles varies from 0.275 mm. to 0.1 mm. by steps of 0.025 mm.

These plates are placed one at s and the other at s_1; they occupy only one-third of the corresponding half of the visual field. The plate s_1 being fixed, the plate s should be at the division 245 mm., in order that the designs of the two plates may be seen by the observer, subtending the same angle.

If the two parts of the visual field are equally illuminated by radiants of the same color, the clearness of the designs is the same in both parts; but if the tube *B* is directed at a radiant whose tint differs from that of the other, and then the equality of the light of

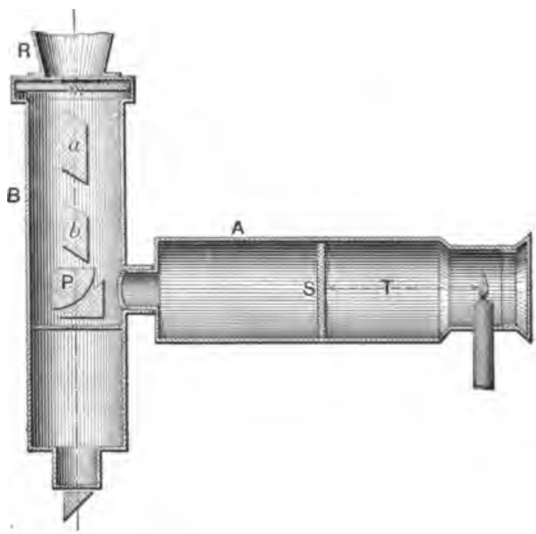

FIG. 86.

the two surfaces is established, the designs will not be equally clear in the two halves of the visual field. The phenomenon is due to the fact, enunciated by Macé de Lépinay, that in the spectrum, as we go from the green, the refrangible colors contribute little to clearness of perception while contributing in a considerable measure to illumination of the surfaces.

Two observations should be made. The plates being put in place, *B* is directed at any radiant; then the photometer is moved backwards or forwards until the designs appear on the two plates with equal clearness.

This point is determined in a way which is precise and independent of the sight of the observer, by at the same time fixing the

two designs, and turning one's gaze from the largest to the smallest circles. It then happens that for a square of a certain order, the white cannot be distinguished from the black; the right point is reached when the square in each of the two plates, for which this distinction becomes impossible, has the same number. At this moment the red glass is placed before the eye, and without further considering the designs, the equal illumination of the surfaces which are outside the photographic plates is brought about by moving the glass *s*. Let us suppose that *s* has been moved *r* cm. from *O*; the value of *k* will evidently be

$$k = \frac{r^2}{(24.5)^2}.$$

This value of *k* is not absolutely independent of the intensity of illumination which has served to establish the equal clearness of the designs; therefore the flame of the standard should be maintained at as invariable a height as possible.

For radiants which include more yellow and less red light relatively to the ascetate of amyl lamp employed as a standard, *r* is found greater than 245 mm. The value of *k* becomes equal to 1 when the light considered has the same color as the flame of the standard.

Weber undertook a second series of experiments to determine the variations of *k* with the color of the radiant. The surfaces are illuminated equally for the red light, *s* remaining fixed at 245 mm., then a green glass is substituted for the red glass, and the new movement ρ of *s* is noted by which the equal illumination of the surfaces is established anew. These measurements are repeated for different tints of the radiant which are obtained by raising a lamp to more or less incandescence.

Under these circumstances the ratio of the intensities *G* and *R* of the green and the red, compared with the same colors of the ascetate of amyl lamp, is expressed by

$$\frac{G}{R} = \frac{(24.5)^2}{\rho^2}.$$

Spectroscopic researches have shown that the red glass used in connection with Weber's photometer, lets pass only those rays included between the wave-lengths 0.687 μ and 0.630 μ, whose maximum intensity corresponds to 0.656 μ, and the green glass only those between 0.577 μ and 0.516 μ, with a maximum at 0.547 μ.

At the same time as Weber, Dr. O. Schumann determined by the aid of Glan's spectrophotometer (§ 66) the value of the ratio of the green and red radiations from an incandescent lamp, compared in the same way with radiations of the same color from an ascetate of amyl lamp; the wave-lengths of the radiations compared were $\lambda = 0.6762\,\mu$ and $\lambda = 0.5574\,\mu$.

These values differing slightly from one another, Weber constructed curves which gave the mean value of k included in the following table:

$\frac{G}{R}$	k	$\frac{G}{R}$	k
0.3	0.50	1.3	1.22
0.4	0.56	1.4	1.28
0.5	0.64	1.5	1.34
0.6	0.72	1.6	1.40
0.7	0.80	1.7	1.46
0.8	0.87	1.8	1.51
0.9	0.94	1.9	1.56
1.0	1.00	2.0	1.61
1.1	1.08	2.1	1.65
1.2	1.15	2.2	1.69

This table is sent out with each piece of apparatus. Practical measurements then offer no difficulty.

We obtain the points d_r and d_g, corresponding to the equality of illumination of the two divisions of the photometer for red and green radiations, by inserting successively in the movable tube plates of red and green glass. We then have $\frac{G}{R} = \frac{d_r^2}{d_g^2}$, and, seeking in the above table the corresponding values of k, we obtain

$$I = akR,$$

a being a factor which depends on the coefficient of absorption of the ground plates of the photometer and on the intensity of the standard flame. It is easy to determine this constant a once for all.

We may remark that the values of k obtained from the preceding table give results which are only precise for the photometric comparisons of radiants whose light has an analogous composition to that which is emitted by incandescent lamps under ordinary circumstances.

If the radiants compared emit light of the same tint, $k = 1$. We may then make the comparison without employing colored glasses; if it is desired to use them, it is sufficient to make an observation with one of them merely, the ratio of the intensities of the red and green radiations being equal to the ratio of the total intensities.

53. L. Weber has recently* perfected his apparatus by the introduction of Lummer and Brodhun's optical screen (§ 31) and by the substitution of polarization apparatus for the screens designed to equalize the illumination of the two halves of the visual field. This new model really belongs to the class of polarization photometers.

Figure 36 shows the arrangement of this new piece of apparatus. The prism-screen of Lummer is put in place of the total reflection prism of the older model. In the movable tube B are found two nicols a and b, whose relative position determines the intensity of illumination of the outer part of the photometric field; the illumination of the inner part of the field is regulated in the ordinary manner by the movement of the opalescent screen s.

Two methods may be employed in making the comparisons. In the first, the nicols are placed in their parallel position ($\alpha = 0°$, $\beta = \pm 90°$, or, $\alpha = \pm 90°$, $\beta = 0°$), so that there is no weakening of the light which traverses them, except what is produced by absorption. Next the illuminations of the two halves of the field are equalized by moving the opalescent screen s.

The second method is preferable. The plate s is left in a fixed position, then the nicol b is turned until the two halves of the visual field are equally illuminated. The weakening effected is easily calculated by means of the equation of Malus, the intensity of the exterior field being given by the formula $\frac{c}{d^2}\sin^2(\alpha - \beta)$. In this formula c represents the constant of the nicol, d the fixed distance of the opalescent plate. We may also determine empirically the weakening corresponding to a determined position of the nicol b and make in advance a table for each piece of apparatus. This second method renders the division of the fixed tube A useless.

The exterior tube R serves to keep lateral light from the bare plate while measurements of illumination are being made.

The two nicols a and b are placed between the plate s' and the prism P; for if one of them, b for example, were placed between the

* *Zeitschrift für Instrumentenkunde*, 1891, p. 7.

prism-screen and the ocular prism, complications in the formulæ would result. A part of the light reflected by the exterior part of the screen and coming from *A* is polarized by reflection; account should then be taken of this fact in the formulæ. The tube *B* is fitted with an ocular prism of total reflection in order to facilitate observations when it is in a vertical position.

Crova's Method.

54. Whereas the method invented by Macé de Lépinay reduces the photometric comparison of two lights of different tints to the comparison of the intensities of two of the component colors, Crova* has invented a method which requires only the comparison of the intensities of a single one. This method depends on a fact verified by a great number of experiments. It may be enunciated as follows: *if two lights of very different tints are compared, the total intensities are to one another as the intensities measured in that part of the spectrum where the wave-length is* $0.582\,\mu$.

The following is the manner in which Crova arrived at this conclusion. He measured by means of the spectrophotometer the luminous intensity of different regions of the spectrum of sunlight and of the light of the carcel lamp, taking in each series as unity the hundredth part of the intensity of the most intense radiation.

Tracing the two curves whose abscissæ are the wave-lengths and whose ordinates are the illuminating powers, the area of each of them represents for each light the total illuminating power. Crova determined with care the ratio of the areas of the two curves furnished by sunlight and the carcel lamp, and obtained the fraction 0.7302. If the ordinates of the curve of intensities of sunlight are divided by 0.7302 while retaining the curve of the carcel lamp, the two areas are then equal, and the curves correspond to equal illuminations. These two curves cut one another at a point whose abscissa corresponds to $\lambda = 0.582\,\mu$; this radiation is then that whose intensity is the same in the two lights when the two illuminating powers are equal.

The preceding comparisons having been made with the light of a radiant of low temperature, as the carcel lamp, whose maximum illumination is at the radiation $\lambda = 0.592\,\mu$, and with sunlight which has the highest temperature of emission, and whose maximum is at $\lambda = 0.564\,\mu$, the preceding conclusions may be applied directly to

* *Comptes Rendus*, Vol. XCIII. p. 512; *Ann. de Chim. et de Phys.*, 6e série, Vol. VI. p. 528; *Lum. Él.*, Vol. XVIII. p. 549.

the usual radiants, whose temperatures of emission are comprised between these limits.

This method cannot be applied to the photometric study of a radiant whose temperature of emission varies with the luminous intensity; incandescent lamps are in this class. The luminous intensity increases with the energy spent in the lamp, but the proportion of the radiations of different wave-lengths varies also. Weber's method, in which the luminous intensity for a single wave-length is measured, is free from this restriction, because the coefficient k is determined experimentally for different degrees of incandescence.

To obtain an exact comparison by Crova's method, it is necessary to use a spectrophotometer, and to bring the middle of the ocular slit to a point on the graduation which corresponds to $\lambda = 0.582\,\mu$. But, in practice, we may work in a more simple and rapid manner by the aid of one of the two following methods.

In the first, we employ the ordinary Foucault screen, or any screen that is viewed by means of a telescope whose objective, of short focal length, allows a very clear image of the disc and of the line of separation of the two regions to be obtained.

In the body of the telescope there is a system of two nicols, placed at right angles, between which there is a plate of quartz 9 mm. thick, perpendicular to the axis; this thickness has been calculated so that its interposition between the two crossed nicols gives rise, in the spectrum of the light which traverses them, to two wide interference bands situated at the two extremities of the spectrum, which in this way almost extinguish the intensity in these regions. In going from these two bands toward the limit of the yellow and the green, where the wave-length is $0.582\,\mu$, the intensity of the transmitted radiations varies in proportion to the square of the cosine of the angle made by the right section of the second nicol with the planes of polarization of the different radiations which have undergone in the quartz plate rotary dispersion; the radiation for which the cosine is equal to unity undergoes no diminution.

The apparatus is regulated so that the radiations in the immediate neighborhood of $0.582\,\mu$ undergo no diminution.

In the second method, which is much simpler, the photometric screen is observed through a special solution which allows only radiations whose wave-length is about $0.582\,\mu$ to pass. This solution, which produces considerable absorption, is more specially applicable to very intense radiants.

The solution is prepared thus:

Anhydrous sublimed perchloride of iron . .	22.321 grams.
Crystallized chloride of nickel	27.191 "

It should be dissolved in distilled water, and the volume of the solution made 100 cc. at a temperature of 15° C.

The liquid is contained in a glass receptacle with parallel faces, the surfaces in contact being ground. The receptacles which Crova used consist of a flat ring of glass 7 mm. thick, ground smooth, against the faces of which are fixed, by simple adhesion, using a drop of distilled water, two thin sheets of plate glass with accurately parallel faces, pressed against the ring by means of two blackened brass plates with four pressure screws; the two plates have circular openings whose diameter is slightly less than that of the receptacle, so as to allow only the rays which traverse the liquid to pass through.

The liquid is introduced by means of a capillary pipette through a small orifice in the ring which is afterwards closed by a ground glass stopper. To avoid the possible breaking of the glass on account of too strong pressure of the screws, it is well to insert between the receptacle and the glass plates, washers of cardboard or leather. Receptacles thus filled last a long time without alteration.

With a thickness of 7 mm. this solution allows to pass only radiations comprised between the wave-lengths 0.630 μ and 0.534 μ, with a maximum near 0.580 μ; if the thickness is increased, these limits approach one another and tend toward a maximum of from 0.580 μ to 0.582 μ, which is the most favorable.

Temperature has a notable effect on the absorbing power of perchloride of iron; in proportion as it increases, the absorption increases in the most refrangible region, and the black screen which appears to cover the spectrum up to the limit of the green advances toward the red. The limits of the wave-length indicated above are then variable with the temperature, but at about 13° C. these limits are sensibly invariable for small variations of temperature.

The Employment of Media of Complementary Color.

55. The study of heterochromatic photometry would be incomplete if we passed in silence the employment of glass of complementary colors, designed to eliminate errors due to differences of color in the lights compared.

Tresca first proposed to insert between the radiants and the screen plates of glass of about complementary colors. He invented this process on the occasion of the photometric measurements of arc lights at the Electrical Exposition of 1881.

A green glass is placed in the path of the reddish rays of the standard, the carcel lamp, and a red glass in the path of the bluish rays of the arc lamp. If the colored glasses that are thus interposed were exactly complementary to the rays from the corresponding radiant, all the rays would be intercepted, and the two fields of the photometer would remain dark; but as glass of *about* complementary color is employed, we shall find on either side a dull gray color whose tone will be sensibly the same on both sides. But if the color of the two fields of the photometer is made equal, it is difficult to see how account may be taken of the quantity of light absorbed, on both sides, by the two colored glasses. Tresca seems to assume *a priori* that these losses are equivalent, *i.e.* that the red rays lose as much in the green plate as the green rays in the red plate, which is anything but proved. If we wish to measure experimentally the value of these losses, we would be brought anew to the comparison of the relative intensities of two very differently colored lights, and that is exactly what we wish to avoid. Tresca's process lacks, then, a precise scientific basis.

A second method based on the employment of glasses of complementary color has been sometimes proposed. It consists in making two measurements by observing the screen successively through two glasses or two special media, which allow only exactly complementary rays of light to pass. The arithmetical mean of the results of these two measurements is then considered the exact result.

This method cannot give exact results, for it is only a rude approximation to the scientific methods of Macé de Lépinay and Weber. We may, however, establish with sufficient ease the conditions which the two colored media and the lights we are comparing should satisfy in order that the process may be strictly exact.

Let us consider two exactly complementary glasses, *e.g.* red and green. Let us designate by I_1 the intensity of the first radiant, by G_1 and R_1 this intensity measured through green and red glass respectively. Let us further designate by I_2, G_2, and R_2 the same elements of the second radiant. We have evidently the following relations:

$$I_1 = G_1 + R_1,$$
$$I_2 = G_2 + R_2.$$

Let us designate by k the ratio of the total intensities:

$$k = \frac{I_2}{I_1}.$$

If the two radiants L_1 and L_2 should emit light of the same quality, that is, have similar spectra, we should have

$$I_2 = kI_1 = kG_1 + kR_1.$$

But this is not the case, the relation being

$$I_2 = aG_1 + bR_1.$$

The two coefficients a and b vary with the quality of the light from L_1 and L_2. If one of these is smaller than k, the other must be greater. Let $a = k - x$; then

$$I_2 = kI_1 = (k - x)G_1 + bR_1,$$

whence
$$b = \frac{kI_1 - (k - x)G_1}{R_1}.$$

In order that the arithmetical mean of the measurements may be exact, $a + b$ should be equal to $2k$, that is, $b = k + x$. But we have

$$b = k + x = \frac{kI_1 - (k - x)G_1}{R_1},$$

and, substituting for kI_1 its value,

$$k + x = \frac{kR_1 + xG_1}{R_1}.$$

In order that this condition may be satisfied, it is necessary that $G_1 = R_1$. These two colored media should accordingly be not only complementary, but they should absorb the same quantity of light: the luminous pencil which has passed through the first medium should have the same value as after having traversed the second. The verification of this property of colored media is as difficult as the comparison of the two radiants, since it is necessary to verify the equality of the two lights, the one yellowish red, and the other bluish green. Moreover, the construction of these two exactly complementary media would not be without difficulty. We have not space to dwell further on this process. [For another method see Appendix B.]

E. PHOTOMETERS BASED ON VARIOUS ACTIONS OF LIGHT.

56. No apparatus in which the action of light on the eye is replaced by its action exerted on physical or chemical phenomena independent of the observer, is suitable for photometric measurements. Such apparatus measures the action of light upon the phenomenon on which the instrument is based, but it does not measure in any way its physiological action. Such are chemical photometers which have, however, their importance from a photographic point of view, selenium photometers, etc.

Although this apparatus does not fulfil the principal condition of photometric measurements, nevertheless it should be mentioned, since it alone permits the effectual automatic registering of photometric measurements. This registering is in fact impossible with ordinary photometric apparatus, since the eye is its principal organ.

Among the photometers of this category, we should mention principally Siemens's selenium photometer and the photometers of Dessendier and of Lion. We should mention, also, the bolometer of which Langley made such extended use in measuring the distribution of energy in the spectrum, and the radiometer of Crooks, of which Olivier* has constructed a special form for photometry.

Selenium Photometer.

57. It is known that the electrical resistance of selenium diminishes under the influence of light; it is the luminous rays which most affect it; calorific rays exert a much less marked effect. Making use of this property, Siemens and Halske† about ten years ago invented their selenium photometer. A selenium tube replaces the screen of the ordinary photometers.

This tube is placed in circuit with a battery and a mirror galvanometer. First it is submitted to the rays of the photometric standard, and the deflection of the galvanometer is noted, then it is turned so as to be exposed to the rays of the radiant to be studied. Next, the distance of the selenium plate to the radiant is varied until the deflection of the galvanometer is equal to that produced by the standard. In this case the illuminations produced by the standard and the unknown light are equal.

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* *Lum. Él.*, Vol. XXVII. p. 560.

† *Lum. Él.*, Vol. IV. p. 367; Vol. VII. p. 38.

Gimé* also has invented a photometric method based on the employment of a narrow ribbon of selenium, rolled up so as to form a surface of considerable size which is placed in the circuit of a constant cell.

Dessendier's Registering Photometer.

58. This photometer is based on the following principle: If a mixture of equal volumes of hydrogen and chlorine is kept in the dark, the two gases do not combine. If this mixture is exposed, there is combination, and as a consequence the formation of hydrochloric acid. This may be absorbed by a chlorine solution whose level tends to rise. Now, the quantity of hydrochloric acid produced being, according to Dessendier, proportional to the quantity of light received by the gaseous mixture, it is sufficient to register the variations of the level of the chlorine solution in order to register the variations of the quantity of light received †.

This photometer registers the chemical action of light on a mixture of hydrogen and chlorine, and not its photometric action; so everything points to its employment in photography, where it has, moreover, been applied to the automatic printing of proofs. It is a long step from this to the application of this apparatus to industrial photometric measurements. If we limit ourselves strictly to registering the luminous intensity of a given radiant, the selenium photometer permits the problem to be solved more simply, since it reduces it to recording the variations of intensity of a current.

Lion's Photometric Balance.

59. Lion's photometer ‡ is based on the decomposition of iodide of nitrogen by the action of light. This substance decomposes slowly with a disengagement of pure nitrogen which varies with the intensity of the incident light. Guiard determined that only the portions of the iodide which are directly struck by the luminous pencil undergo this decomposition, it being purely superficial.

Lion has arranged an apparatus based on this principle; it is composed of two equal quantities of the reagent prepared under the same conditions. Above the liquid are two gas chambers of the same volume.

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* *Lum. Él.*, Vol. XXIV. p. 85, 1886.

† *Lum. Él.*, Vol. XXXIII. p. 407.

‡ *Bulletin des sciences physiques*, Vol. III. p. 149, 1890.

A capillary tube bent twice and fitting closely in rubber stoppers puts into connection the liquid in the two receptacles. After having filled it with liquid, an air bubble is introduced to serve as an index (Fig. 37).

The receptacles being opaque, windows of equal dimensions permit the illumination of equal surfaces of iodide of nitrogen in each of them. By the aid of the mirrors M, M' inclined at 45° we illuminate one of the windows by the photometric standard, and the other by the radiant to be studied. The index will not remain stationary unless the volumes of nitrogen disengaged in the same time are equal; that is, unless the windows are equally illuminated.

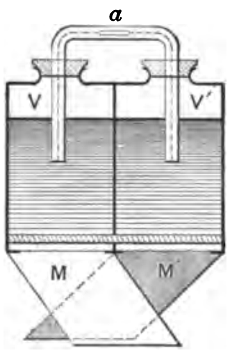

FIG. 37.—Lion's Photometric Balance.

Keeping the position of the standard the same, it is sufficient to move the other radiant until this condition is realized, which permits us to apply the law of the squares of the distances.

Lion has perfected this elementary apparatus in order to eliminate the causes of error. We may refer to Lion's memoir for the description and theory of this photometer, though making the same reservation as for the preceding apparatus.

Pupillary Photometer.

60. We know that the diameter of the pupil diminishes as the intensity of the light which strikes the eye increases; the object of this is to protect the eye against the injurious action of too intense luminous excitations.

If we knew the diameter of the pupil which corresponds to a given intensity of the luminous pencil penetrating the eye, we should have in this organ a photometer susceptible of considerable precision. For this we ought to be able to measure exactly and rapidly the diameter of the pupil. The pupillary photometer of Gorham* permits this.

This apparatus (Fig. 38) is composed of a bronze tube 6 cm. long and 4.5 cm. in diameter, closed at one end by a disc which has near its edge small holes arranged in pairs along the radii of the

Lum. Él., Vol. XIV. p. 458.

disc; the distance between the two holes of any pair varies progressively from 1.8 to 9.8 mm. The tube is closed by a cover which is slit along a radius, with so narrow an opening that it allows only the two holes of one pair to be seen at a time; this cover is movable around the tube, and has an index which indicates the distance between the observed holes.

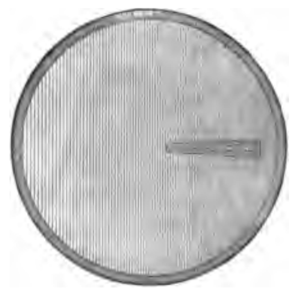

FIG. 38. — Pupil Photometer.

To make a measurement with this apparatus, we look at the source of light through the two holes exposed by the slit in the cover; we then see two points of light in appearance quite like that of a double star; the cover is turned until a pair of spots of light is found whose edges seem to touch. The diameter of the pupil is then found to be indicated by the division on the scale.

To make photometric measurements it is necessary first to point at the standard of light placed before a white background, next to point at the source of light to be studied, and to move until the spots of light seen with the same position of the cover seem to touch again.

Before proceeding to make measurements, the apparatus, that is one's eye, should be standardized, by determining the diameter of the pupil for different luminous intensities, obtained for instance by observing the photometric standard at variable distances.

This process is already old. Lambert determined in the last century the variations of the diameter of the pupil by observing at variable distances a circular opening pierced in a shutter and directed toward the sky. He thus obtained the following values, which may give an idea of the elements which are called into play in this phenomenon:

Distance of the Eye from the Radiant.	Visual Angle of the Radiant.	Diameters of the Pupil in Lines (2.4 mm.).	Distance of the Eye from the Radiant.	Visual Angle of the Radiant.	Diameters of the Pupil in Lines (2.4 mm.).
1	8°.36′	1.14	6	1°.26′	2.31
2	4°.20′	1.50	7	1°.14′	2.53
3	2°.53′	1.70	8	1°.05′	2.78
4	2°.10′	1.89	9	0°.58′	2.89
5	1°.44′	2.08	10	0°.56′	3.15

Wheatstone's and Masson's Photometers.

61. Photometers may also be constructed based on the duration of luminous sensation. Wheatstone and Masson, in particular, have constructed photometers of this kind.

Wheatstone's photometer consists of a polished ball of steel fixed on a disc placed eccentrically on a toothed pinion, which engages with the inner circumference of a toothed wheel. The ball may be moved rapidly by means of a crank which acts upon a series of cogs; if, during the movement, a pencil of light falls on the apparatus, the eye will perceive, in consequence of the reflection on the movable ball, a closed figure composed of epicycloids. It is sufficient to illuminate the apparatus by two radiants to obtain two figures whose equality of luminous intensity is determined by moving either or both of the radiants.

Masson* in his photometer uses a disc on which alternately black and white sectors are painted; this disc appears uniformly gray when set in rapid rotation, while if it is suddenly illuminated, it seems stationary, and the sectors appear distinctly black and white. Placing the radiants at a sufficient distance the illumination becomes too weak for the sectors to be distinguished, and the disc again appears uniformly gray. The photometric measurement consists in successively moving the two radiants until the disc appears uniformly gray; the intensities of the two radiants compared are then proportional to the square of the distances.

F. SPECTROPHOTOMETRY.

62. The general problem of photometry, that is, the comparison of differently colored radiants, can only be solved *in an approximate manner* by the methods previously described. To solve it *exactly* we must compare the ratios of the intensities of each of the simple rays which compose the light emitted by each radiant. Now this comparison may be effected by the aid of spectrophotometers alone. But the difficulty and length of spectrophotometric measurements interfere with this apparatus ever coming into common use in industrial practice.

The description and study of spectrophotometers exceed then the limits of this work. However, to be complete, we shall give from

* *Ann. de Chim. et de Phys.*, 3e série, Vol. XIV. p. 137.

Crova * a cursory glance at the principal apparatus, while referring to the original memoirs for a complete study.

63. Govi † was the first to publish the description of an analyzing photometer. The lights to be compared are received on two rectangular prisms placed before the slit, then reflected on an achromatic lens and dispersed by a prism. The two spectra in juxtaposition are received on a strip of glass covered with starch, identical with that of the Foucault photometer. This glass is covered by an opaque screen with a slit, which allows only the light of a single color of the two spectra to pass. Equality of illumination of the two divisions is brought about by suitably varying the distances of the two radiants. Govi also proposed to polarize them at right angles and to equalize the intensity by the rotation of a Nicol analyzer, as Arago had already done in his photometric researches.

64. Later Vierordt ‡ published an account of a spectrophotometer which he applied to the qualitative analysis of colored substances dissolved in liquids. Vierordt's spectrophotometer is an ordinary spectroscope whose slit is formed on one side by a continuous plate, on the other by a plate identical with the first except that it is cut into two equal parts, each of which is moved by a micrometer screw; thus two slits of unequal width are obtained which give in the spectroscope two superposed spectra of different intensities. If the two half-slits receive light of unequal intensity, or, indeed, if one receives light directly, and the other the same light modified by the absorption which it has undergone in passing through a colored medium, we may equalize the intensities of the rays of the same wave-length in the two spectra by suitably varying the sizes of the two half-slits; the intensities are then in inverse ratio to the size of the slits.

This arrangement is sufficient when the intensities to be compared are not very different; when they are very different, we must enlarge one of the two half-slits a great deal, and the corresponding spectrum becomes more and more impure because of the superposition of rays of different refrangibilities at one and the same point of the spectrum; the two colors to be compared can then no longer be made identical. In this case, Vierordt used glasses slightly

* *Ann. de Chim. et de Phys.*, 5e série, Vol. XIX. p. 472.

† *Comptes Rendus*, Vol. L. p. 156, 1860.

‡ *Poggendorffs Annalen*, Vol. 140, p. 172. 1870.

smoked, which he placed in the path of the more intense light, so as to render the intensity of its spectrum little different from that of the other; equality of intensity is then obtained by a slight variation in the width of the slits.

65. It then becomes necessary to determine for each of the smoked glasses the coefficients of absorption corresponding to various simple rays. Trannin* has constructed a more convenient photometer. He made use of the phenomenon of the disappearance of the complementary fringes of the two lights polarized at right angles. This principle had already been applied by Babinet, Wild, and by other physicists. The two lights, reflected by two rectangular prisms placed before the two halves of the slit of the spectroscope, are first polarized by a Foucault prism, then traverse a quartz plate parallel to the axis, and finally a Wollaston prism, which gives two images, polarized at right angles, of each half of the slit. The dispersion prism therefore gives four spectra, two of which, polarized at right angles and coming from the two halves of the slit, are partially superposed in the middle of the field. The insertion of the quartz has for its object the production, in the four spectra, of bands whose intensities are complementary in the two spectra polarized at right angles; they ought, therefore, to disappear in the region where the two spectra are superposed, when we have equalized the intensities of the two spectra at the point considered. We arrive at this result either by varying the distance between the two lights and the instrument or by interposing between the Wollaston prism and the dispersion prism a Foucault prism, which by a suitable rotation produces equality of the two intensities.

66. Glan† has invented a spectrophometer whose construction is analogous to that of Trannin's instrument, but which differs from it in the method adopted for obtaining equality of the intensities of the two contiguous spectra.

The slit of the spectroscope is divided into two equal parts by a transverse strip of blackened brass. We obtain thus two spectra separated by an obscure interval. A Wollaston prism placed behind the objective of the collimator doubles the image of each half-slit so as to have, not two, but four spectra polarized two and two at right angles. For a suitabl- width of the transverse brass strip the

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* *Journal de Physique*, Vol. V. p. 297, 1876.

† *Wiedemanns Annalen*, Vol. I. p. 353, 1877.

lower spectrum of the upper half-slit is tangent to the upper spectrum of the lower half-slit, and we conclude that, by a suitable rotation of a nicol movable on a divided circle, placed between the Wollaston prism and the dispersion prism, we may obtain equality of intensity of the two adjacent monochromatic regions in the two contiguous spectra of the lights compared. Let I_1 be the intensity of the ray λ of the lower spectrum of the upper half-slit, I_2 that of the same ray in the upper spectrum, a_1 and a_2 the coefficients of diminution due to the refractions and absorptions which the two pencils undergo in the apparatus, and α the angle between the principal sections of the Nicol prism and the Wollaston prism; we shall have

$$I_1 a_1 \cos^2 \alpha = I_2 a_2 \sin^2 \alpha.$$

If the intensity of the first ray changes and becomes I_1', the intensity of the other remaining constant, there must be a rotation α' to re-establish the equality of the intensities; we have, then,

$$I_1' a_1 \cos^2 \alpha' = I_2 a_2 \sin^2 \alpha',$$

whence

$$\frac{I_1'}{I_1} = \frac{\tan^2 \alpha'}{\tan^2 \alpha}.$$

In order the better to appreciate the equality of intensity of the two contiguous divisions, we rid ourselves of all extraneous light by means of a slit formed of two plates of blackened brass, which are movable in the focal plane of the telescope, and whose separation is regulated so as to admit only the rays to be compared.

But in order that this comparison may be made with precision, the two luminous divisions must be brought into contact without being separated by a line either light or dark.

Now the two spectra to be compared have undergone unequal deviation in the direction perpendicular to their length because of the special dispersion which the Wollaston prism has given them. In both, the violet is more deviated than the red, and, as the deviations take place for both in opposite directions, exact contact is only obtained for a determinate region of the spectra, in the middle for instance; while toward the violet the two spectra overlap, and toward the red they are separated by a dark interval. However, to obtain contact of the two divisions belonging to any region of the two spectra, we move the slit of the objective of the collimator suitably back or forth, which varies the ratio of the width

of the brass strip to the distance of the images of the two half-slits in the Wollaston prism.

When this instrument is used to compare two radiants, we should cover one of the half-slits of one rectangular prism, which totally reflects, along the axis of the instrument, the light of one of the sources placed at the side, while that of the other is received directly on the other half-slit.

Crova's Spectrophotometer.

67. Crova * has modified Glan's spectrophotometer so as to make it more precise. Following is the description of the apparatus.

The lamp to be studied, L' (Fig. 39), is placed in front of a circular opening in a rectangular box, blackened within, and illuminates the lower part of the slit F, whose width may be regulated

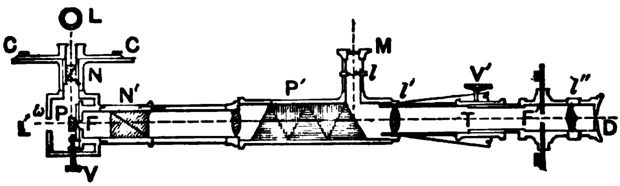

FIG. 39. — Crova's Spectrophotometer.

at pleasure by means of the screw V. The light emitted by the standard L, which has traversed a nicol N, movable on its graduated circle CC, is received through a lateral opening in the box, on a prism of total reflection P, which covers only the upper half of the slit F.

The rays emanating from the two sources, after passing through the slit F, traverse a fixed Nicol analyzer N', and a system of prisms P' forming a direct vision spectroscope, and are displayed in two superposed and contiguous spectra.

These spectra are examined by means of an eyepiece l'', at the extremity of a tube M'', which can be moved laterally by means of a rack and pinion V', so as to traverse the whole field of the spectrum. We rid ourselves of all light, other than the radiations which we wish to compare, by means of a slit F' between two blackened

* *Ann. de Chim. et de Phys.*, 5e serie, Vol. XIX. p. 495 and Vol. XXIX. p. 556.

brass plates moving in the focal plane of the telescope $l'l''$, and whose separation is governed at will.

In this apparatus we annul the elliptical polarization due to total reflection by employing a double total reflection prism. That the instrument may serve for the photometric study of the rays of the spectrum, and especially that it may be calibrated, a micrometer M is placed at the side as in all direct-vision spectroscopes.

CHAPTER III.

PHOTOMETRIC STANDARDS.

Introduction.

68. The first photometric standard was the candle which Bouguer constantly employed in his researches.

As long as there were only luminants of low power to be measured, the candle sufficed as a unit of comparison, no account having been taken, to be sure, of constancy and comparability.

For photometric measurements of the illuminating power of gas-lights, Dumas and Regnault saw the necessity of a standard at the same time more constant, more comparable with itself, and more intense. They then adopted the carcel lamp, which gave in their hands exact results and which, since then, has been generally employed in France for all industrial photometric measurements. In France, Giroud, and in England, Methven, sought to represent the standard of light by means of a flame burning ordinary illuminating gas, such as is furnished for consumption; but these standards depend in a great measure on the composition of the gas.

Vernon-Harcourt of London sought to produce a gas of constant composition by utilizing, as a combustible, air carburetted by volatile carburets of hydrogen extracted from petroleum, principally by pentane. The burner used is a candle-burner of well-defined dimensions, and the flow of gas is automatically regulated.

There should be mentioned also among the standards of light produced by combustion, the ascetate of amyl lamp of von Hefner-Alteneck, which gives a light of very satisfactory constancy, but of too slight intensity. In this lamp of free combustion, the wick is immersed in ascetate of amyl, which gives better results than other hydrocarbons, benzine and ligroin* for instance.

In petroleum lamps which are universally used, the rise of oil in the wick takes place solely by capillary action; considerable varia-

* "That part of petroleum which has a boiling-point between 90° and 120° C." — *Cent. Dictionary.*

tion results in the intensity according to the height of the liquid in the reservoir. Further, the composition of commercial petroleum is far from being constant.

In order that the photometric standard furnished by the preceding apparatus may be utilizable, the flame must always be the same, the combustible and supporter of combustion must have a constant composition, and combustion must take place under invariable conditions. Now flame standards are sensitive in a great measure to every modification in the state of the supporter of combustion. Thus a candle and a carcel lamp depart very rapidly from the normal condition when they are put in a somewhat small room containing several observers.

Further, the illuminating power of a luminous body depends on its temperature; the former increases very rapidly with the latter, so that the higher the temperature of a luminous body is raised, the more necessary is it to obtain constancy in it, without which the luminous intensity would be essentially variable.

Now, in a flame, constancy of temperature is not easy to realize, for it requires that the mixture of combustible and supporter of combustion should always be made under identical conditions. If this mixture is not perfect and invariable, the temperature varies, and the brightness even more.

Finally, a flame is always transparent, and the quantity of light emitted varies with the degree of transparency. In order that a flame may emit constant luminous radiations, it is not sufficient for the temperature to be invariable; it is necessary in addition that the transparency of the flame and its thickness undergo no change.

To furnish a satisfactory unit of light, a flame should then satisfy an aggregation of conditions quite difficult to realize completely. These are the difficulties which determined the International Commission on Electrical Units to reject definitely, as the absolute standard, standards of combustion, and to adopt the platinum standard proposed by Violle.

The advantages of a standard of light based on the incandescence of a body raised to a high temperature, for instance to the temperature of fusion of platinum, have been recognized for a long time by all physicists. As early as 1844 Draper indicated the possibility of taking as unity the light emitted by platinum wire made incandescent by the passage of an electrical current, and later, in 1859, Zoellner also conceived the same idea. It was taken up again in 1878 by Schwendler, in Calcutta, who made numerous attempts

with apparatus constructed with a view to realizing this photometric standard.

Schwendler's platinum standard is excellent in theory, but cannot give satisfactory practical results, because of the modifications which platinum undergoes as a consequence of slightly prolonged incandescence. Under the action of the electrical current continuous modifications are produced in the platinum wire to which changes in electrical resistance correspond and, consequently, with the same current, changes in temperature.

Incandescent lamps, as they are now made, are free from a part only of these defects; the luminous intensity for a constant current varies, although very slowly; but the energy spent in the lamp is divided differently, according to the nature of the carbon, into calorific energy and luminous energy.

At the International Congress of Electricians in 1881, Violle proposed as the absolute standard of light the light emitted normally by 1 sq. cm. of platinum raised to its fusion point and about to solidify.

The phenomenon here employed has the advantage of being constant and susceptible of being always exactly reproduced. A liquid metal which is on the point of solidifying, and which is furthermore unalterable, like platinum, constitutes a body of fixed temperature. The temperature remains in fact invariable as long as any part of the mass remains liquid. If this metal is unalterable, like platinum, it will always have the same emissive power. With a given surface, it will always emit the same quantity of light. The quality of this light depends on the temperature; platinum, being the most refractory of ordinary metals, will be the one which, at its point of fusion, will give the whitest light.

As a consequence of the researches undertaken by Violle, which have demonstrated the exactness of the arguments by which he supported his proposition before the congress, the International Commission on Electrical Units, in session at Paris in 1883, definitely adopted as the absolute standard of white light the standard proposed by this French savant.

The Absolute Standard and Secondary Standards.

69. Before studying in detail the principal photometric standards, we must examine carefully the relative importance of an absolute standard and secondary standards. The difference between these two classes of standards is analogous to that which exists,

for instance, between the original meter and the original kilogram in iridium-platinum, and their ordinary copies in iron, brass, or wood.

The original meter and kilogram were constructed according to their definition with all the exactness that modern methods afford, and they have been kept in such a way as to guarantee perfect invariability. Since then national standards have been established which serve in each country for the comparison of the usual units of commerce and industry.

It should be the same with the unit of light, always with one difference. The prototypes of the metric system may be materially represented, and this representation is invariable at a given place. It cannot be the same with the absolute standard of light or with secondary standards; they must be constructed each time it is desired to use them, and to support them it is necessary to spend a certain quantity of energy.

The value of the standard of light depends on the energy spent and on the conditions under which this transformation of energy is effected. The only thing which may be done is to employ apparatus of definite dimensions, giving, under fixed conditions, the same luminous intensity.

The absolute standard and secondary standards should be as nearly as possible independent of the dangers mentioned above; this condition is necessary for the absolute standard; it renders its construction and reproduction difficult, so that its employment in industrial practice would not be dreamed of. Thus the employment of secondary standards becomes necessary. It is sufficient, in fact, to construct the absolute standard in a fixed place, and to make all the comparisons of secondary standards under the most varied conditions of working. The choice of an absolute standard being made judiciously, we may afterwards in secondary standards profit by all the improvements and simplifications taught by prolonged use, without introducing confusion into the measurements, since the value of secondary standards is always expressed as a function of the absolute standard. We employ as a unit of comparison the standard whose luminous intensity gives the most facility or precision in the experiments, according to the nature of the radiants to be compared. These secondary standards are then multiples and submultiples of the absolute standard, in the same way as in the metric system the kilometer is a multiple and the centimeter a submultiple of the meter.

It suffices if the secondary standard is of sufficient constancy for current practice. While we may reasonably require of the absolute standard a constancy of $\frac{1}{2}$ per cent, we may be satisfied with the secondary standard if its constancy proves to be within 2 per cent. These considerations show, indeed, that a uniform secondary standard is not necessary, and, in certain cases, would be even disadvantageous, since the known relation of each to the absolute standard brings about complete unity in photometric measurements.

The Mechanical Equivalent of the Unit of Light.

70. The standard of light being defined, it is interesting to investigate the quantity of energy in the luminous rays which it emits. This quantity is easy enough to determine.

It is enough to measure by an air thermometer the energy corresponding to the totality of the rays emitted by the radiant, then, by a thermo-electric pile, to measure the ratio between the energy of the luminous rays and that of all the rays. A simple multiplication then gives the energy corresponding to the luminous rays or the mechanical equivalent of the light emitted by this radiant.

This is the process which Tumlirz* employed to determine the mechanical equivalent of the light emitted by the flame of the Hefner ascetate of amyl standard. He found

$$k = 0.00361 \text{ small calorie per second,}$$

or transforming,

$$k = 151{,}500 \text{ ergs per second,}$$

which corresponds to the electrical work of a current of 0.1226 amperes in a conductor of 1 ohm.

This may also be expressed as follows: if the flame of the ascetate of amyl lamp is placed at a distance of 1 m. from an element of surface 1 cm. square, so placed that its normal is horizontal and passes through the center of the flame, the quantity of light which strikes this element of surface corresponds to a quantity of heat of 0.000000361 small calorie per second or 15.15 ergs.

If the eye replaces the element of surface and if we suppose that the pupil has an opening of 3 mm., the quantity of light which penetrates the eye corresponds to 1.07 ergs per second; this work

* *Wiedemanns Annalen*, Vol. XXXVIII. p. 640.

would require a year and 89 days to raise the temperature of a gram of water one degree C.

The value of the mechanical equivalent of the ascetate of amyl standard being determined, a simple photometric comparison allows us to find the mechanical equivalent of other standards of light. Tumlirz found in this manner the following value for the German candle:

$$k = 0.00447 \text{ small calorie} = 187{,}900 \text{ ergs.}$$

THE CARCEL STANDARD.

71. The carcel lamp is a simple modification of the Argand lamp; we know that in 1787 its Genevese maker produced a revolution in illumination, by replacing the flat wick burning openly in the air by a round wick giving passage through it and around its edge to a double current of air, produced by a metal chimney placed above the flame. Soon this metal chimney was replaced by a glass tube having at the top of the flame a constriction which forces the air into closer contact with the flame and which thus brings about complete combustion.

In 1800, Carcel made an important modification of the Argand lamp, giving it a very regular feed of oil. In this modification the oil reservoir is placed in the base of the lamp and the oil is raised to the level of the top of the wick by means of clock-work which operates two small pumps in the standard. The quantity of oil raised should be greater than that which is required for combustion, and the excess falls back to the reservoir; the wick, constantly wet with oil at the point where combustion is taking place, is charred very slowly and gives an almost constant light.

Since the mechanism is apt to get out of order, a regulating lamp is preferably employed, in which the pressure of a spring on a piston produces the same effect as the clock-work; the flow of oil is rendered sensibly constant by means of a small tube, fixed to the piston, in which there fits a fixed regulating rod which offers less obstruction in proportion as the piston is lower and the pressure of the spring less.

At the time of their photometric studies relative to the illumination of light-houses, Arago and Fresnel employed the oil lamp. Fresnel showed that by using certain precautions great constancy may be obtained within certain limits. It is well to remember the following interesting detail to show what care should be used in pho-

tometric measurements. Fresnel insisted, himself, on cleaning and caring for the lamps which he used; he took, further, the greatest precautions to insure the constancy and comparability of this standard.

The carcel lamp was next adopted by Dumas and Regnault for the photometric tests of the gas illumination of Paris. The good results obtained on this occasion and the authority of the two savants led to a general adoption of this unit of light by the gas companies in France. The conclusions of Dumas and Regnault were published after a long series of researches made in the municipal laboratory of Paris by Audoin and Bérard*; these last have shown the conditions which must be realized to insure the constancy and comparability of the carcel standard.

72. The luminous intensity of the carcel lamp depends on many circumstances of which the principal are the height of the wick, its nature, and the height of the constriction of the glass above the level of the wick.

Audoin and Bérard investigated the influence of each of these conditions on the intensity and consumption of the carcel burner.

TABLE I.

Height of the Wick.	Fine Wick.		Medium Wick.		Coarse Wick.	
	Consumption of Oil per Hour.	Consumption of Gas calculated to equal the Carcel burning 42 gr.	Consumption of Oil per Hour.	Consumption of Gas calculated to equal the Carcel burning 42 gr.	Consumption of Oil per Hour.	Consumption of Gas calculated to equal the Carcel burning 42 gr.
mm.	grams.	liters.	grams.	liters.	grams.	liters.
4	27	96	30	155	32	99
6	33	175	36	193	36	159
8	38	196	42	185	42	192
10	40	190	42	200	45	194
12	35	170	40	193	48	212
14	38	177	40	....	51	216
16	36	180	45	186	48	189
18	31	153	..	....	42	192

Table I. shows the results of tests made with various heights of the wick; Table II. shows the results of tests made with a constant

* *Ann. de Chim. et de Phys.* 3e série, Vol. LXV.

height of wick of 7 mm., but varying the position of the chimney. The relative luminous intensity is indicated by the number of liters of gas consumed by the gas-burner to give an equal light.

TABLE II.

Height of the Bend above the Level of the Wick.	Fine Wick.		Medium Wick.		Coarse Wick.	
	Consumption of Oil per Hour.	Consumption of Gas calculated to equal the Carcel burning 42 gr.	Consumption of Oil per Hour.	Consumption of Gas calculated to equal the Carcel burning 42 gr.	Consumption of Oil per Hour.	Consumption of Gas calculated to equal the Carcel burning 42 gr.
mm.	grams.	liters.	grams.	liters.	grams.	liters.
− 2	18	24	18	11	15	23
+ 3	25	63	21	57	27	....
7	36	187	39	161	48	175
12	39	199	42	200	50	186
19	42	151	45	175	51	164
24	46	315	45	161	54	140
29	flares.	....	51	133	.	....

The preceding figures show that the intensity of the carcel burner depends:

1°. On the height of the wick; as this increases, the consumption of oil and the luminous intensity increase up to a height of 10 mm. for the medium wick; above this limit the two quantities diminish.

2°. On the wick adopted; the medium wick is the best, for it gives the greatest luminous intensity for an equal consumption.

3°. On the height of the constriction of the glass chimney above the level of the wick; the elevation of the constriction tends to increase the consumption of oil in an always increasing proportion, but there exists a height of the chimney's neck which corresponds to a maximum of illuminating power. Thus with the medium wick in these tests, the height of the bend should be 7 mm. above the level of the wick.

Dimensions and Working Conditions of the Carcel Lamp.

73. The dimensions and working conditions of the carcel standard, as given by Dumas and Regnault in their practical instructions for gas testing, are given below (Fig. 40).

Extreme diameter of the burner	23.5 mm.
Interior diameter (for the interior current of air) . .	17.0
Diameter of exterior current of air	45.5
Total height of the glass chimney	290.0
Distance of the bend from the base of the chimney . .	61.0
Exterior diameter at the bend	47.0
Exterior diameter at the top of the chimney	34.0
Mean thickness of the glass	2.0

The wick adopted is the medium one, called the light-house wick; the strand is composed of 75 threads and weighs 3.6 grams per decimeter. The wicks should be kept in a dry place, or preferably in a box with a double bottom containing unslaked lime.

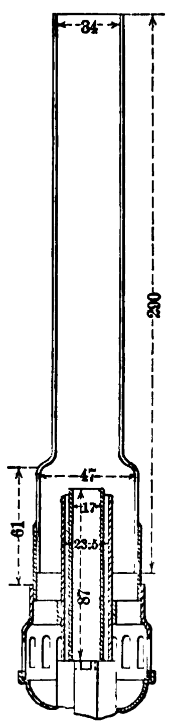

FIG. 40.—Burner of the Carcel Lamp.

The carcel lamp burns well-purified rape-seed oil. According to Crova, the composition of the rape-seed oil which is used in these lamps is apt to undergo only insignificant variations, for it is furnished by the grain of a particular vegetable, and its purity is more easy to control than that of other combustibles, such as stearic acid, spermaceti, paraffine, petroleum, and gas. This oil is purified by adding a small quantity of sulphuric acid, which coagulates the mucilage which it naturally contains and renders it more limped and more fluid.

The opinion of Crova seems somewhat optimistic, although its author has had occasion to employ the carcel lamp frequently in the course of his excellent photometric work. It should be remarked, in fact, that vegetable products are rarely of constant composition and are easily affected by external causes and even by the action of time alone, and that the process of manufacture is not absolutely definite.

74. For any combustible there is in general no definite relation between the quantity of material burned and the light produced; but adopting the dimensions given above for the burner and the chimney of the carcel lamp, and using a medium wick, it is noticed that the quantity of light increases in direct proportion to the consumption of oil, when this consumption

is in the neighborhood of 42 grams per hour; however, the consumption should not be below 38 grams, nor above 46 grams, for this proportion to hold good.

Two lamps having the same diameter of wick and the same capacity may differ in their consumption of oil and their illuminating power; further, it is known that the temperature, the movement of the air, the duration of the illumination, and the fullness of the lamp, have an influence on the quantity consumed. We should then, before using the carcel lamp as a standard, submit it to a series of tests to obtain the exact conditions under which it should be placed to obtain a nearly constant consumption.

These conditions obtained, we may proceed to definite photometric measurements. For each experiment we should insert a new wick which is trimmed even with the wick-holder; next the lamp is filled up to the beginning of the gallery, it is afterward lighted, keeping the wick at first 5 or 6 mm. high, and then the chimney is put on.

The consumption of oil is regulated by raising the wick to a height of 10 mm. and the chimney so that the bend may be at a height of 7 mm. above the level of the wick. To easily realize these conditions, we make the lower point of the small contrivance which is fitted to the wick-holder level with the wick itself, and the upper point level with a diamond scratch on the neck of the chimney.

The consumption and intensity increase slowly during the first half-hour, because of the heating of the burner; at the end of this time a constant state is established which lasts more than an hour; it is during this period that the photometric observations are made; then the consumption and intensity begin to decrease slowly in proportion as the wick becomes charred.

75. The lamp should consume 42 grams of oil per hour; when the consumption falls below 38 grams or rises above 46 grams, the tests should be rejected. When the consumption is maintained between these limits, the luminous intensity is reduced by a simple proportion to that which corresponds exactly to 42 grams per hour.

To regulate th consumption of the lamp, we suspend it at the end of one of the arms of a balance having on the other arm a counterweight; equilibrium being obtained at any given time, a weight of 10 grams is placed on the side of the lamp; when this weight of oil is consumed, the balance returns to its position of equilibrium; now the point of the balance has a notch which at the moment of equi-

librium causes the fall of a hammer; this, striking a bell, notifies the experimenter, who reads on a seconds counter the time necessary for the burning of 10 grams of oil. The apparatus was constructed by Deleuil, who gave it a very practical form. (Fig. 54, p. 165.)

The normal consumption of the carcel standard being 42 grams per hour, it requires 14 minutes and 17 seconds to burn 10 grams. The seconds counter permits the determination in each experiment of the rate per hour at which the lamp consumes oil and indicates whether we are within the prescribed limits. If, for instance, the counter indicates 15 minutes 30 seconds, the proportion

$$10 : 15.5 :: x : 60$$

gives immediately 38.7 grams as the rate per hour of consumption of oil.

The carcel standard is only comparable with itself on condition that we adopt the dimensions indicated above. In the carcel lamp, the layer which emits the light has the form of a hollow cylinder included between two cylindrical layers, one outside, the other inside, where the combustion of hydrocarbons takes place at a very high temperature without the deposit of carbon. In the intermediate layer which radiates the light, the hydrocarbons are dissociated with the formation of solid incandescent carbon; this last is raised to a temperature higher in proportion as the temperature of the two non-luminous layers between which it is found is higher, which happens when the draught of the chimney is very active. This shows well the importance of an exact determination of the conditions of combustion.

Practical Value of the Carcel Lamp.

76. Opinions are much divided as to the practical value of the carcel standard; while French engineers think its qualities unrivalled, in other countries other photometric standards are used. In connection with this, the discussions which took place at the International Congress of Electricians, in 1881, and at the International Conference of 1882, are very interesting.

The French representatives, Dumas, Allard, Crova, etc., all insisted on the advantages of the carcel lamp, — advantages scarcely recognized by foreign savants, who put this standard on a par with the candle.

The numerous experiments of Crova showed that two very differ-

ent carcel lamps, compared within an hour, give indications whose variations amount to from 2 per cent to 3 per cent at most.

According to Leblanc*, the employees charged with testing the gas at the municipal bureau of Paris easily acquire such experience that they regulate the consumption of the oil very exactly between 41 and 42 grams per hour. It follows, from the measurements made daily in this bureau, that the carcel lamp, well cared for and carefully manipulated, can give good results, quite comparable with one another.

However, this agreement does not happen with lamps, wicks, and rape-seed oil of different origins; it is probably to this that we must attribute the great differences which have been several times found to exist by foreign engineers who have used the carcel lamp in their photometric comparisons, but without following exactly the very minute directions given by Dumas and Regnault.

It is of the greatest importance to observe exactly all the precautions mentioned by these two scientists; then only may we have results truly comparable to those of other observers, working under analogous conditions.

Leblanc recommends the use of two lamps, which are used on alternate days; when a lamp remains unused for several days, the oil thickens and the mechanism gets out of order.

We have considered so far only the opinions of those who favor the employment of the carcel lamp, after having used it for a long time. However, we must not be blind to the fact that this standard has also serious inconveniences.

To give an idea of them, we cannot do better than to cite the following passage from a communication of Hartley † to the British Association of Gas Managers, in 1880:

"The very great number of tests that I made in 1867 with a carcel lamp does not encourage me to give credit to the indications of any lamp in which vegetable oil is used. I say vegetable oil because some recent experiments with lamps burning paraffine oil have shown great uniformity in their illuminating power.

"The objections which I have to standard lamps are that they must be kept in a state of perfect cleanliness; the wick must be renewed very often, if not each time the lamp is used (this last point is essential with the carcel lamp); the wick as well as the

* *Procès-verbaux de la Conférence Internationale*, 1882, p. 145.

† *Journal des usines à gaz*, 1882.

chimney must be adjusted with the greatest care and exactness, and finally, when all this has been done, there is no certainty that the quantity of oil consumed will not be greatly in excess of the regulation quantity.

"This variation in the consumption would have no effect if, as my experiments have shown me, the quantity of light emitted did not often increase in a much more rapid proportion than the consumption of oil. It is, furthermore, very difficult to keep the consumption of the lamp as low as the regulation rate."

In spite of all the fault that may be found with the carcel lamp, it has remained, nevertheless, a practical standard of light which has rendered great service in industrial photometric comparisons; its intensity and color are about the same as those of the gas-burners generally employed.

In the comparison of arc lights, the question becomes complicated, for the flame of the carcel lamp is still too red to render insensible the differences of color which enter so largely into photometric comparisons.

CANDLES.

77. The candle, although giving poorer results than the other standards of light with respect to the constancy of the light emitted, has enjoyed up to the present the greatest favor.

Petroleum lamps and other oil lamps furnish a quantity of light which depends on the dimensions of the wick, than which there is nothing more varied, while there exists a certain uniformity in the composition and dimensions of the candles which commerce produces in such great quantity. Hence the employment of this radiant as the usual photometric standard.

Practice has recognized the use of four different candles furnishing, even under identical conditions, unequal quantities of light. It is necessary, then, when speaking of candles, to specify the one meant; this is a precaution which unfortunately has not been observed.

These four kinds of candles are:

1°. The Stearine Star candle, which is used in France along with the carcel lamp.

2°. The London Standard Spermaceti candle, which is used in England and the United States.

3°. The candle of the Union of German gas-men, "Vereinskerze," a paraffine candle, which is much employed in Germany and Austro-Hungary.

4°. The Munich Stearine candle, in form slightly conical, which is in use in Germany along with the paraffine candle.

There should be added also the *decimal candle* defined by the Congress of Electricians, in 1889, as the twentieth part of the absolute platinum standard.

Combustion of the Candle.

78. The luminous intensities of these candles, burning under normal conditions, have been determined many times by a great number of observers who have compared them with one another or with the carcel lamp. The values obtained are very different; it is the same with the conclusions relative to the variability of the luminous power of the candle.

Thus while Schwendler says that he has found variations of from 40 to 50 per cent in the intensity of an English candle, Hugo Krüss says that, taking certain precautions, one may easily succeed in keeping the light emitted by the flame of a paraffine candle constant within 5 per cent.

The luminous intensity of the flame of a candle, whatever be the nature of the combustible of which it is made, depends on the form and nature of the wick, which vary greatly with the mode of manufacture.

The wick is generally formed of many strands of cotton, braided and saturated beforehand with a solution of boric acid. When the candle is lighted, the wax melts, rises by capillarity in the wick, and is decomposed into products rich in hydrocarbons which burn, and into carbon which is precipitated in a solid and finely divided state in the middle of the flame.

The gaseous envelope, in contact with the air, burns completely without precipitation of carbon, at a very high temperature and without sensible emission of light. The temperature of this layer is very high, and certainly approaches, according to Crova, that of the fusion of platinum.

The carbon precipitated in the middle of the flame at a very high temperature undergoes combustion, which is produced with a great elevation of temperature, accompanied by a strong emission of light by the incandescent carbon.

The axis of the flame, relatively cold, is composed of pyrogenous products not yet dissociated; it is in this axial part and in the luminous envelope that the upper part of the wick is found which undergoes gradual carbonization; as it bends over, it approaches the outer part, where it is completely burned; the boric acid which the wick contains then melts and vitrifies the ashes of the cotton in the form of small globules, whose weight bends the wick outside the flame, and thus brings about its complete combustion.

The wick, then, undergoes continual changes in form and position; hence the variations in the state of the flame which is unequally chilled and modified in its form: this may be proved by placing before a Foucault photometer a carcel lamp and a candle. If the lamp is so regulated as to insure the greatest possible constancy, we may follow, on the photometric screen, the variations of intensity of the candle, and determine their agreement with the corresponding form and position of the wick.

The influence of torsion in the wick is also very sensible; as far as possible, the wick, quite regularly braided and made of a well-determined number of strands, should be placed without torsion in the middle of the candle; if this condition is not rigidly complied with, the curvature of the wick in the flame changes continually in direction, sometimes even abruptly; there then result very considerable variations in the luminous intensity of the flame.

The movement of the air exerts a very great influence on the light of the candle; if the air is even slightly stirred, the variations are very great.

If, to avoid these disturbances, the candle is enclosed in a blackened box, having openings planned to remove the products of combustion and to admit fresh air, the ascending movement of the air in the box also exerts a very notable influence on the composition and intensity of the light emitted.

The more rapid the movement, the more the exterior non-luminous envelope of the flame develops, the higher also the temperature of the middle layer, which radiates the light, rises, and its mass becomes less, so that as the draught increases, the reddish yellow of the flame becomes more and more white and less and less bright. This effect may be shown by exaggerating it; it is sufficient to surround the candle with a large glass tube acting as a chimney, whose draught produces in a more marked manner the effects indicated above. It is, then, necessary to place the photometric candle in air perfectly quiet and free; but these are conditions difficult to

realize in industrial practice. Variations of the temperature of the photometric chamber and of the barometric pressure modify the conditions of combustion of the candle: consequently they have an influence on its luminous intensity.

In order to become as independent as possible of the variations indicated above, the quantity of material burned by each candle per hour was specified. However, it was not long before it was noticed that this condition is not sufficient in all cases, and there was then added the height which the flame must have during the measurements in order that the luminous intensity may be constant.

The Star Candle.

79. The French Star candle burns 10 grams of material per hour. Péclet, in 1830, compared the first candles made by de Milly; he found that they gave a light whose intensity was equal to $\frac{1}{7}$ carcel. Candles of this quality are no longer to be found; the best candles made in France do not equal more than $\frac{1}{8}$ carcel.

According to Monnier, the employment of the Star candle as a photometric unit requires not only a consumption of 10 grams per hour, but a height of flame of 52.4 mm. These candles come 5 or 6 to the package.

The candles with 5 in a package weigh 100 grams each; their dimensions are: total length, 306 mm.; length of cylindrical part, 290 mm.; diameter above, 20 mm.; diameter below, 22 mm.; the wick is composed of 81 threads.

The candles with 6 in a package weigh 83.3 grams; their dimensions are: total length, 274 mm.; length of cylindrical part, 258 mm.; diameter above, 20 mm.; diameter below, 21.5 mm.; the wick has 81 threads.

The comparisons of Star candles with other candles not being numerous on account of their comparatively small employment, we shall give immediately the most probable values of this standard of light as a function of the normal carcel lamp.

Monnier found the following mean values. A Star candle equals 0.136 carcel with a mean consumption of 10 grams per hour, and 0.136 carcel also with a height of flame of 52.2 mm. For candles with 6 in a package, the corresponding values are 0.131 and 0.132 carcel.

In the same way 1 normal carcel equals 7.4 candles of 5 to the package, or 7.6 candles, 6 to the package, taking as the normal candle that which consumes 10 grams of stearine per hour, or that which gives a flame 52.5 mm. in height.

The English Candle.

80. The English photometric candle is the spermaceti candle, 6 to the pound, burning 2 grains of material per minute, or 120 grains (7.776 grams) per hour. Schwendler says 8.26 grams. The dimensions of the candle are: length, 252 mm.; diameter at top, 20 mm.; at bottom, 22.5 mm.; mean weight, 75.7 grams.

When the real consumption of the candle differs from this figure, and is between 114 and 126 grains per hour, we assume that the illuminating power is proportional to the consumption, and correction is made by means of a simple proportion. The wick is made of three strands of cotton, each containing from 18 to 21 threads, according to the brand.

The height of flame adopted is 45 mm. The composition and purity of spermaceti are liable to considerable variation, according to the source and method of refining.

Thus Heisch and Hartley mention the fact, with the proof, that spermaceti candles now give more light for the same weight of matter burned than formerly. This is due to small improvements in the wicks or to progress in the treatment of spermaceti.

The German Candle (Vereinskerze).

81. The German Association of Gas and Water Industries adopted in 1868 as photometric candle a paraffine candle of 6 to the pound, having a uniform diameter of 20 mm.; its length is 314 mm., and its weight 83.6 grams. The melting-point of the paraffine employed is 55° C. The wick is made of a twist of 25 threads of cotton; a meter of wick weighs 668 mg. The illuminating power of the candle depends on the height of the flame; unity corresponds to a flame 50 mm. high.

The melting-point of paraffine is quite variable, and oscillates between 55° and 65°; it may even reach 80°, or fall to 44°. These variations in the melting-point oblige the manufacturers to add, in certain cases, from 10 to 15 per cent of stearine. It is necessary then, in order to have a constant candle, to be sure that its composition corresponds closely to the conditions of purity mentioned above.

The Munich Candle.

82. The Munich candle conforms to the type of candles specified in the contract made between the city of Munich and the gas

company. They are stearine candles; their form is slightly conical; they are 20.5 mm. in diameter at the top, 23 mm. at the base, 31 cm. long, and weigh 108.9 grams on the average; the wick is made of 50 threads.

They should consume 10.2 grams to 10.6 grams of stearine per hour, without smoking and without requiring snuffing; the height of the flame is 56 mm.

Variations in the Luminous Intensity of the Candle with the Height of the Flame and the Consumption of Combustible Material.

83. The consumption of the candle is not sufficient to characterize its luminous intensity; the height of the flame should also be indicated. The employment of candles requires the maintenance of the flame at its normal height; this result is obtained by snuffing the wick at intervals sufficiently close for the flame to keep its normal height during the time of measurements; for the variations of this normal height are exceedingly slow when the wick has attained its normal state soon after having been snuffed.

As doing this produces a perturbation in the combustion of the candle, it would be preferable to wait until the flame reaches its normal height; but the delay would in general be too long, and thus much time would be lost, the normal height adopted for candles not corresponding exactly to the height of the free flame.

As an example we give in the following table* the results found by Krüss with a certain number of candles:

MUNICH STEARINE CANDLES.

Prescribed Height, 52 mm.

	Limits of the Height of the Flame.	Mean Height of Flame.	Mean Deviation from the Mean.	Sum of the Successive Variations in the Height of the Flame.
No. 1	53 to 60 mm.	55.09 mm.	± 1.07	58 mm.
No. 2	51 " 57 "	54.2 "	± 1.02	50 "
No. 3	51 " 59 "	55.15 "	± 1.27	49 "
No. 4	49 " 54 "	50.65 "	± 0.93	41 "
Total or Mean . .	49 to 60 mm.	54.0 mm.	± 1.98	198 mm.

* *Journal für Gasbeleuchtung*, 1883, p. 511.

GERMAN CANDLES (VEREINSKERZE).

Prescribed Height, 50 mm.

	Limits of the Height of the Flame.	Mean Height of Flame.	Mean Deviation from the Mean.	Sum of the Successive Variations in the Height of the Flame.
No. 1	51 to 63 mm.	54.0 mm.	± 1.35	63 mm.
No. 2	49 " 56 "	52.5 "	± 1.52	61 "
No. 3	47 " 55 "	50.8 "	± 1.62	62 "
No. 4	50 " 60 "	55.3 "	± 1.73	90 "
Total or mean . .	47 to 63 mm.	53.1 mm.	± 1.98	276 mm.

ENGLISH SPERMACETI CANDLES.

Prescribed Height, 44 mm.

	Limits of the Height of the Flame.	Mean Height of Flame.	Mean Deviation from the Mean.	Sum of the Successive Variations in the Height of the Flame.
No. 1	46 to 52 mm.	49.8 mm.	± 1.20	36 mm.
No. 2	46 " 50 "	47.5 "	± 0.92	41 "
No. 3	45 " 50 "	47.8 "	± 0.75	39 "
No. 4	44 " 49 "	45.5 "	± 0.83	41 "
Total or mean . .	44 to 52 mm.	47.67 mm.	± 1.57	157 mm.

These results show the differences which exist between the prescribed height of flame and that which is obtained in reality; for stearine candles, the most frequent height of flame is between 54 and 56 mm., for paraffine candles between 52 and 54 mm., and for spermaceti candles between 47 and 48 mm.

The figures in the first column show also that the spermaceti candles differ less from one another than the others; the figures in the fourth column give, on the contrary, information as to the amount of the variations of the height of the flame and its regularity. Thus it is seen that the English spermaceti candle is much superior to the two others, as to the regularity and small amount of variations of each candle. Certain measurements made at the electro-technic station of Munich, under the direction of Voit, confirm Krüss's conclusions.

These conclusions clearly show that it is necessary to snuff the wick to obtain the normal height of flame.

We also give the results obtained by many observers concerning the mean values and the variations of the height of the flame of different candles.

MEAN HEIGHTS OF THE FLAME (IN MM.).

Candle.	Rudorff.	Schiele.	German Commission.	Krüss.	Monnier.	Voit.
Stearine . . .	56.0	50.3	60.8	54.0	55.0	59.3
Paraffine . . .	50.0	50.0	51.2	53.1	50.8	50.4
Spermaceti . .	52.2	52.0	....	47.7	46.0	44.8

VARIATIONS IN THE HEIGHT OF THE FLAME (PER CENT).

Candle.	Rudorff.	Schiele.	German Commission.	Krüss.	Voit.
Stearine	5%	8%	35%	20%	5%
Paraffine	8	20	35	30	4
Spermaceti	7	17	....	17	3

Some measurements were made at the electro-technic station of Munich to determine the variation of luminous intensity with the height of the flame; and, further, an attempt was made to express this luminous intensity as a function of the height H, by the formula

$$I = a + bH.$$

The results obtained are given below; they are very interesting, but the measurements are not numerous enough to draw from them absolute conclusions.

Candle.	Height of Flame.	Number of Measurements.	Formula Expressing the Luminous Intensity.
Munich . . .	47–55	87	$I = 0.0068 + 0.0192\,H \pm 0.035$
	42–62	149	$I = 0.0120 + 0.0190\,H \pm 0.064$
German . . .	39–53	197	$I = -0.0300 + 0.0206\,H \pm 0.058$
	42–57	138	$I = 0.0350 + 0.0193\,H \pm 0.043$
English . . .	32–52	119	$I = 0.0077 + 0.0223\,H \pm 0.050$
	32–49	81	$I = 0.0121 + 0.0222\,H \pm 0.082$

Measurement of the Height of the Flame.

84. The height of the flame being so important an element, it should be possible to measure it easily.

Direct measurement of the height of the flame by means of a pair of compasses is scarcely practicable, because of the proximity of the observer and contact of the measuring instrument. The employment of the cathetometer gives excellent results; but this instrument is reserved for laboratory work of great precision.

The height may be measured by placing a divided scale behind the candle and observing it with a telescope. On this principle, Krüss has constructed the following apparatus* which gives good results (Fig. 41).

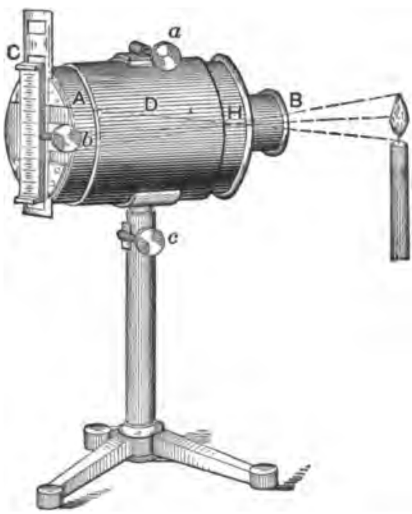

FIG. 41.—Optical Scale for Measuring the Height of the Flame.

At one of the ends of the tube A is the achromatic objective B, while the other end is fitted with a glass scale C.

The distance from the center H of the objective to the divided scale should be equal to twice the focal length of the objective; the longitudinal movement of A is governed by the screw a, and the vertical movement of the glass scale by the screw b.

To make a measurement, the apparatus is so placed that its

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* *Journal für Gasbeleuchtung*, 1883, p. 717.

objective is at the same distance from the candle as from the divided scale; next, by means of the screws *a* and *b*, the image of the candle is regulated until it is clearly visible on the glass scale. At this instant the image of the flame is equal to the original in size, and its height is read directly on the scale.

Precise measurements of the height of the flame of the candle present some difficulty on account of its irregularity. The lower edge of the flame cannot always be seen; this edge, of bluish color, is frequently hidden by the edges of the paraffine or stearine cup which is formed about the wick. Further, it is rarely of the same height on different sides. Finally, the top of the flame generally has three points, especially when the height is considerable; the middle point is larger and longer than the lateral points; but very frequently one of these flares, especially when the flame is about 50 mm. high. Measurements are then impossible.

Measurement of the Consumption of Candles.

85. In general, for normal candles, a definite hourly consumption is prescribed; the height of the flame plays a still more important *rôle;* in fact, unless the candle burns freely and uniformly, a fixed consumption of material is out of the question.

Now in all photometric observations it is necessary to snuff the candle; at every operation of this kind its consumption is changed, so that the height of the flame, whose variations may be followed, should be the characteristic element of the luminous intensity of the candle, and not its consumption, which is modified at each snuffing of the wick.

Below are some figures given by Krüss concerning the consumption of various candles first burning freely, then snuffed at regular intervals.

Candle.	Burning freely.	Snuffed.
Stearine	10.20 grams.	8.78 grams.
Paraffine	7.34 "	7.61 "
Spermaceti	7.265 "	7.45 "

The consumption of material may be measured by means of Deleuil's apparatus employed for the carcel lamp. We give, besides

(Fig. 42), the description of a particular balance* constructed by Krüss and specially adapted to determining the consumption of candles.

The arms of the balance are in the ratio of 1 to 2; the two candles which generally serve as a double standard, in order to diminish the variations of luminous intensity, are placed at the end of the shorter, so as to diminish proportionally the vertical displacement of the flame during the comparison. The double candlestick *A* may move vertically within wide limits. The box *C* encloses a Leclanché cell, one of whose terminals is connected

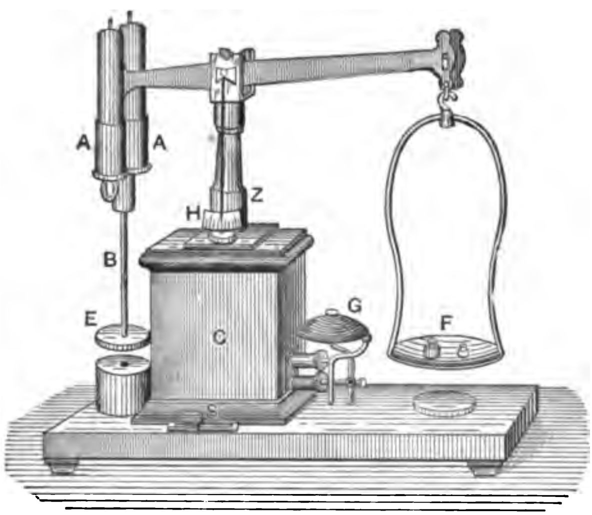

Fig. 42.—Candle Balance.

through the electro-magnet of the bell *G* to the index *Z*; the other terminal is connected to a small pin *H*, which, when it is in contact with the index *Z*, closes the circuit of the cell, and this rings the bell. The method of employment of this apparatus is the same as that of Deleuil's balance.

Another apparatus to register the consumption of a candle has been employed frequently, by Monnier, among others. It is Elster's areometer, whose construction is well known.

To make a measurement, when the candle has reached its normal condition, the pan is loaded until the areometer rests on the bottom

* *Journal für Gasbeleuchtung*, 1885, p. 345.

of the vessel; the candle in burning becomes lighter, and the areometer rises; when it passes the zero of the scale, the seconds-counter is started, and the areometer is allowed to rise, the position of the index being noted from minute to minute, which permits one to ascertain whether the consumption of the candle is regular.

Fusion-Point of Stearine.

86. A knowledge of the height of the flame, combined with that of the hourly consumption of the candle, permits us to calculate the luminous intensity in terms of its normal value. These two points, the first especially, are of the greatest importance. A third should, however, not be neglected; it is the degree of purity of the material of which the candle is made. In this regard the point of fusion and of solidification furnishes a valuable basis on which to measure the value of the material of the candle, for the presence of foreign bodies modifies this temperature considerably.

Below are the values of the fusion-point of common candles, according to the measurements of Krüss and Rudorff: the figures are the mean of a very large number of measurements.

Candle.	Krüss.	Rudorff.
Munich stearine	54.0°	53.0° to 56.6°
German paraffine	53.7°	49.0° to 54.0°
English spermaceti	43.7°	43.5° to 44.3°

The method most used to obtain the fusion-point of greasy materials consists in enclosing the substance to be studied in a capillary tube, then heating it in a water-bath, measuring the temperature of the latter.

The fusion-point is considered to be the temperature at which the substance becomes transparent or detaches itself from the tube and begins to move. This process is not exact.

Another more exact method has been proposed by Loewe. An electric bell and a bath of mercury are placed in the circuit of a cell; a thermometer and two platinum wires are introduced into the mercury; one of the wires, ending in a ball, is covered with a thin layer of the greasy substance, and the circuit is established as soon as the fusion-point is attained.

Luminous Intensity of Standard Candles.

87. It is difficult to express numerically the relative luminous intensity of various candles, there being as many different values as observers. One of the principal causes of these differences should be sought in the lack of constancy in the luminous intensity of these photometric standards.

We have already said that Schwendler found variations of 40 per cent in luminous intensity; we must believe, however, that he took account neither of the height of the flame nor the consumption of the candle.

At the other extreme is Krüss, who claims to have been able to maintain the constancy of the light emitted by the candle within 3 per cent; this figure is evidently more favorable than can be obtained in industrial tests, where one can scarcely observe all the minute precautions used by this renowned German specialist. But let us mention the results of other observers.

Dibdin, after extended investigation of luminous standards, in his report to the Metropolitan Board of Works of London, claims that candles give uniform results only by accident. Thus, in 454 measurements made with candles, 154, or 34 per cent only, gave results differing by less than 10 per cent from the mean.

Heisch and Hartley, in an investigation of the same subject, found deviations of from 1.3 to 16 per cent from the mean, the mean deviation being about 7 per cent. On the other hand, Foucart found that, in his experiments, the intensity of the Star candle varies from 9.9 per cent above to 13.9 per cent below the mean, the total variation being 23.8 per cent.

An English commission, including Williamson, called attention, in its report to the Board of Trade, to the fact that candles taken in two different packages, or even in the same package, coming from the same factory, may give variations of from 14 to 15 per cent.

We see then that it is scarcely possible under these conditions to give exact figures for the relative values of the different candles. So we shall limit ourselves to briefly indicating in the following table the relative values obtained by different observers.

HEIGHT OF FLAME OF 44.5 MM.

Candle.	Rudorff.	Buhe.	Krüss.	Monnier.
Munich	100.0	100.0	100.0	100.0
German	107.9	106.4	106.0	87.5
English	108.7	108.7	104.5	78.4

HEIGHT OF NORMAL FLAME.

Candle.	Schilling.	Krüss.	Voit.	Schiele.
Munich	100.0	100.0	100.0	100.0
German	88.7	97.6	96.5	92.0
English	90.7	85.8	94.4	. . . . .

We give in conclusion the results obtained by Monnier relative to the value of the luminous intensity of ordinary candles, indicating the height of the flame and hourly consumption; all the numbers are expressed in terms of the normal carcel lamp.

Candle.	Height of the Flame.	Consumption (mean hourly).	Value (mean in Carcels).
English	46.0 mm.	7.8 grams.	0.120
German	50.0 "	7.5 "	0.134
Munich	55.0 "	10.4 "	0.153
Star	52.4 "	10.0 "	0.134

Petroleum Lamps.

88. The measurement of the intensity of very intense radiants can scarcely be made by direct comparison with the standard, candle or carcel lamp, because of the great difference in the intensities; it is advantageous to employ an intermediate photometric standard whose constancy is sufficient throughout the duration of the tests, and which has a luminous intensity between that of the standard and that of the source studied.

First among the light sources which realize the conditions required of an intermediate photometric standard should be mentioned

petroleum lamps. Round-wick petroleum lamps are nowadays universally in use. The most simple form is that which is identical with the carcel lamp, — a round wick with a double current of air and constricted glass chimney.

We cannot give a detailed description of the principal types of petroleum lamps*, of which there is a great variety. We should bear in mind, however, that the majority of them have been brought out without taking sufficient account of the rational principles which are at the basis of the construction of lamps which should give a fixed and intense light with a minimum consumption of petroleum.

Among the round-wick lamps should be mentioned those which have a central disc designed to enlarge the flame and to promote combustion; the chimney should not be constricted, to obtain a maximum luminous output.

The petroleum lamp has many advantages over the carcel lamp. First, it has no pump designed to raise the oil to the top of the wick; petroleum being very fluid, it rises in the latter by capillarity with sufficient rapidity.

It has been found, for instance, that a mineral oil having a density of 0.85 rises in the wick with sufficient rapidity to feed a normal flame, even when the height to be ascended reaches 200 mm. Further, the flame is quite at the top of the wick, so that the latter chars much more slowly; it is not necessary to renew it for each measurement, and it is sufficient to clean it by wiping with a rag, without cutting it.

It appears from precise measurements that the increase in the density of the petroleum, which is produced after the lamp has burned for some time, has no sensible influence on the luminous intensity. The diminution of this depends principally on the conditions under which the lamp works. When first lighted, the petroleum is near the flame, and the wick is not yet charred; the oil and the lamp are still cold. At the end of four or five hours the level of the oil has lowered considerably, but its temperature and that of the lamp have increased, which is a compensation. If we represent graphically the variation of the luminous intensity with the time, we obtain, in general, a curve which rises in the beginning and then descends very slowly.

* See *Dingler's Journal*, Vol. CCLV. p. 39; Vol. CCLXIII. p. 374; Vol. CCLXVII. pp. 145 and 265.

At the International Congress of Electricians, in 1881, Wiedemann extolled the petroleum lamp for photometric measurements; in general, all who have used it as an intermediate standard have nothing but praise for it.

Thus von Hefner-Alteneck warmly recommends the use of the petroleum lamp with a round burner or with the intensive burner; the photometric measurements made in the Siemens and Halske laboratory have shown that this standard gives a uniformity quite sufficient for the majority of industrial measurements; the quantity of petroleum used has very little influence on the variations of luminous intensity; the intrinsic intensity increases, however, with the fluidity of the liquid.

Owing to government regulations as to mineral oils, the refined petroleum of commerce is of a sufficiently uniform quality. As, however, we compare at each measurement the intensity of the petroleum lamp with that of the photometric standard, the composition of the petroleum has no influence except as regards the constancy of the light emitted.

Krüss compared with one another two ordinary round-burner lamps of equal dimensions. In the space of one hour, the greatest variation in the luminous intensities of these lamps was 1.7 per cent, and the mean variation was 0.5 per cent.

Liebenthal obtained less satisfactory results by comparing a petroleum lamp with the von Hefner-Alteneck acetate of amyl lamp; the mean error of each observation was found to be about 3 per cent. It is possible, however, that the majority of these variations should be attributed to the variations of the Hefner standard.

The remarks with respect to the influence of the height of the flame upon the luminous intensity of candles and of the carcel lamp are evidently applicable to petroleum lamps. However, the variations in the height of the flame in the last are very slight.

We should, in general, give to the flame the maximum height which it may have without flickering; in this case, the variations in the height of the flame are the least sensible, and the lamp burns under the best conditions.

It is, moreover, easy to guard against variations of luminous intensity produced by variations in the height of the flame. The petroleum lamp serves only as an intermediate standard, and its absolute luminous intensity has no importance from a practical point of view. For this reason, we may very advantageously employ

a species of screen of the Methven genus (§ 96), which allows only the central part of the flame of the lamp to be seen. Variations in the height of the flame then exercise only an insensible influence on the luminous intensity of the standard. The most simple screen consists of a blackened metallic tube, with an opening of determined size, which is placed on the lamp coaxially with the ordinary glass chimney. The dimensions of the opening in the opaque tube are so chosen as to let pass only the rays of light emitted by the central part of the flame, little affected by variations of height.

In closing, we give certain data on the luminous intensity of several petroleum lamps, — data which we have selected from a work by Heim * upon common lighting apparatus, and from a research of Dolinin and Alibegow †, on lamps fed by Caucasian mineral oils (Nobel kerosene, density 0.822, at 15° C.; Ragosin naphtha, density 0.858, at 15° C.).

Heim.	Diameter of Wick-holder in mm.	Intensity in Normal Candles.	Consumption of Oil per Hour per Candle.
Ordinary round burner	25	16.1	3.37 grams.
Victoria burner with central disc,	30	19.2	3.30 "
" " " " "	62	67.3	3.40 "
Cosmos "	30	22.9	3.70 "

The above lamps consumed refined American petroleum (density 0.796, at 18° C.).

Among the best central-disc burners should be mentioned the Mondbrenner of Schuster and Baer with a wick-holder of fourteen lines (1 line = 2.256 mm.); fed with kerosene, it gives a luminous intensity of 14.88 candles for an hourly consumption of 3.56 grams per candle. [Dolinin and Alibegow.]

This burner has a central disc, and, at the base of the wick-holder, lateral channels planned for cooling the metal work. The greatest variation of intensity observed with this lamp was 1.32 candles (from 14.36 to 15.68 candles).

* *Lum. Él.*, Vol. XXVI. p. 220.

† *Dingler's Journal*, Vol. CCLXVII. p. 265.

THE HEFNER ACETATE OF AMYL LAMP.

Benzine Lamps.

89. In the carcel lamp, as in candles, the wick is one of the principal causes of irregularities of the flame. It should then be suppressed. But to obtain a standard flame without a wick, or one in which its influence is reduced to a minimum, recourse should be had to easily combustible liquids, which become volatilized by the heat of the flame and burn in the form of vapor, unlike rape-seed oil, petroleum, etc., which require the direct action of the flame on a wick saturated by the liquid.

Benzine or ligroin lamps have been employed as secondary photometric standards. Thus in 1877 Eitner* declared himself pleased with the use of a small lamp fed with benzine which gave satisfactory results; the round and compact wick is placed in a very thin tube of brass and extends about 10 mm. beyond it; the latter is moved by means of a rack and pinion in a second tube which serves to limit the flame; the wick, 7.5 mm. in diameter, is brought to the level of the outer tube, and the combustion of the benzine is effected without charring the wick too much. A platinum sight serves to regulate the height of the flame. The luminous intensity of this lamp is in the neighborhood of one candle. Ordinary benzine lamps may render excellent service in certain photometric studies.

There are to be found, for instance, in the market, small benzine or spirit lamps which give good results as photometric standards, as Uppenborn† has shown.

By means of a grooved sheet of metal, the glass is so placed that the upper edge has a constant determined height above the wick-holder (*e.g.* 45 mm.). The edge of the glass then serves to sight the top of the flame, and its height is maintained constant by means of the rack and pinion.

When the height of the flame is kept constant, these small lamps furnish light of remarkable constancy; it is sufficient to compare them from time to time with one of the usual standards.

Eitner's lamp, as well as all those which are based on the combustion of mineral oils, is affected by the same cause of error. These

* *Centralblatt für Electr.*, 1885, p. 711.
† *Lum. Él.*, Vol. XXVIII. p. 532.

liquids are not of well-determined chemical composition, but are mixtures of different substances having different boiling-points and variable compositions; they cannot be obtained in conditions which are always identical. They have further the disadvantage of not burning uniformly, the combustion at first being at the expense of the most volatile materials; there remains finally a product volatilizable with difficulty, which requires other conditions of combustion to furnish the same flame.

These considerations and a great number of tests of different liquids induced von Hefner-Alteneck * to take as a combustible acetate of amyl. This liquid is fluid and possesses a very intense, agreeable odor. It may be easily obtained pure by distilling crystallizable acetic acid, or an acetate, with sulphuric acid and amylic alcohol. It is manufactured in great quantities for perfumery; its boiling-point is very constant at 138° C., and its price is not high.

The Hefner Lamp.

90. The von Hefner-Alteneck lamp is a simple spirit lamp (Figs. 43 and 43 *bis*); the inventor retained the wick because the lamp is manipulated more easily, and because, further, the wick does not char in burning acetate of amyl; its object is, in fact, simply to suck up the liquid which is disengaged as vapor when the temperature reaches 138° C.

The wick-holder is a German silver tube, 8 mm. in interior diameter, 0.15 mm. thick, and 25 mm. high.

The normal intensity of the lamp is determined by the height of the flame; this height is normally 40 mm., or five times the diameter, measured from the top of the wick-holder; it is regulated by raising the wick more or less in the latter. The normal height is kept by means of a sight fitted to the lamp.

The flame should burn freely in the air, without a glass chimney; however, a straight glass tube is sometimes used, 88 mm. in height, and 55 mm. in diameter; under these conditions the luminous intensity of the 40 mm. flame diminishes 2 per cent.

The wick should be made with great care; it should exactly fill the German silver tube without being crowded. One may make it himself by placing parallel to one another ordinary cotton threads, until the required diameter is reached. It is not advantageous to

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* *Lum. Él.*, Vol. X. p. 501; *Electr. Zeitsch.*, Vol. IV., 1883; Vol. III. p. 20, 1884.

employ, as has been proposed, a wick whose end is made of threads of amiantus [a fine kind of asbestos]; the complication which results is not compensated by the slightly greater uniformity of the light

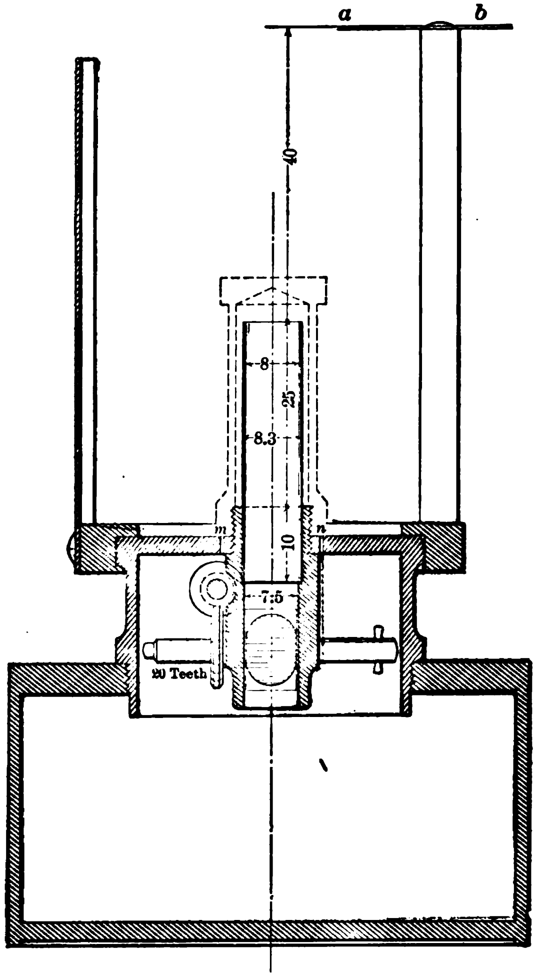

FIG. 43.—Hefner Lamp.

thus obtained; and, further, it is not proved that the intensity of the latter is unaltered; moreover, the amiantus does not at all dispense with cutting the wick from time to time.

The normal height of flame of 40 mm. was adopted because the

lamp then gives a light equal to that of an English candle whose flame is 43 mm. in height. However, Bunte* found that the von Hefner-Alteneck unit corresponds to the English candle with a flame 45 mm. in height, and Liebenthal concluded, from more than 200 comparisons, that the flame of the acetate of amyl lamp, 37 mm. in height, has the same luminous intensity as that of the English candle 43.2 mm. in height. These differences are not exaggerated if we

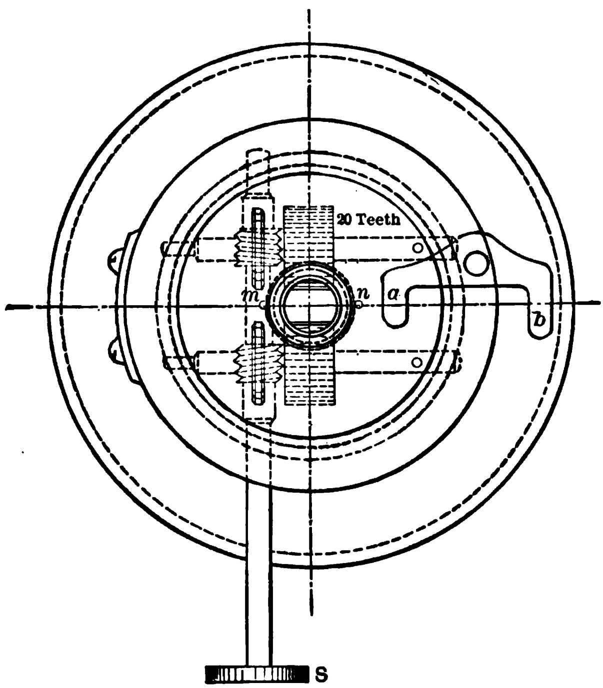

FIG. 48 *bis.*—Hefner Lamp.

take account of the want of uniformity of these two light standards, and the difficulties of measuring with exactness the height of the flame.

Some comparisons, made with the greatest care with perfected apparatus, have been effected by the commission on photometric standards of the German Society of Gas Engineers, and by Lummer and Brodhun, of the Physico-Technical Institute of Berlin.

* *Journal für Gasbeleuchtung*, 1885, p. 796.

Below are the results obtained, which are the mean of a great number of measurements made with different lamps and candles.

1 German candle equals	1.223	Hefner units	(German Commission).
	1.162	" "	(Lummer).
1 English candle equals	1.129	" "	(German Commission).

The difference of 6 per cent between the results of the commission and those of Lummer and Brodhun shows well the difficulties of obtaining photometric standards with free combustion.

The intensity of the Hefner standard is too small, especially now that the tendency is more and more toward employing intense radiants; the color of the flame is somewhat red, on account of its relatively low temperature. It is the richest in red rays of all light standards. On this account, also, we cannot use it with advantage in the photometry of incandescent and arc lamps.

Variations of the Hefner Standard.

91. The sole advantage of the acetate of amyl lamp is to be found in its great constancy; on this point all those who have used it agree. Thus, Liebenthal determined from a great number of measurements that the mean error of one observation varies between 0.5 and 1.5 per cent. In 225 observations by Dibdin with the Hefner lamp, the result differed from the mean by a quantity less than 1 per cent in 206 (90 per cent of the measurements).

The causes which influence the value of the acetate of amyl standard proceed principally from the state of the surrounding air. The vitiated air of a room has a sensible influence on the illuminating power of the lamp; this is easily noticed by comparison with an incandescent lamp. The influence of the variations of the barometric pressure in a given place has not been determined; it ought, however, to be sensible on carrying the lamp to elevated places; besides, this influence is sensible in all standards based on combustion.

The temperature of the place of observation has no influence on the intrinsic value of the light standard; we notice only that the variations of the flame are a little greater when the temperature is slightly lower.

According to the directions of von Hefner-Alteneck, measurements may be commenced ten minutes after lighting; but this space

of time is somewhat short, and it is better to wait at least forty-five minutes *.

The illuminating power of the acetate of amyl lamp varies greatly with the height of the flame, which is slender, very unsteady, and whose height it is difficult to measure. In this is found one of the great inconveniences of the Hefner standard. It is necessary for photometric measurements that a special observer should indicate the moment when the normal height of the flame is attained.

The most exact process of measurement is based on the employment of the cathetometer, but we can only take time for this solution in the scientific researches of the laboratory. Under these conditions the height of the flame may be measured with a mean deviation of 0.2 mm. Much attention is necessary, however, to arrive at this result, because of the influence of the edge of the flame.

The height of the flame as measured by means of the sight fitted to the lamp has a mean deviation of 0.5 mm.; deviations of 1 mm. are, however, possible. We may easily adapt to the Hefner lamp an apparatus for measuring the height of the flame analogous to that which we described in speaking of candles †.

The lamp has a screen at the side which supports at its upper part a tube closed at one end by an achromatic lens; a second tube which moves on the first carries a sheet of ground glass on which a millimeter scale is engraved; its fortieth division corresponds with the center of the lens, and is 40 mm. above the end of the wick-holder: the image of the flame is projected on this glass, and its variations are followed without difficulty. This permits the measurement of the height of the flame with a mean deviation of 0.3 mm., which corresponds to a deviation of 0.8 per cent in the luminous intensity.

This mean deviation of from 0.2 to 0.3 mm. has to do with the measurements of one and the same observer. The results of two observers are less concordant ‡, and differ very frequently by from 0.5 to 0.6 mm.

Liebenthal § found that the luminous intensity of the acetate of

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* *Journal für Gasbeleuchtung*, 1890, p. 33.
† *Zeitschrift für Instrumentenkunde*, 1890, p. 133.
‡ *Zeitschrift für Instrumentenkunde*, 1890, p. 131.
§ *Lum. Él.*, Vol. XXVII. p. 413, and Vol. XXXI. p. 113.

amyl lamp is expressed for a height of flame of less than 40 mm. by the formula

$$I = 1 - 0.038\ (40 - h).$$

For heights greater than 50 mm. the formula becomes

$$I = 1 + 0.025\ (h - 40).$$

Below 40 mm. the intensity increases more rapidly than the height; above, on the other hand, the increase is sensibly proportional.

The dimensions of the wick-holder indicated by Von Hefner-Alteneck correspond to the maximum luminous intensity, for Liebenthal found a diminution of about 1 per cent on increasing or diminishing by 2 mm. the diameter of the German silver tube. As to the free height of this tube, a difference of 1 mm. produces a variation of only 0.2 per cent.

Notwithstanding these favorable results, it is certain that difference of make produces considerable difference in the luminous intensity of the Hefner standard. Thus six standard lamps compared by a special commission of the German Society of Gas Engineers with the best lamp belonging to the Physico-Technical Institute at Berlin produced luminous intensities expressed by the following figures: 0.987, 0.993, 0.993, 0.965, 1.016, 0.981. These numbers represent differences of from +1.6 per cent to −3.5 per cent. The same commission found also a difference of 2.9 per cent between the illuminating powers of two lamps, one constructed by Siemens, the other by Krüss at Hamburg.

Finally lamps of Siemens, Krüss, and the Physico-Technical Institute of Berlin gave differences included between +8.9 per cent and −3.2 per cent.

Under these conditions, it is difficult to adopt the Hefner lamp as an absolute standard when one of the principal advantages claimed in its favor, the facility of identical reproduction, does not exist.

Influence of the Purity of the Acetate of Amyl.

92. The influence of impurities in the acetate of amyl on the luminous intensity of the Hefner standard has been investigated with great pains by Liebenthal.

It is known that acetate of amyl, $C_7H_{14}O_2$, whose boiling-point is between 138° and 140° C., is obtained by the action of sulphuric acid on amylic alcohol, $C_5H_{12}O$ (boiling-point 129° to 133° C.), and

vinegar (boiling-point 117° to 119° C.). This ether very frequently contains variable quantities of amylic alcohol, acetic acid, and water. It may be purified by a fractional distillation, but this long operation should be made with great care, because of the slight difference between the boiling-points of the impurities.

If these impurities are considerable, four parts of the liquid should be mixed with one part of a concentrated solution of common salt plus a small quantity of calcined magnesia; shake well and repeat this operation several times; next separate the common salt solution, and shake the liquid again with pulverized chloride of calcium; it only remains to rectify the acetate of amyl at a temperature of about 80° C.

Liebenthal found that impurities diminish the illuminating power; for amylic alcohol, however, the diminution is so slight that it may be neglected in practice; the presence of 20 per cent of amylic alcohol lessens the illuminating power by 1.1 per cent only. Acetate of amyl saturated with water gives also an illuminating power practically equal to the normal value; the diminution is 0.5 per cent. A diminution of 3 per cent in the illuminating power has been found with a sample of acetate of amyl containing 10 per cent of alcohol and 5 per cent of water, but this change is not found in practice.

It follows then from these measurements that the composition of acetate of amyl does not cause the illuminating power of the Hefner lamp to vary for a height of 40 mm., except within very narrow limits.

The influence of impurities is more considerable on the stability of the flame. In proportion as the impurities of the liquid increase, the flame becomes less and less quiet, and is affected with very slight oscillations, which may, however, in certain cases, be perceptible to the naked eye. The instability of the flame may be explained by assuming that the impure liquid, unalterable in a closed vessel, is altered little by little, in proportion to its combustion; it is further noticed by the production of verdigris and the charring of the wick; the latter then cannot draw up the liquid regularly.

The composition of the liquid should be tested with care before proceeding to measurements, and in case of doubt, it should be submitted to special distillation.

Below is the method indicated by Bannow* for verifying the purity of acetate of amyl.

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* *Electrotech. Zeitschr.*, 1891, pp. 122, 177, 193.

1°. The density of the liquid should be between 0.872 and 0.876 at 15° C. (test for alcohol).

2°. The mixture of equal volumes of acetate of amyl and benzine (or sulphuret of carbon) should remain clear and liquid (test for amylic alcohol or ethyl hydrate; the water separates in globules under the action of the sulphuret of carbon).

3°. On shaking in a graduate 1 cc. of acetate of amyl with 10 cc. of 90 per cent alcohol and 10 cc. of water, a clear and liquid solution should be obtained (test for toluene, etc., not determined by the second test).

4°. A drop of acetate of amyl should leave no greasy trace on a sheet of white paper after evaporation (test for oils, tar, and other greasy materials).

The Hefner standard has been adopted in Germany by the Society of Gas Engineers, and official sanction will soon be given it by the Physico-Technical Institute of Berlin. It may be asked, in consequence of the numerous sources of error in this light standard, if the advance brought about by replacing the candle by the Hefner lamp is sufficiently great to justify the introduction of this new absolute standard. (See Appendix C.)

The Giroud Standard.

93. In 1882, Giroud proposed a new photometric standard designed primarily for the testing of gas, but which, in certain cases, may be used with advantage in electric photometry. This standard is based on the combustion of gas in a determined burner, under a constant and well-defined pressure.

Audouin and Bérard* investigated carefully the circumstances which have an influence on the light emitted by a gas-burner, among which should be mentioned, in addition to the composition and pressure of the gas, the dimensions, form, and nature of the burner employed.

The Giroud standard† consists of two lights, one of which serves to determine the constancy of the other, while permitting measurements of its variations. These two lights are fed by ordinary illuminating gas, which is procured with as much facility as oil, stearine, etc., if not more. One of the two burners of the standard is a candle-burner with a single opening of 1 mm.; the other is an

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* *Ann. de Chim. et de Phys.*, 3e serie, Vol. LXV.

† *Journal des usines à gaz*, 1881 and 1882.

Argand burner. The lights produced are in the ratio of 1 to 10, and since they result from the same gas, they remain in the same ratio, regardless of accidental variations entering into the quality of the latter.

Every change in the real intensity is manifested in the standard by a change in the surface of the flame or in its dimensions. It is impossible to determine this change in dimensions in the Argand burner, while it is easy to recognize and measure it with the single-aperture burner, in which the slightest change is shown in a very sensible manner in the length of the flame. It amounts then to a single measurement of this length; if it remains constant, it follows that the intensity of the candle-burner and consequently that of the Argand burner, are not modified; if the former varies, the extent of this variation measures the change undergone by the Argand burner whatever be the number of units the burner represents.

Giroud's Candle-burner (bec-bougie) Standard.

94. Giroud has given to the candle-burner the value of $\frac{1}{10}$ carcel, that is, the value which is assumed approximately for the candle, and to the Argand burner the value of one carcel.

Invariability of the length of the flame is necessary during the measurements. The consumption of gas should then be constant, which necessitates the employment of a special meterage, effected by the rheometer of Giroud.

The rheometer is an instrument for continuous meterage, applicable to the flow of both liquids and gases. Figure 44 represents it in working size for ordinary gas-burners.

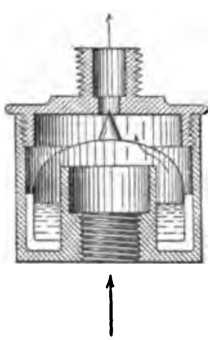


FIG. 44.—Giroud Rheometer.

The current indicated by the arrows raises a movable barrage whose vertical movement should be quite free; it is formed for this purpose by a cap gliding without friction in a bath of glycerine which serves to lute it. This cap is pierced by an orifice which has nothing in common with that of the burner, and at this orifice there is established a velocity which has nothing in common with that of the current. It follows that we may place on the rheometer any burner, or even none at all, without affecting the flow through the rheometer.

Below is the description of the candle standard of Giroud, of which Fig. 45 gives a vertical section.

It is in fact only a rheometer whose valve is under instead of above the cap *C*. The conical cover of the valve is soldered at its base to the tube *A* which plunges in the glycerine bath of the chamber *E*. The inner section of this tube is exactly equal to that of the opening forming the seat of the conical valve; the rod which connects the cone to the cap is a small open tube. In this way the movable part, consisting of the cap, the cone, and the tube *B*, rests suspended in equilibrium in the current and cannot move except in a vertical direction, under the pressure of the gas; it is shown that this resisting pressure is constant, for it is expressed by the formula,

$$P - P' = \frac{p}{S - s},$$

P being the pressure of the gas, *P'* the counter-pressure produced by the burner, *p* the weight of the movable part complete, *S* the surface of the cap *C*, *s* the inner section of the tube *A*.

When the pressure $P - P'$ increases, the cap *C* lifts, and the conical cover of the tube *A* partly closes the opening, through which a smaller quantity of gas then passes; equilibrium is re-established when the normal pressure $P - P'$ has been attained anew. The consumption only depends then on the diameter of the opening of the cap *C* and on the nature of the gas.

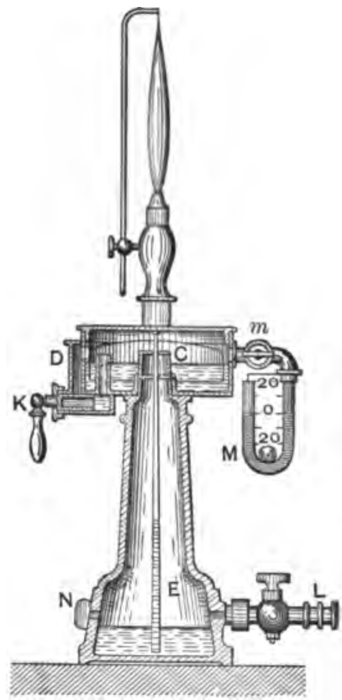


FIG. 45.—Giroud Candle-burner (*bec-bougie*) Standard.

The sight shown in the cut is only a crude representation; the measurement is made from the burner to the extreme top of the flame, the place where the red point which terminates it ceases to be visible.

The rheometric opening in the cap is of such a diameter that it is always necessary to make use of the cock *K* to obtain the volume which will give the normal height of the flame with the ordinary flow of gas.

The burner is of soapstone, giving the freest access to the air at the point where the flame begins. The opening of this burner should be 1 mm.; this may be determined by a gauge.

Giroud adopted a height of flame of 67.5 mm., for which the candle-burner gives a luminous intensity of exactly $\frac{1}{10}$ carcel. There is between the height and the intensity of the flame of the gas-burner of single opening a ratio which remains constant for heights between 45 and 120 mm. It is the same as in the carcel lamp, where there is between the weight of the oil consumed and the luminous intensity a ratio which remains constant for consumptions comprised between 40 and 44 grams of oil per hour.

Below are some figures in regard to this:

Height of the Flame of the Candle-burner (*bec-bougie*) in Millimeters.	Intensity in Carcels.	Height of the Flame of the Candle-burner (*bec-bougie*) in Millimeters.	Intensity in Carcels.
45	0.0505	75	0.1165
50	0.0614	80	0.1275
60	0.0835	90	0.1495
65	0.0945	100	0.1715
67.5	0.1000	110	0.1935
70	0.1055	120	0.2155

The variation in luminous intensity is then 0.0022 per mm. of variation in the height of the flame.

The opening should be exactly 1 mm. in diameter; a variation of 0.05 mm. produces a variation in luminous intensity of 3 per cent with the same quantity of gas; it is very easy to determine this diameter within 0.01 mm., so that we may be sure of the luminous intensity within 1 per cent.

The influence of the composition of the gas on the intensity of the candle-burner has been found negligible, care being taken to maintain the flame at the normal height of 67.5 mm., by a suitable regulation of the rheometer; with an inferior quality there simply results a more considerable consumption of gas.

The same remark applies to variations in the density of the gas; the consumption alone varies with this latter. For a density of 0.4, that of air, the consumption of the candle-burner is 25 liters per hour.

As to the influence of the atmospheric pressure, Giroud has shown that it is negligible; it is the same with variations of temperature between 15 and 25° C.

The flame of the candle-burner is then an absolute standard whose intensity has been so made by definition as to equal $\frac{1}{10}$ carcel, but which is always susceptible of being exactly reproduced.

Relative Photometric Standards of Giroud.

95. However, the advantages of the Giroud standard are greatest in the comparison of intense radiants, for which we then use the relative standards whose brief description follows.

It is an observed fact that in Paris, for instance, when the flame of the absolute standard has been brought to 67.5 mm. by means of the cock *K*, the rheometric consumption is 25 liters per hour of standard gas at a density of 0.4.

We may easily reproduce this rate of flow with an apparatus provided with a cock *K*, if we make in the cap a rheometric opening of suitable dimensions, with which we need not concern ourselves further.

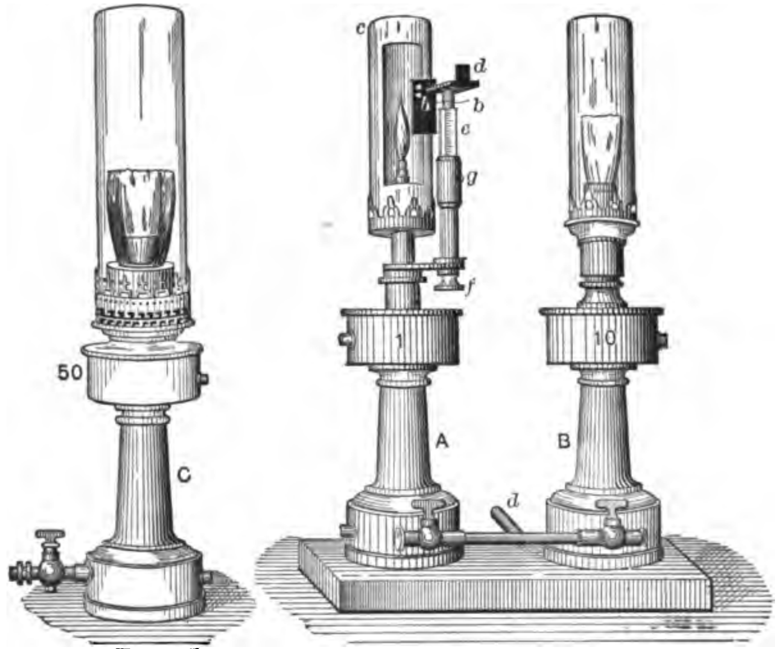
The two photorheometers *B* and *C*, in Figs. 46 and 47, are fitted with burners suitably chosen, and regulated so as to give that volume of gas, whatever it be, which is necessary in order that the burners may give, one 10 times, the other 50 times, as much light as *A* gives, when the same gas supplies them all.

If the gas is standard, *A* should be $\frac{1}{10}$ carcel, *B* 1 carcel, and *C* 5 carcels; if the gas is not standard, the flame *A* should be shorter or longer than 67.5 mm., and each millimeter difference will represent a change of 2.2 per cent in the value of the standards *B* and *C*.

To measure the flame of the candle-burner exactly, a sight of which Fig. 47 gives a sufficient idea, is used; the line of sight is determined by the lower edge of the screen *a*, and by the point of tangency of the two circular holes in the screen *b*. It does not appear, however, that this method of measuring the height of the flame is sufficiently exact. At least this is the conclusion from numerous measurements made by Heisch and Hartley under the authority of the Gas Institute.

The measurements of Heisch and Hartley * showed that the Argand-Giroud burner of 1 carcel for a length of flame of the candle-burner of 67.5 mm., equalled 10 English candles when the flame of the candle-burner was 69.5 mm., the gas employed having an illuminating power of 15.8 candles.

* *Journal des usines à gaz*, 1884 and 1885.



FIGS. 46 and 47. —Giroud Photorheometer.

Here are some results among others, which show the agreement realized between the direct measurements of the luminous intensity of the Argand-Giroud burner and the intensity calculated according to the instructions of the inventor.

Illuminating Power of the Gas *.	Height of the Flame of the Candle-burner.	Luminous Intensity (real) of the Argand Flame.	Luminous Intensity (calculated) of the Argand Flame.	Difference in per cent.
Candles.				
15.47	69.0	10.31	9.89	+ 4.2
15.80	69.5	10.00	10.00	0.0
15.80	69.5	10.13	10.00	+ 1.3
15.64	69.5	10.17	10.00	+ 1.7
15.94	68.5	10.31	9.78	+ 5.4
15.90	69.7	10.23	9.90	+ 2.7
16.06	71.6	10.80	10.46	+ 3.2
15.98	72.0	10.85	10.55	+ 2.9

* Let us recall briefly what is meant by illuminating power (*titre*) of gas. In France the illuminating power of gas is fixed by stipulating the consumption

The mean deviation is 2.5 per cent. The conclusion of Heisch and Hartley is favorable to the Giroud standard. These two engineers think in fact that, in ordinary practice, with a better arranged sight, the errors of the instrument should not exceed 3 per cent, the illuminating power of the gas varying between 15 and 16.5 candles.

Uppenborn also made some measurements* of the luminous intensity of the Giroud candle-burner of one opening, which tend to prove that the rheometer does not regulate the flow of gas, and consequently the height of the flame, so exactly as is generally supposed. Uppenborn found that the luminous intensity of the candle-burner increases slowly after lighting for a period often extending over two hours. His values for the Giroud candle-burner in terms of the Hefner standard, for different heights of the flame, are

Height of Flame in Millimeters.	Luminous Intensity in Hefner Units.	Variations of the Luminous Intensity per Millimeter of Flame.
81	1.369	......
71	1.143	0.0226
61	0.965	0.0178
51	0.838	0.0127

THE METHVEN STANDARD SCREEN.

96. John Methven announced in 1878, to the British Association of gas managers, that he had discovered that the portions of flames of gas of different qualities, burned so as to obtain perfect combustion in similar Argand burners, have equal illuminating powers.

Methven further determined the particular parts of an Argand flame which emit the same quantity of light on supplying the burner with gas of different qualities so as to give a three-inch flame; he also found the means of avoiding the radiation toward

necessary to obtain the luminous intensity of a carcel lamp burning 42 grams of purified rape-seed oil per hour. In England it is fixed by determining the number of candles corresponding to a consumption of 5 cu. ft. (141.6 liters) of gas per hour. In the first case there is employed a Bengel burner of 30 openings, in the second, an Argand burner (Sugg's London Argand No. 1).

* *Lum. Él.*, Vol. XXVIII. p. 330.

the photometer of all rays other than those belonging to the constant portion of the flame.

The original method of Methven was as follows: in front of the burner, between it and the photometer, is placed a blackened copper screen. A rectangular opening, one inch high by a quarter of an inch wide, is made in the screen, which thus permits the passage of a quantity of light equal to that of two spermaceti candles only.

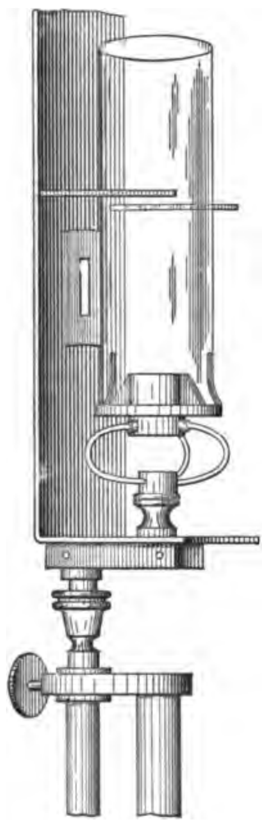


FIG. 48. — Methven Screen.

The first tests were made using ordinary gas; later Methven * undertook a series of measurements to determine the influence of the illuminating power of the gas on the intrinsic intensity of the flame, and thus obtained, by an appropriate carburetting of the gas, an intrinsic intensity exactly comparable with itself, and, besides, a constant luminous standard.

He found that gases, originally of different qualities, produce the same quality of light when they are carburetted and burned with the same burner in a flame 2.5 inches high.

The hydrocarburet employed is a product of petroleum generally known by the name of gasoline. This has a very low specific gravity and a very low boiling-point, and emits its vapors at the ordinary temperature, so that gas passing over its surface completely evaporates it at a temperature of 10° C.

The carbureter employed by Methven is composed of many troughs of fine wire gauze, fixed on an inclined plane in a small rectangular chamber; the necessary quantity of hydrocarburet is determined and flows from a reservoir into the highest trough; the volatilization commences at once; that which does not evaporate in the highest compartment falls into the second, and so on. The gas, on simply passing above the troughs in question, becomes saturated and is

* *Journal des usines à gaz*, 1883.

afterwards consumed by the burner. As the quantity of gasoline which flows into the carbureter is determined with care, it becomes completely volatilized, and thus a carburetted gas of uniform quality is obtained. Below is shown the effect of the carburetting produced by this apparatus on gas of different qualities.

Illuminating Power of the Gas before Carburetting.	Illuminating Power of the Gas after Carburetting.	Variations.
10.51 candles.	62.86 candles.	+ 0.54 candles.
13.82 "	60.38 "	− 1.94 "
16.25 "	61.92 "	− 0.40 "
18.85 "	62.49 "	− 0.17 "
20.44 "	63.77 "	+ 1.65 "
Mean	62.32 candles.	

The carbureter should be placed in a bath at about 10° C., in order to compensate for the cold produced by the evaporation of the oil.

The Methven screen, in its most recent form, is composed of a movable plate carrying two silver screens; one of them, having a rectangular opening 1 inch (25.4 mm.) long by 0.233 of an inch (5.92 mm.) wide, is designed for use with ordinary non-carburetted gas; the other, which is used when gas, carburetted as indicated above, is burned, has an opening 0.585 of an inch (14.86 mm.) long by 0.310 of an inch (7.87 mm.) wide.

The burner employed is Sugg's ordinary Argand burner with a glass chimney (Standard London Argand, No. 1). The base of the opening, when using ordinary gas, is 1 inch (25.4 mm.), and when using carburetted gas, 0.96 of an inch (24.38 mm.) above the top of the burner; the horizontal distance from the center of the flame to the screen is 1.5 inches (38.10 mm.). The height of the flame from the burner is 3 inches (76.2 mm.) for carburetted gas; these two heights of the flame are defined by a double set of horizontal wires forming sights fixed on the screen. The chimney of the Argand burner is 2 inches (50.8 mm.) in diameter and 6 inches (152.4 mm.) in length.

The Methven standard, being very simple in construction, is not subject to derangement, and its use is extremely easy. When employing ordinary gas, it is much better than the candle in an open-air photometer. The indications of the Methven screen with

carburetted gas are very concordant. Thus Dibdin, in a very extensive investigation of the different light standards, found, out of 283 experiments, 211 or 74 per cent, in which the results did not differ by 1 per cent from the mean.

The Giroud burners and the Methven screen may render marked service as intermediate standards, always on the condition of having their values determined in comparison with the absolute standard.

By supplying these burners with gas stored in the gas-holder of the laboratory at the commencement of the experiment, we have at our disposal a source of light which remains constant for many hours, and whose employment requires no delicate manipulation. This is of considerable advantage, and makes these burners, notably that of Giroud, valuable auxiliaries in photometric researches.

Vernon-Harcourt's Pentane Standard *.

97. The objections which may be made to photometric standards in which ordinary illuminating gas is employed, evidently cease if we are able to have gas of constant composition. It is this constancy of composition which Methven sought to realize, in a certain measure, by replacing ordinary gas by carburetted gas.

A more perfect solution of the problem has been found, after numerous trials, by Vernon-Harcourt; the combustible employed by him is air carburetted by means of volatile carburets of hydrogen, products of petroleum. The carburet is prepared by a fractional distillation of gasoline, previously washed with sulphuric acid and caustic soda. The liquid decanted is distilled four times, successively at 60°, 55°, 50°, and lastly at 50° again. The product obtained is composed of hydrocarburets of the paraffine series, C_nH_{2n+2}, principally pentane, C_5H_{12}, mixed with its homologues, tetrane, C_4H_{10}, and hexane, C_6H_{14}. Its specific gravity at 15° C. varies between the limits 0.628 and 0.631.

We may employ air carburetted with pentane in two ways: in one of the standards the gas is prepared in advance and stored in a special gas-holder; in the other, it is prepared in proportion and at the same rate as it is burned, which is precisely what takes place in the case of a lamp or candle.

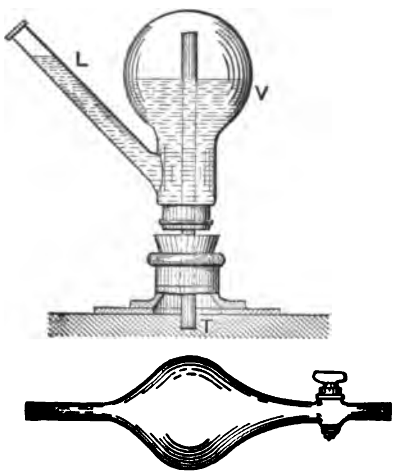
The following is the way, according to Monnier †, of preparing standard carburetted air after the directions given by Vernon-Harcourt.

* *Elect. Review* (London), May 6, 1887.

† *Journal des usines à gaz*, 1883.

To prepare the standard carburetted air, the hydrocarburet is left to mix with the air by the diffusion of its vapor in the proportion of 3 cu. in. of liquid to 1 cu. ft. of air measured at a pressure of 76 cm. of mercury, and at a temperature of 15° C.; the proportion of pentane, when in a state of vapor, is 7 volumes of pentane mixed with 20 volume of air.

The gas-holder (Figs. 49 and 50) is composed of a cylindrical receiver 7 cu. ft. in capacity, suspended and balanced in an annular cistern filled with water. A graduated scale, fixed to the receiver, serves to measure the volume of the gas. In the cap, which is plane



FIGS. 49 and 50.

to avoid waste space, are found two tubes: one gives access to a thermometer; in the other is fixed a glass receiver of special form.

To prepare the mixture, we blow into the receiver 3 cu. ft. of air measured at 76 cm. pressure of mercury and at 15° C.; a graduated pipette which contains 9 cu. in. is filled with pentane. The point of the pipette being introduced into the lateral tube *L* of the vessel *V* (partly full of water), the cock is opened; the pentane flows in and rises to the upper part of *V* and enters the receiver by the tube *T*. The last part of the pentane is driven in by blowing gently. When all the liquid has entered, the pipette is withdrawn, and the last part of the pentane is forced into the receiver by pouring some water into the lateral tube *L*. The diffusion is complete in five

or six hours; at this time, the volume of gas should be 4.05 cu. ft., with an allowable variation of 1 per cent.

The vapor of the hydrocarburet employed to carburet the air should be very slightly soluble in water, so that the composition of the gas does not become altered by contact with the water in the gas-holder: this condition excludes the employment of olefiant gas, ether, or benzine, which are quite soluble in water, while pentane is very slightly so, 100 volumes of boiled water dissolving only 0.92 of a volume of pentane vapor.

This solubility is sufficient to slightly modify the composition of the first gas prepared in a receiver over pure water; but, after an operation or two, the water is saturated; and as variations in the composition of the gas may only come from a temperature change in the coefficient of solubility, they are very slight.

The vapor of pentane behaves like a perfect gas within the ordinary limits of temperature and pressure. The mixture of air and pentane vapor constitutes then a gas of constant composition.

The burner adopted by Vernon-Harcourt is a candle-burner of yellow copper, because of the ease with which this metal is worked. The opening is circular; it is made in a copper plate 0.05 of an inch thick and should be 0.25 of an inch in diameter.

This dimension may be reproduced within 0.001 of an inch; that is, with an error of less than 0.8 per cent of the total section of the opening. The greatest diameter compatible with steadiness of the flame was chosen. The body of the burner is 4 inches long and 1 inch in diameter.

The normal height of the flame is 2.5 inches measured from the opening of the burner; under these conditions, the normal consumption of gas is between 0.48 and 0.52 cu. ft. per hour. The height of the flame is shown by a platinum wire carried on a rod parallel to the burner.

The gas is measured by means of a small meter, and its consumption is further regulated by a sensitive regulator.

Under the above conditions, the Harcourt standard gives a luminous intensity exactly equal to that of a spermaceti candle.

For the needs of industrial photometry, we may obtain a quite intense standard of light, about ten candles, of great constancy, by using the pentane air gas of Harcourt in an Argand burner (Sugg's Standard London Argand, No. 1) combined with the Methven screen. It is this which has been done by Dibdin among others.

It is even sufficient, in order to carburet the air in the Methven carbureter, simply to pass it over the liquid pentane.

Dibdin found that under these conditions the height of the flame may vary between 2.5 and 6 inches without the quantity of light emitted upon the screen varying in a sensible manner. The normal height of the flame is then 3 inches, but a slight variation in this height hardly affects the standard. However, the pentane Argand does not afford any control over the working of the burner, so that we should hardly dream of employing it with safety in photometric measurements, still less of giving it a legal sanction.

Laboratory Form of the Pentane Standard.

98. The air-gas lamp, in which the gaseous mixture is prepared at the time of combustion, has undergone some quite important modifications since the time when Vernon-Harcourt brought it out in its first form. This gave results which were little concordant, if we may judge from the numerous measurements of Heisch and Hartley, while the final form presented to the British Association in 1887, reproduced the light standard with very great exactness; this is at least the conclusion from Dibdin's report, to which we shall return later.

Following is the description of the laboratory form of the pentane standard, as made by Woodhouse and Rawson of London (Fig. 51).

The admission of the gas to the burner is made exclusively under the action of gravity and without a regulator. The air and vapor are mixed in a reservoir *M*, whence they descend to the burner. At a certain point, the diameter of the tube through which the gas flows is reduced, and there is between this reduction and the height of the reservoir such a relation that, when the mixture, in the above indicated proportions, is introduced into the tube, it is burned with a flame 2.5 inches high.

The pentane is introduced in the liquid state into the globe *M*, whence it flows into the reservoir *B*, where it is vaporized; the vapor passes through *C* and *H*, and next descends by its own weight in the vertical tube which leads to the interior of the reservoir *R*, where the mixture passes through the cock *D* and reaches the burner *FG*.

To regulate the velocity and regularity of the flow of the mixture, the pentane vapors are passed through a thermometric tube which is stopped up for a greater or less distance by a platinum wire attached to a screw *O*. We may thus give to the pentane the velocity of entrance necessary to make the flame exactly 2.5 inches high.

The level of the pentane in the reservoir has naturally a certain influence, of which account is taken in the following manner. The square box shown in the figure contains a rubber balloon filled with water and connected by a flexible tube to the reservoir; when the level of the pentane is too low, the balloon is compressed by means

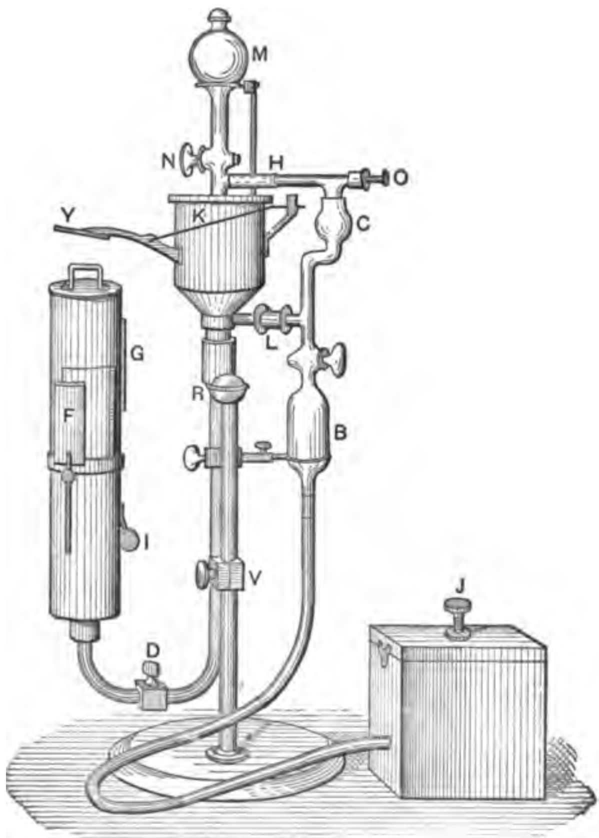


FIG. 51. — Pentane Lamp (Laboratory Model).

of the screw *J*, and a certain quantity of water is made to enter the reservoir *B* below the pentane. In the opposite case this screw is loosened, and the water is allowed to descend again.

The heavy copper disc *Y* suspended above the flame at a variable height serves to compensate for the influence of the external temperature; the chimney *G* protects the flame from currents of air.

Industrial Form of the Pentane Standard.

99. Vernon-Harcourt has simplified this apparatus still more, so as to make it portable and much less complicated without taking away any of its precision.

Vernon-Harcourt's new lamp, instead of burning a mixture of air and pentane vapor, burns the vapor of pentane alone; the flame is surrounded by a chimney which produces sufficient draught and steadiness; it is as white as in the original form of the pentane standard, which is a very important condition.

The new standard lamp, represented in Fig. 52, has a form analogous to that of ordinary alcohol burners, with a metal chimney in addition. The metallic chimney producing a strong current of air gives the necessary steadiness to the flame and increases its temperature, which also gives it a whiter color.

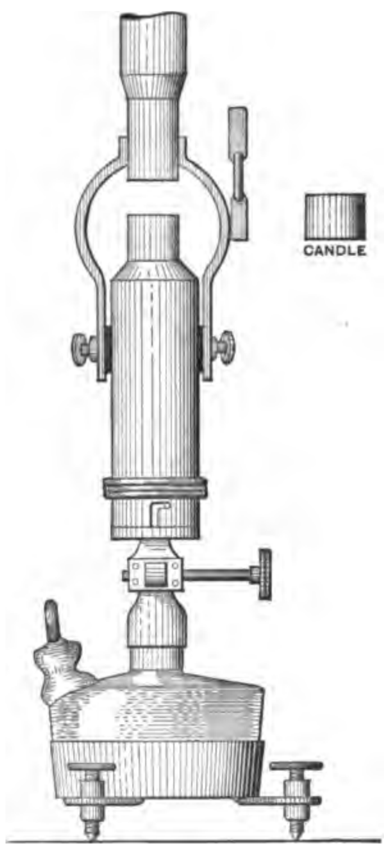


Fig. 52. — Pentane Lamp (Industrial Model).

Use is made of a wick, which would be a serious disadvantage if combustion took place at its end; this is not the case, for it simply serves to raise the pentane by capillarity from the lower reservoir to the place in the wick-holder where it is vaporized under the influence of the heat produced by the combustion of the vapor, 5 or 8 cm. higher. The wick enters with slight friction in a tube open at both ends and itself surrounded by a metallic covering much larger, intended to keep the temperature more constant. The combustion of the pentane vapor takes place at the end of this outer tube; the whole is surrounded by a third tube much larger, which

contracts in its upper part so as to have only the diameter of the glass chimney. This chimney is enlarged at its upper end and is fixed to the metallic envelope of the lamp by means of the two movable arms shown in the figure.

The working of the lamp is as follows: on raising the wick-holder and heating the inner tube a little, the pentane vapor is immediately disengaged and lighted. The outer covering with the chimney is then put on; the flame immediately rises, because of the increased draught, and its end enters the chimney. This has, at a height of 10 mm., two horizontal slits diametrically opposite, so that, by looking across, the height of the flame may be exactly regulated.

Since the movable chimney is regulated in height, and the height of the flame is determined exactly by the two slits mentioned above, the portion of the flame included between the lower envelop and the chimney emits a perfectly definite quantity of light.

We know that the quantity of light emitted by the central part of a flame is only very slightly affected by variations in the height of the later. Harcourt and the makers have carefully determined the dimensions of the lamp which correspond to a luminous intensity of one candle and two candles. By varying the height of the chimney, we may obtain any luminous intensity within the limits of the power of the apparatus.

The height of the chimney is verified by means of a special cylindrical gauge of the same diameter as the lower part of the chimney, which is placed between the latter and the outer tube of the base; the chimney is then fixed in this position by means of regulating screws.

The variations in the height of the flame are very slight; they become insensible ten or fifteen minutes after lighting. The base of the apparatus is made horizontal by means of a level, and a small mirror placed behind the slits facilitates observation of the height of the flame.

The new pentane lamp is more easy of manipulation than the acetate of amyl lamp of Hefner-Alteneck, since variations in the height of the flame have only a very slight influence on the luminous intensity; the flame is, further, much whiter.

100. There is a difference in the results obtained with Harcourt's air gas, among the observers who have investigated this standard with care; there is evidently a question of personal preference here.

Thus while Heisch and Hartley favor rather the Methven screen, Dibdin, by reason of very extended and careful comparative measurements, considers that Harcourt's pentane lamp is, of all the industrial photometric standards, the one whose luminous intensity is the most constant and fixed.

We quote the following from his report to the Metropolitan Board of Works of London:

"The pentane flame fulfils all the conditions which may be imposed on it. Experiments showed that the preparation of carburetted air was easy and not dangerous, the measurement of the volume of air consumed simple and precise, the regulation of the height of the flame very exact, that this possessed all the constancy desirable when operated with care, and finally, that the color of the light was identical with that of a gas-flame.

"Throughout the tests there was no noticeable defect in the stability of the light. The only precaution to be taken is to avoid currents of air."

A variation of 2 per cent in the luminous intensity of the pentane flame is exceptional, while with candles, for instance, a variation of 10 per cent is a common thing.

As to the value of the pentane standard, Dibdin found that it was sensibly equal to an English candle.

To sum up the investigation of different light standards based on combustion, we give here a table which recapitulates the numerous observations of Dibdin. He includes in the first column the total number of experiments made on each standard, in the second the number of those whose results differ from the mean by less than 1 per cent, and finally in the third the ratio of these numbers.

Candle	454	154	34 %
Keate's lamp*	244	98	39
Pentane (original form)	468	373	80
Methven	282	211	74
Pentane-Argand	243	212	87
Acetate of Amyl lamp	225	206	90
Pentane lamp	154	150	97

* Keate's lamp is used in England alone; we shall not dwell on its details, for it only differs from the carcel lamp in the oil used. It burns paraffine oil in place of rape-seed oil; but its conditions of working are analogous to those of the carcel lamp, and all that we have said concerning the latter is applicable to this.

These results represent to some extent the respective values of the different photometric standards; but before being accepted as final, this classification should be confirmed, for exactness of photometric measurements depends greatly on conditions peculiar to the observer.

STANDARDS BASED ON THE INCANDESCENCE OF CARBON AND OF PLATINUM.

Schwendler Standard.

101. The Schwendler standard is composed of a platinum strip in the form of a horseshoe; this strip being cut from a sheet of large dimensions, there are kept two ends of considerable surface connected to the terminals of the apparatus; the heating of the conductor thus takes place solely in the horseshoe part.

The light unit adopted by Schwendler (P.L.S.) is the quantity of light emitted by a platinum strip of the preceding form, 2 mm. in width, 36.28 mm. in length, 0.017 mm. in thickness, weighing 0.0264 mg., traversed by a constant current of 6.15 amperes.

The causes which influence the quantity of light emitted by a platinum strip heated by the electric current are too numerous for one to count on the constancy of the light emitted, if one is limited to controlling simply the constancy of the current. The variations in the emissive power, as well as the diminution of the section of the platinum strip under the influence of a prolonged incandescence, are factors of which it is difficult to take account. These circumstances have, from the beginning, inspired a certain distrust of the Schwendler standard, so that its use has not spread either in industrial measurements or in scientific researches.

The Incandescent Lamp as an Absolute Standard.

102. Preece, considering that incandescent lamps of a given type, coming from the same maker, present only insignificant differences among themselves with respect to luminous intensity and efficiency, thought that there might be obtained, by means of the incandescent lamp, a very convenient photometric standard sufficiently exact for the majority of industrial measurements. It would then be sufficient, as in the Schwendler standard, to maintain the current at a determined intensity.

This idea was advanced anew by Edison, some time after Preece's communication; the variations in the emissive power of the carbon filaments, sometimes very sensible from one lamp to another, and the greater or less transparency of the bulb, are so many factors opposed to the adoption of a unit of this kind.

We do not wish to say by this that one ought, *a priori,* to avoid the use of incandescent lamps in photometric measurements. In certain cases it is, on the contrary, very advantageous to use one of these lamps as a secondary standard. But the adoption of a typical incandescent lamp as an absolute standard will not take place very soon; for its inconveniences are not of such a nature as to be overcome without considerable improvement in its construction. However, if the invariability of the luminous intensity of an incandescent lamp for a constant expenditure of energy can ever be attained, the incandescent lamp will furnish a very convenient absolute photometric standard; it might then be defined by the nature and dimensions of the filament and the energy absorbed, expressed for instance in ergs per second.

The Incandescent Lamp as a Secondary Standard.

103. Until within the last few years, industrial photometric measurements were principally made to verify the luminous intensity of gas-lights. It is in this that we may find the explanation of the great number of photometric standards based on the combustion of ordinary gas or a special gas in a typical burner. These standards are very convenient in the photometric laboratories of gas manufactories, where one has at his disposal all the accessory apparatus indispensable to their successful operation. It is not so in industrial electrical laboratories where one has at his disposal generally a simple connection with a gas supply. But these laboratories possess, on the contrary, a complete outfit for the production and regulation of light by means of incandescent lamps. It is, then, natural to have recourse to these lamps as secondary standards in measurements of electric photometry.

An incandescent lamp furnishes light of constant intensity for quite a long time, provided that the number of watts expended in the filament remains invariable. It is easy to realize this condition by furnishing the lamp with the current from a battery of accumulators. It is in this manner only that we may obtain sufficient constancy without continually occupying ourselves with regulating

the current. It is easy to determine the conditions which must be satisfied by the current which supplies an incandescent lamp, designed to serve as a secondary photometric standard.

As a first approximation, the luminous intensity of an incandescent lamp is proportional to the cube of the power W expended in the filament (§ 132),

$$I = aW^3.$$

By differentiating we obtain

$$dI = 3aW^2dW.$$

This equation enables us to calculate the variation in the luminous intensity dI which corresponds to the variation dW in the energy absorbed by the filament.

For an Edison lamp of 100 volts giving 16 candle power with an expenditure of 56 watts (3.5 watts per candle), the value of a is equal to about 0.00009. The value of dW which produces an error dI of 1 per cent, i.e. 0.16 candle power, is then

$$dW = \frac{dI}{3aW^2} = 0.19 \text{ watt.}$$

In order that the incandescent lamp may be used as a photometric standard, we must be able to count on a constancy within at least 1 per cent. We must have at our disposal, in the case considered, apparatus allowing the expenditure of energy to be maintained constant within 0.2 watt in 56 watts, i.e. 0.4 per cent.

Accumulators alone satisfy this condition. The current furnished by a dynamo, though having an excellent regulator, is not sufficiently constant. As to primary batteries, their electromotive force and resistance vary too much for them to be considered, while a battery of accumulators on being discharged slowly has a remarkable constancy of resistance and of electromotive force for several hours; in certain cases its variations cannot be discovered, even with the most precise measuring apparatus.

It is usual to have recourse to a low-voltage lamp, using considerable current, unless there is at one's disposal a battery of accumulators of 50 cells, which gives the 100 volts required for lamps of the kind most used. It is true that, if low-voltage lamps are used, the current is higher, and the constancy of the cells of the battery of accumulators is less.

The measurement of the energy absorbed by the standard lamp,

i.e. the simultaneous measurement of voltage and current, may be made by means of apparatus which every electrical laboratory possesses. It should not be forgotten that this measurement must be exact in order to obtain the real value of the luminous intensity. Its accuracy should be many times (at least 4 times) that which we wish to obtain in the constancy of the luminous standard. It is useless to dwell on the experimental arrangements to be used in this measurement, for they are principally determined by the apparatus at hand. However, it may be well to say that the simplest arrangement for measuring the intensity of the current appears to be the measurement of the difference of potential between the ends of an invariable known resistance. It is at the same time the most convenient, since the measurement of the difference of potential between the terminals of the lamp may be made with the same apparatus. It is further the method employed by Lummer and Brodhun* in their researches on the employment of Siemens 65 volt, 16 c. p. lamps as secondary photometric standards. These investigations showed that these lamps gave a sufficiently constant light for all practical needs; further, for quite a long period, these lamps have a sensibly constant luminous efficiency, i.e. they absorb the same quantity of energy to produce the same luminous intensity. It follows from this that the standardizing of the incandescent lamp by means of the chosen absolute standard does not need to be repeated as frequently as seems necessary on first thought.

Violle Absolute Standard.

104. As a consequence of Violle's researches, the conference on electrical units adopted the proposition of this physicist to take as the photometric unit the quantity of light emitted by a determined portion of the surface of a bath of fused platinum.

The resolution adopted by the Conference is as follows:

The unit of each simple light is the quantity of light of the same kind emitted normally by 1 sq. cm. of surface of melted platinum, at the temperature of solidification.

The practical unit of white light is the quantity of light emitted normally by the same source.

One of the first conditions is to have perfectly pure platinum; the presence of foreign bodies not only alters the temperature of

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* *Zeitschr. für Instrumentenkunde*, 1890, p. 119.

fusion, but may also cause the formation of oxides tarnishing the surface of the bath and modifying its emissive power. Perfect purity is not difficult to obtain, and, moreover, the same platinum may serve indefinitely.

To produce the fusion of platinum Violle* employed the furnace designed by Deville and Debray in their work on the metallurgy of this metal. This apparatus consists of a piece of lime hollowed out

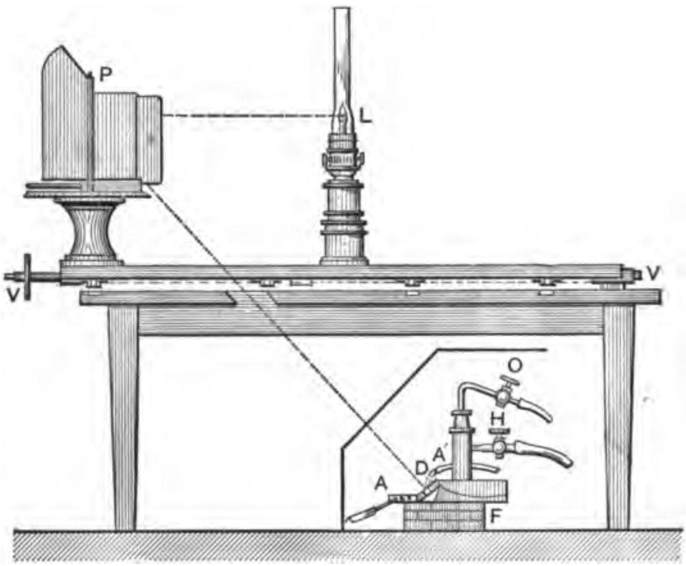


FIG. 58.—Apparatus for the Reproduction of the Violle Standard.

to receive the platinum and having a cover also of lime, with a blowpipe (using illuminating gas and oxygen) passing through it; a temperature well above the fusion point of platinum (1775° C.) is thus obtained. When the platinum is melted, it is brought below a diaphragm having an opening of determined area which may be any fraction of a square centimeter; a multiple or a sub-multiple of the absolute unit is thus directly obtained. The hollow diaphragm of copper or platinum is constantly traversed by a current of cold water.

The photometric measurement should be made at the moment of solidification. Following is the manner in which Violle conducted

* *Lum. Él.*, Vol. XIV. p. 475.

the operation: the gas is shut off and the melted metal is allowed to cool; the luminous intensity diminishes at first rapidly, then more slowly and next becomes stationary, only to begin again to decrease some seconds later after a flash; the moment of making the measurement is thus well determined.

Figures 53 and 54 show the photometric arrangements which Violle used to compare his standard with the carcel lamp. The photometer of Fig. 53 is a Rumford photometer, used in lighthouses, arranged to make the comparison with the light emitted at 45° by the melting platinum; that in Fig. 54 is a Foucault photometer

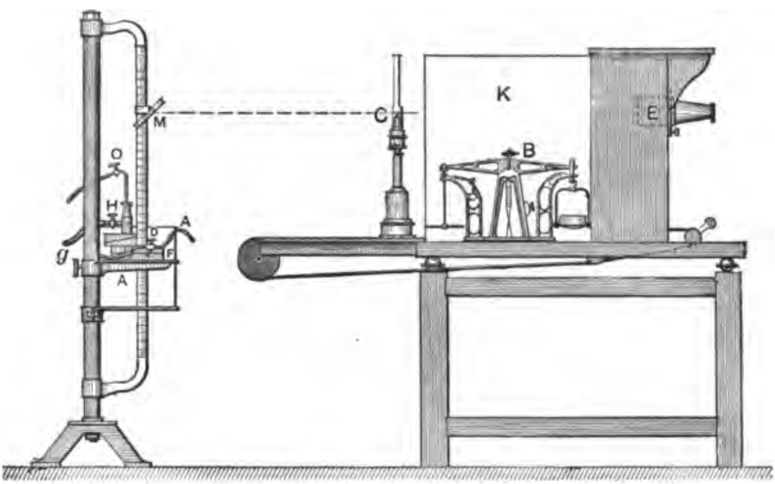


FIG. 54. — Apparatus for the Violle Standard.

with an automatic balance for the carcel lamp; with this photometer is measured light emitted normally and reflected by a mirror at 45°.

Proceeding in this way, Violle found the absolute standard, as defined by the International Conference, equal to 2.08 carcel units.

The principal objections which have been made to the Violle standard are: the complication of the apparatus, the difficulty of measuring, and the high price of the platinum, which must be employed in quite large quantities. From a practical point of view, it was evident from the first that the absolute standard would not be commonly employed; the International Conference in adopting the proposition of Violle cared more for the unification of photometric standards than for the creation of a unit of light directly useful in photometric measurements.

However, it should not be concluded that Violle gave up the introduction of his apparatus into industrial practice; but while the practitioners who sought to simplify the absolute standard have employed very small masses of platinum, thus sacrificing exactness to convenience, Violle has aimed to obtain an apparatus industrial and at the same time fulfilling the promises of exactness of the original apparatus.

Simplified Model of the Violle Standard.

105. According to the plans of Violle, Carpentier constructed the apparatus of which Fig. 55 gives the perspective. The fusion of platinum is obtained by the combustion of illuminating gas in

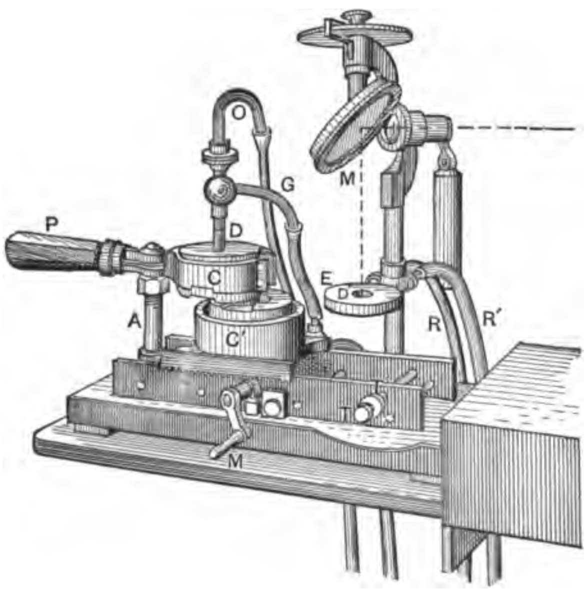


Fig. 55. — Violle Standard (Simplified Form).

oxygen and takes place in a crucible of lime. This crucible is composed of two parts: the lower part has a cavity in which the ingot of platinum is placed; the upper part serves as a cover, and is also hollowed out to correspond to the lower part.

The two blocks of lime of the lower and upper parts of the crucible are mounted in iron; the cover of the crucible has a circular channel *D* which serves to carry the gas. This channel has two

concentric tubes d and d'; the inner tube corresponds to the tube O bringing oxygen, and the exterior tube corresponds to the tube G through which the illuminating gas comes.

The valves admitting the gases are operated by two concentric rods which are moved by means of the buttons T and T'; each of these rods works a rack which gears with a toothed sector in which the valve terminates.

When the platinum is melted, the cover is gently raised from the crucible by pressing on the handle P and moved to one side by turning it about the axis A by means of a lateral pressure on the handle P. We then give to the crucible C a gentle movement back and forth by oscillating the crank M which works the rack F; in this way it is ascertained whether the fusion of the platinum is complete and the surface perfectly clear. By turning the crank M, the crucible is then brought under the screen E. This screen of copper has a circular opening 1 sq. cm. in area; it is hollow in order that a current of cold water, carried by the tubes R and R', may keep it from becoming too hot.

The quantity of light emitted by the surface of the platinum, at the instant of the solidification of the metal, and which traverses the opening O of the screen, is then exactly equal to the absolute unit of light.

The mirror M, which may be regulated by means of a rotation about two axes perpendicular to one another, serves to reflect the light in the desired direction.

The oxygen necessary for the fusion may be obtained in the ordinary manner in proportion as it is needed; but this solution, which is quite acceptable in a laboratory, is not at all practicable for common use. For the latter, it is simpler to employ the oxygen under pressure, which is now to be obtained at a comparatively low price. This oxygen is stored in cylinders under a pressure of 50 atmospheres; as the pressure of the gas on leaving the pipe does not exceed that of a few centimeters of mercury, the pressure of the oxygen should be reduced to the same value. This reduction of pressure is obtained by means of a special regulator fitted to the cylinder.

An ordinary cylinder about 0.30 m. in diameter, and 1.50 m. in length, suffices for a great number of measurements. Thus Violle used the same cylinder for the numerous tests which he made for the benefit of visitors to the photometry room, at the Exposition of 1889.

The crucible is very easily manipulated, and its installation presents no difficulty. It is sufficient to have a gas connection near by, and to have at one's disposal a cylinder of oxygen. It is put into operation very rapidly, and the fusion of a block of platinum of 1 kg. is obtained in about a quarter of an hour.

It is known that the absolute unit of light is obtained at the moment of solidification of the platinum; this is indicated in a precise manner by a characteristic flash which is produced regularly and surely when a mass of platinum of about 1 kg. is employed; it cannot be attained with the same regularity and the same certainty if the mass of platinum is much smaller. This flash is very well observed directly; it is still better appreciated on the photometer by following the variations of the light emitted by the standard up to the time when it emits its flash; it is the reading which is made at this exact moment which should be counted. The latter is obtained very surely with a little experience, for it is determined instinctively by a comparison of the observations made immediately before the solidification with those made immediately after.

The reading made, it is sufficient to replace the crucible under the oxyhydrogen flame; complete fusion is again obtained at the end of several minutes, and a new measurement may be commenced.

A condition essential to exactness of the measurements is the absolute purity of the metal in fusion and the complete absence of films on the surface of the liquid metal. When these appear, they are removed either mechanically or by a chemical reduction.

The great importance of the platinum standard is a consequence not only of its constancy, and the fact that it can be exactly reproduced each time, but also of the quality of its luminous radiations. From the point of view of electric photometry in particular, the composition of the light of the absolute standard is comparable with that of the light of incandescent lamps under ordinary circumstances and, although in a less degree, with that of the arc-light.

The photometric investigation of the arc-light can only be made with exactness when the composition of the light emitted by the source compared sufficiently resembles the voltaic arc; for this reason the platinum standard in the industrial form appears destined to render considerable service to electric photometry, especially as this form of the apparatus is still susceptible of considerable simplification.

It is sufficient, in fact, to apply to the fusion of platinum the

processes of electric fusion in order to simplify not only the apparatus, but especially its installation and manipulation. Electricians would familiarize themselves much more rapidly with the platinum standard were they not obliged to have recourse to oxyhydrogen fusion, for electric fusion may be effected rapidly and easily with the resources of every well-equipped electro-technical laboratory.

As electric fusion takes place generally by means of very intense currents, it would be easiest to employ the current furnished by a battery of accumulators charged in series and discharged in parallel.

The Violle-Siemens Standard.

106. W. Siemens sought to attain the legal standard in such a way as to satisfy the requirements of practice while conforming as much as possible to the legal definition. However, the simplification attained by Siemens was at the expense of exactness, in this way that the apparatus does not exactly fulfil the conditions which the International Conference resolved upon.

The platinum is taken at its point of fusion and not at its point of solidification. It is not known for certain whether there exists any difference between the points of fusion and solidification of platinum. As to the constancy of the light emitted at this moment, it has not been perfectly proven; thus Cross has observed that the luminous intensity is somewhat greater if one employs platinum which has been melted many times; the influence of this is insensible on the point of solidification, which is an additional argument in favor of Violle's method.

Figures 56 and 56 *bis* represent horizontal and vertical cuts of the apparatus; it consists essentially of a small metallic box, one of whose sides is pierced with a conical opening; the surface of the smallest circle measures exactly one-tenth of a square centimeter. Immediately behind this window there is a very thin (0.02 mm.) ribbon of platinum 5 or 6 mm. in width, which extends beyond the edge of the opening in all directions. Through the platinum ribbon there is passed an electric current whose intensity goes on increasing gradually; the brightness of the light emitted through the conical opening increases also gradually up to the instant when the platinum melts and the brightness suddenly disappears.

This progressive increase in the intensity permits the operator to balance in the photometer at each instant the illuminations of the lamp studied and of the platinum standard. The quantity of light

emitted by this apparatus, at the instant of the fusion of the platinum, is equal to a tenth of the absolute standard, i.e. about 0.2 carcel.

The International Congress of Electricians of 1889 having decided to give the name of decimal candle (*bougie décimale*) to the twentieth part of the absolute platinum unit, the Violle-Siemens standard has thus a luminous intensity equal to two decimal candles.

A special mechanism, inside the case, managed by the handle *g*, is designed to bring a new ribbon of platinum before the window in place of the one which has been melted; the experiment may then be repeated without loss of time.

Liebenthal*, in the course of very extended investigations of the Siemens standard, modified to some extent the original apparatus;

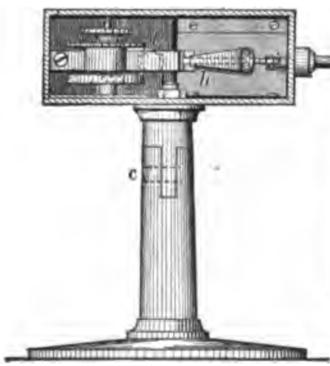


FIG. 56. — Violle-Siemens Standard (Vertical Section).

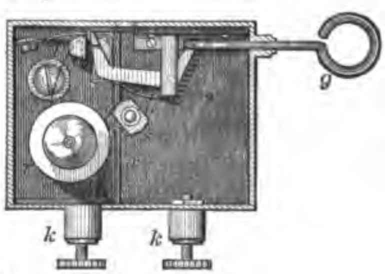


FIG. 56 *bis*. — Violle-Siemens Standard (Horizontal Section).

the small metallic box *K* bears the metallic plate *A*, insulated from the frame, on which there is a coil *R* of platinum ribbon; this plate bears, further, the small movable guide *m*, and the fixed guide *M* of greater size; one of the terminals *P* of the apparatus is also connected with this plate, while the other *Q* is fixed directly in the side of the metallic box. The platinum ribbon, after having left the cylinder *R*, passes by the guides *m* and *M*, against which it is pressed by the spring *f*, and before the opening *D* whose area is exactly 0.1 sq. cm.; it is then grasped by the clamp *S*; a key which governs the guide *m* allows the tension of the platinum ribbon to be varied.

* *Lum. Él.*, Vol. XXXI. p. 116.

The current brought to the terminal *P* enters the platinum ribbon at the guide *M* principally, then is conducted by the spring *Z* to the terminal *Q*. When the circuit is broken because of the fusion of the platinum, the rod *g* is pressed, which pushes the movable clamp *S* upon the plate *A*, where it opens; when the rod *g* and the movable clamp *S* are drawn back, the clamp closes and draws along a new piece of the platinum ribbon to be used; with 1 gram of metal, costing about 60 cents, 50 measurements may be made.

The first measurement of Liebenthal gave good enough results; but soon quite large errors appeared, caused by irregularities in the spring *f*, which rested on the guide *m* only; the portion of the platinum ribbon traversed by the current thus being longer, fusion took place near *M* and not opposite the opening *D*.

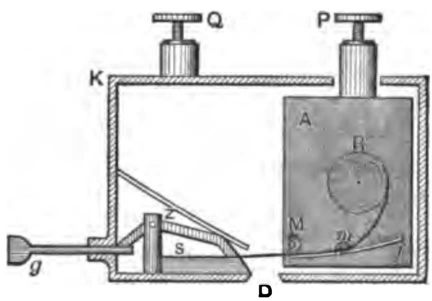


FIG. 57. — Violle-Siemens Standard (Liebenthal Form).

Liebenthal was then led to somewhat modify his apparatus; doing away with the contact at *M* or at *m* not having given good results, a simple screw, insulated from the metal box and pressing the platinum ribbon against *M*, replaced advantageously the original arrangement. The guide *m* was also done away with, because the tension of the ribbon could be regulated well enough by the clamp *S*, and experience, furthermore, showed that the degree of tension of the platinum ribbon had only a very slight influence on the luminous power of the lamp. To obtain good contacts, the cylinder *M* must further be covered with a strip of platinum; with these modifications, the apparatus worked successfully from that time.

When rolling the platinum on the cylinder *R*, care should be taken to avoid wrinkling the ribbon, for the least break displaces the point of fusion and modifies the light emitted.

Increase in the intensity of the current produces at first a rapid increase in the luminous intensity; it then becomes slower as the instant of fusion approaches; finally, fusion takes place suddenly, and the light disappears.

The last photometric setting which is made before extinction is alone valuable.

Liebenthal made a certain number of comparisons with Hefner's acetate of amyl lamp, taking with the latter all the precautions enumerated above and employing a liquid carefully rectified.

He found that the mean deviation of a photometric comparison between the acetate of amyl lamp and the platinum standard was about 2.9 per cent, while other measurements gave 0.9 per cent for the mean deviation of the comparisons of the acetate of amyl lamps.

These variations, as Liebenthal elsewhere remarks with justice, do not result alone from such large real variations in the luminous intensity of the platinum standard, due for instance to variations in the molecular structure and in the emissive power of the metal; they are due in great part to the difference in color of the two sources of light. At the moment of fusion, the platinum standard emits a much whiter light than that of the acetate of amyl lamp; the measurements are then affected by all the causes of error which render the comparison of two differently colored luminous sources so difficult.

The luminous intensity of the platinum standard in terms of the acetate of amyl lamp was also determined many times by Liebenthal. He found that 1 Violle-Siemens unit = 1.757 Hefner units.

Whatever may be the merits of the Siemens standard and the ingenuity displayed in the details of the apparatus, it is important not to lose sight of two principal defects.

The observer must make the setting at the precise moment when fusion takes place; now at this same moment the emission of light ceases suddenly. The photometric setting must then be made, so to speak, on the wing.

Further, the Siemens standard does not exactly reproduce a tenth of the absolute standard of Violle; for the observation is made at the moment of fusion and not at that of solidification of the platinum, which may produce a quite considerable difference with respect to the quantity of light emitted; further, very slight lack of uniformity in the section of the ribbon and of homogeneity in the metal is enough to cause very sensible variations.

107. To conclude, we give a double-entry table which sums up all the comparisons of different photometric standards made by Violle.

	Violle Units.	Carcels.	Star Candles.	German Candles.	English Candles.	Hefner-Alteneck Lamps.
Violle units . .	1.000	2.08	16.1	16.4	18.5	18.9
Carcels . . .	0.481	1.00	7.75	7.89	8.91	9.08
Star candles .	0.062	0.130	1.00	1.02	1.15	1.17
German candles	0.061	0.127	0.984	1.00	1.13	1.15
English candles	0.054	0.112	0.870	0.886	1.00	1.02
Hefner-Alteneck lamps . . .	0.053	0.114	0.853	0.869	0.98	1.00

CHAPTER IV.

GENERAL EQUIPMENT AND AUXILIARY APPARATUS OF PRACTICAL PHOTOMETRY.

108. The preceding chapters have been devoted to the study of photometric apparatus and units of measurement. There remains to be found the best manner of arranging them for measurements and the precautions which must be taken to obtain satisfactory precision.

Photometry Room.

109. In every laboratory where it is desired to make photometric measurements, there should be set apart a special place, judiciously chosen, and satisfying the following fundamental conditions.

The photometry room should be large enough. It is a great error to crowd the photometric apparatus into a small room, especially when photometric standards based on combustion are used, e.g. carcel, Hefner, petroleum lamps, candles, etc.

The luminous intensity of these photometric standards varies greatly with the degree of purity of the surrounding air; it diminishes in proportion as the quantity of carbonic acid gas in the air increases. For instance, at the end of an hour the products of respiration of two people are sufficient to vitiate the air in a large room so as to produce a very noticeable diminution in the luminous intensity. Further, the presence of lights brings about an elevation of temperature which contributes to render the measurements less exact, owing to the fatigue of the observer. This can be remedied only imperfectly in a photometer room of small dimensions by introducing a little stronger system of ventilation. No attempt should be made to use ventilating apparatus during the measurements themselves, for the photometric standards should be free from even the smallest currents of air. Recourse should be had to it between measurements, which, however, produces each time a disturbance in

the regulation of the standard lamp. We do not know then whether the luminous intensity of the lamp returns to its initial value or not.

The photometry room should have its walls painted a dull black, and it should be possible to obtain complete darkness. This condition is indispensable if the photometric screen is to receive only such light as comes from the two sources to be compared. It moreover permits the eye to rest in the interval between measurements.

The light necessary for reading the apparatus and making notes should be furnished by lamps of small intensity, fitted with reflectors which do not allow the light to be diffused over the room.

During the setting of the photometer, the observer should be shielded from luminous rays coming directly from the two sources. For this there may be used either screens or a black cloth like that used by photographers. The latter has, however, the objection that it makes the observer very warm.

Personal Errors.

110. Like all measurements in which the personality of the observer enters directly, photometric measurements are affected by personal errors which may be very considerable. In a general way the personal error is proportionately less when the observed phenomenon is precise and leaves the observer no chance for doubt. In photometric measurements the determination of the equality of the illumination of the two parts of the screen depends very largely on the judgment of the observer, especially when there is a difference in tint. From this some sensible personal errors must result; this was discovered as soon as the exactness of photometric measurements permitted. This fact was proven only recently in a precise manner by Nichols*. The following is the method used by him, a method which may serve as an example for researches of this kind.

Three incandescent lamps of 16 candle power at 110 volts, chosen so as to have as nearly as possible the same luminous intensity, are placed in shunt, using a battery of accumulators of 120 volts, the circuits separating beyond a rheostat which regulates the intensity of the current.

One of the lamps L_1 being taken as a standard, the two others, L_2 and L_3, are successively compared with it, so as to obtain the

* *Lum. Él.*, Vol. XXXIII. p. 414.

ratio of their luminous intensities, $\frac{I_2}{I_3}$. It is found, for instance, in this way, that

$$\frac{I_2}{I_3} = 1.0032 \pm 0.0015.$$

Next, while always preserving the same intensity of current through the lamps, we determine the ratio $\frac{I_2}{I_3}$ by directly comparing the luminous intensities of the two lamps, the lamp L_2 being placed at the right, the lamp L_3 at the left of the screen.

To avoid errors due to slight variations in the intensity of the currents, the lamps studied were given 12 instead of 16 candle power; for, at this intensity, variations of current are much less felt in the luminous intensity.

The ratio $\frac{I_2}{I_3}$ was determined by ten different observers. The following table gives the values obtained in this manner, as well as the corresponding personal error, which was calculated by assuming as the real value of $\frac{I_2}{I_3}$ the quantity 1.0032, obtained by double comparison. Each number is the mean of the results of ten different measurements; the probable error of this mean was determined.

Observer.	Value of $\frac{I_2}{I_3}$.	Personal Error.
A	1.0590 ± 0.0040	− 0.0558
B	0.9701 ± 0.0044	+ 0.0331
C	1.0021 ± 0.0022	− 0.0189
D	1.0191 ± 0.0072	− 0.0159
E	1.0182 ± 0.0039	− 0.0150
F	1.0902 ± 0.0057	− 0.0870
G	1.0733 ± 0.0053	− 0.0701
H	1.0293 ± 0.0042	− 0.0261
I	1.0297 ± 0.0050	− 0.0263
J	1.0220 ± 0.0027	− 0.0188

It is then seen that the ratio $\frac{I_2}{I_3}$ was found by all the observers, except one, greater than 1.0032, the value obtained by indirect comparison which eliminates personal errors. The personal error of the ten observers, then, varies between -0.0870 and $+0.0331$; i.e. between -8 per cent and $+3$ per cent.

111. These measurements were made by means of the Bunsen photometer, having two lateral Rudorff mirrors. The observer viewed the two images of the spot, the left image with the left eye, the right image with the right eye.

Nichols assumes that in this case the personal error is due in a great part to a difference in sensitiveness of the eyes of the observer; the latter then judges that the screen is at too great a distance from the lamp whose rays fall directly on the side of the spot which is observed by the less sensitive eye. According to this explanation, the right eye must have been more sensitive than the left for the nine observers who had a negative personal error, while the inverse must have been true for the tenth (*B*).

Afterwards some observations were made with one eye, the other being blind-folded; these observations became more difficult, but on the other hand more sure, and likewise the observer felt more confidence in them. Further, the results obtained with the left eye were identical with those obtained with the right. Thus the values obtained by the observers *A* and *B*, which differed at first by 8 per cent, are absolutely concordant when only one eye at a time is used in the observations.

Below are some significant figures:

Observer.	Eye.	$\frac{I_2}{I_1}$	Personal Error.
A	Right.	1.0028 ± 0.0010	0.0004
A	Left.	1.0001 ± 0.0019	0.0031
B	Right.	1.0001 ± 0.0017	0.0031
B	Left.	1.0031 ± 0.0018	0.0001

The preceding shows then that the personal error is far from being a negligible quantity in photometry; the only means of remedying it is to employ photometers which allow monocular observations.

The Photometric Bench.

112. In the study of the Foucault photometer (§ 20), we described the complete equipment and the apparatus for measuring and regulating the distances d_1 and d_2 of the radiants from the screen. We did the same for the Bunsen photometer (§ 24).

It is, however, proper to add to these descriptions, for the photometric bench is an essential part common to the majority of the numerous photometric apparatus which we have described in the second chapter.

The photometric bench is an optical bench strongly and carefully constructed. Its object is to permit the measurement of the distances d_1 and d_2 from the screen to the two radiants. It has a divided scale on which the positions of the screen are read.

In the majority of cases, the two lights to be compared are fixed at the ends of the bench, and the reading is effected by moving the screen alone. This method is the most advantageous, as the observer regulates at will the position of the screen so as to obtain the most precise setting.

The intensity of the standard being I_1, that of the source studied I_2, we have the equation,

$$I_2 = \frac{d_2^2}{d_1^2} I_1.$$

The length of the photometric bench being represented by l, and the distance d_1 from the screen to the standard by x, we have

$$d_2 = l - x,$$

and the preceding equation becomes

$$I_2 = \frac{(l-x)^2}{x^2} I_1.$$

It is advantageous to calculate a table of values of the fraction $\frac{(l-x)^2}{x^2}$ for the length l of the photometric bench employed. This table may be calculated for values of x varying from millimeter to millimeter in the parts of the bench most frequently used.

113. The table of values of the coefficient $\frac{(l-x)^2}{x^2}$ is of great service in the standardizing of incandescent lamps. It is well

known that there should be determined for each lamp the voltage which really corresponds to the nominal luminous intensity. The lamps are then classified according to their voltage.

There is employed as a photometric standard an incandescent lamp whose voltage is maintained constant and whose corresponding luminous intensity is exactly known.

Let us assume that the lamp to be studied should give a luminous intensity of I_2 candles. This intensity will really be obtained when the screen occupies on the photometric bench the division x determined by means of the equation

$$I_2 = \frac{(l-x)^2}{x^2} I_1,$$

whence

$$x = \frac{l}{1 + \sqrt{\frac{I_1}{I_2}}}.$$

A single example will suffice to show the use of this formula.

Let us suppose that the standard I_1 gives 12 candles at 65 volts. What position must be given to the screen in order that the lamp studied may give 16 candles at the moment when the photometric setting is exact?

Let us assume that we have $l = 300$ cm., which is a very practical value. We shall have

$$x = \frac{300}{1 + \sqrt{\frac{16}{12}}} = 139.5 \text{ cm.}$$

We may calculate a table for x in terms of the ratio $\frac{I_2}{I_1}$ and for the length of bench used.

Following is a table of this kind calculated for $l = 300$ cm. The table is arranged as usual with the tens in the left-hand vertical column and the units in the top row. In the numerical example preceding we had $\frac{I_2}{I_1} = \frac{16}{12} = 1.33$. Looking up 1.33 in the table, we find $x = 139.3$.

x.	0	1	2	3	4	5	6	7	8	9
50	25.0	23.8	22.7	21.7	20.8	19.8	19.0	18.2	17.4	16.7
60	16.0	15.4	14.7	14.2	13.6	13.1	12.6	12.1	11.6	11.2
70	10.8	10.4	10.0	9.7	9.3	9.00	8.69	8.39	8.10	7.83
80	7.56	7.31	7.07	6.84	6.61	6.40	6.19	5.99	5.80	5.62
90	5.44	5.27	5.11	4.95	4.80	4.66	4.52	4.38	4.25	4.12
100	4.00	3.88	3.77	3.66	3.55	3.44	3.35	3.25	3.16	3.07
110	2.98	2.90	2.82	2.74	2.66	2.59	2.52	2.45	2.38	2.31
120	2.25	2.19	2.13	2.07	2.01	1.96	1.91	1.85	1.80	1.76
130	1.71	1.66	1.62	1.58	1.53	1.49	1.45	1.42	1.38	1.34
140	1.306	1.271	1.238	1.205	1.173	1.142	1.113	1.083	1.055	1.027
150	1.000	0.974	0.948	0.923	0.899	0.875	0.852	0.830	0.808	0.787
160	0.765	0.745	0.726	0.706	0.688	0.669	0.652	0.634	0.617	0.601
170	0.585	0.569	0.554	0.439	0.524	0.510	0.496	0.483	0.470	0.457
180	0.444	0.432	0.420	0.409	0.397	0.386	0.376	0.365	0.355	0.345
190	0.335	0.326	0.316	0.307	0.298	0.290	0.282	0.273	0.265	0.258
200	0.250	0.243	0.235	0.228	0.221	0.215	0.208	0.202	0.196	0.190
210	0.184	0.178	0.172	0.167	0.161	0.156	0.151	0.146	0.141	0.137
220	0.132	0.128	0.123	0.119	0.115	0.111	0.107	0.104	0.100	0.096
230	0.093	0.089	0.086	0.083	0.080	0.076	0.074	0.071	0.068	0.065
240	0.063	0.060	0.057	0.055	0.053	0.050	0.048	0.046	0.044	0.042
250	0.04	0.038	0.036	0.035	0.033	0.031	0.030	0.028	0.027	0.025

114. To conclude, we give a description of the photometric bench of the Physico-Technical Institute of Berlin, which was used in the researches of Lummer and Brodhun.

It consists of two steel bars more than 2 m. in length; these have a thickness of 25 mm., a height of 50 mm., and the distance between them is about 100 mm. Under these circumstances bending of the bench is not to be feared. Three cars roll on these bars with a very easy movement. They may be stopped by means of a lever in any position on the bench. Each of them has a vernier which reads to about 0.3 mm. on the millimetric divisions engraved on the upper face of one of the bars.

The body of each car consists of a rather thick metallic sheet with an opening in which a steel tube may be moved vertically. On these tubes are fixed the photometric box and the supports of the two luminous sources respectively. Care is then taken to regulate each vernier so that its zero may coincide with the axis of the vertical tube.

Further, the car of the photometric box has an arrangement

which allows it to be moved rapidly 2 or 3 cm., which is indispensable for verifying the exactness of the photometric setting.

Benches planned for industrial measurements need not necessarily be constructed with so great care; but they should comply to some extent with the principal conditions mentioned above.

Equipment of the Photometric Laboratory.

115. It is not possible to give in advance plans and details for the equipment of a laboratory of photometry, for they depend too much on the object for which the laboratory was built. There is in this regard a fundamental difference between a laboratory for research or for instruction, and an industrial laboratory for testing. We shall not occupy ourselves with the former kind except to mention some remarkable installations.

One of the first electro-photometric laboratories built in all particulars according to a determined plan is that which served at the

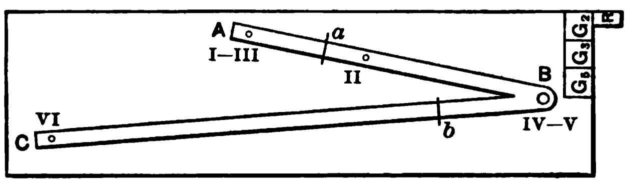


FIG. 58. — Photometry Room at the Munich Exposition.

tests of the Committee of Experiments at the Electrical Exposition at Munich in 1882. Figure 58 gives the arrangement of the apparatus. AB and BC are two photometric benches 6 and 12 m. in length, respectively, on which may be moved the Bunsen screens a, b. The standard adopted was the candle, with various intermediate standards, *e.g.* the candle-burner (*bec-bougie*) of Giroud, shown at II, regulating the Argand burner at III, and an intensive Siemens burner shown at V. The Argand burner served to measure incandescent lamps placed at IV, and the intensive Siemens burner to measure arc-lamps placed at VI. The supply of the three gas-burners was measured by three meters G_2, G_3, G_5 regulated by the general regulator R.

The standard candle is placed at II, the incandescent lamp at I, and a large petroleum lamp occupies the place of the Siemens intensive burner at V.

The arrangement of the various intermediate standards shows the order in which the measurements should be made. First the candle-

burner II is compared with the candle I, then the candle-burner II with the Argand burner III; and, finally, this last with the incandescent lamp IV. The light sources II, III, IV burn throughout the measurements. They are masked by a screen when not in use.

To measure the intensity of the arc-lamp at certain inclinations, there is fixed to the end C of the photometric bench a mirror movable about a horizontal axis. The lamp VI is then raised to well-determined heights, corresponding to inclinations of 15°, 30°, etc., so that the rays may be reflected by the mirror parallel to the axis of the photometer.

The striking thing about this equipment is the number of intermediate standards furnished by gas-burners. Nowadays one would rather have recourse to incandescent lamps.

The equipment of the laboratory of the Electrical Exposition at Vienna in 1883 only differed from that at Munich by doing away with the gas-burners employed as intermediate standards.

At Philadelphia, the committee of the Franklin Institute employed with the greatest success the Methven screen (Fig. 48) combined with a burner of the Argand type, while at Antwerp, as at Paris in 1881, use was made of the carcel lamp. For arc-lamps, however, an intensive Siemens burner was used at Antwerp as an intermediate standard.

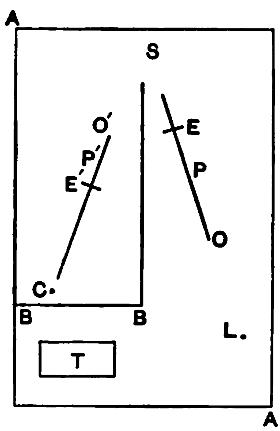


FIG. 59. — Photometry Room at the Antwerp Exposition.

Figure 59 shows the equipment of the photometry room used by the Committee on Tests at Antwerp. This room was divided by a partition BBB of black cloth. The photometric bench P served to compare the arc-lamp L with the Siemens burner placed at S; a second bench placed at P' served to measure the intensity of the Siemens burner in terms of the carcel standard. Observations were made simultaneously with both apparatus.

Among the number of best equipped laboratories of photometry should be cited the one which D. Monnier installed in 1883 for the Association for the Study of Electricity, formed by the principal French gas companies. The photometric part of this laboratory is provided with the latest apparatus.

An industrial laboratory for testing does not necessarily include

all the apparatus of a laboratory for investigation or instruction, but the general plan remains the same as that which is given above, if the measurement of the luminous intensity of arc-lamps is to be included. If measurements are restricted to incandescent lamps, the equipment may be considerably simplified.

The choice of the photometric screen is very important. Up to the present, the Foucault screen and the Bunsen screen have had the preference. But the Lummer and Brodhun screen is much superior to them and ought to be adopted as far as possible.

As to auxiliary apparatus, there is room for choice among those whose description will follow.

Dibdin's Radial Photometer.

116. Arc and incandescent lamps emit quantities of light varying with the direction of the luminous rays; this variation is much greater with these than with ordinary gas-lights. Therefore it is of the greatest importance to be able to measure the intensity of a radiant in any direction.

Attention was called to this for the first time by Allard in his memoir on the intensity and range of lighthouse beacons.

The method which presents itself is to turn the entire photometric bench so as to place it in the direction of the luminous rays. It is this which Ayrton and Perry (§ 39) realized in their dispersion photometer. Weber's photometer (§ 52) also realizes this condition.

The apparatus, being movable about a vertical axis, may be turned in any azimuth. As the tube which serves to determine the direction of the photometer from the light studied is movable about a horizontal axis, it follows that it may be set equally well at any inclination.

Ordinary screens, the Bunsen screen for instance, may, however, be employed for the measurement of the inclined rays, account being taken of the fact that the illumination of the screen does not depend on the distance alone of the luminous sources, but also on the angle of incidence of the rays which fall on the screen. Naturally account should also be taken of the loss of light due to absorption and reflection; this loss increases with the angle of incidence of the rays.

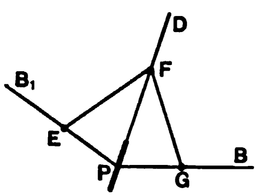
Dibdin found, for instance, that for the Bunsen screen which he employed it was:

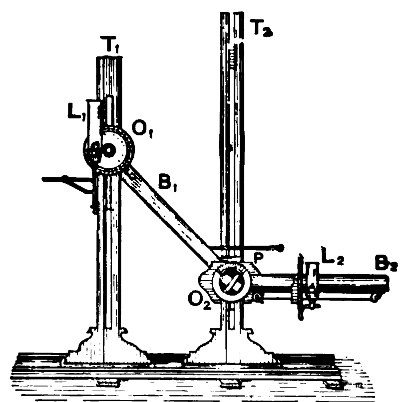
5 %	for an angle of incidence of	22.5°.
12	" " " " " "	45°.
68	" " " " " "	67.5°.

Unfortunately these numbers are only of value for Dibdin's particular screen, and should be determined anew in each particular case.

This correction may be eliminated by so placing the photometric bench that the rays strike the two sides of the screen at the same angle. Hartley * first proposed to make the screen movable about an axis, so as to place it always in the bisecting plane of the dihedral angle formed by the two radiants and the photometer.

Dibdin realized this arrangement in the radial photometer †, the arrangement of which is given in Figs. 60 and 61.





FIGS. 60 and 61. — Dibdin's Photometer.

Upon a horizontal base rest two vertical guides; the guide T_1 is fixed and carries the light to be studied L_1 on a block movable in a slot. The guide T_2, which may be moved horizontally, supports at P the photometric screen.

The two guides further carry two arms which are hinged at O_1 and O_2. The arm B_1 has an index which shows on a divided circle at O_1 the angle which the rays from L_1 falling on the photometric screen make with the horizontal. At O_2 there is further a sector divided into half-degrees. To make a measurement, the index of the screen is placed on the division of O_2 which corresponds to the

* *Lum. Él.*, Vol. X. p. 58.
† *Lum. Él.*, Vol. XXX. p. 227.

number on O_1. The photometric standard L_2 is supported on the divided arm B_2, along which it may be moved at will. The length of the arm B_1 being constant, the distance of the radiant L_1 from the screen P is invariable, so that the arm B_2 may be graduated directly in candles.

Dibdin's radial photometer may be simplified by making the movements of the screen depend on those of B_1 and B_2, so that the screen is always placed in the bisecting plane of the angle formed by B_1 and B_2. This simplification is obtained in the apparatus shown in the preceding figure by means of the articulations EF and GF. The graduated sector O_2 may then be omitted.

Rousseau's Radial Photometer.

117. At the time of the photometric measurements of the arc-lamps exhibited at Antwerp in 1885, Rousseau* invented an apparatus which is much like that of Dibdin, but which provides for the employment of the Rumford photometer. Below is a description of this apparatus (Fig. 62).

The lamp A is suspended between two uprights, and a mechanism operated by the crank W enables it to be raised and lowered. On these uprights there is also fixed a circular box E; from the center of this box two rods diverge, one, G, horizontal, the other, F, inclined, each carrying a movable mirror N and M. At the center of the box E is found a white screen O, carried on the rod OH; this forms one of the diagonals of an articulated quadrilateral $OKHI$, so that the screen O always makes equal angles with the direction of the rods G and F.

When it is desired to use this apparatus, the lamp A is placed behind the graduated circle E, so that the light is opposite the center O, and at a distance as small as the form of the lamp studied will permit. The light emitted by this source is reflected by the mirrors M and N (cut from the same glass), which project on the white screen the shadows of the two rods m and n, also fixed on the rods G and F. One of the mirrors being fixed, the other is moved in or out until equality of the shadows projected is obtained.

Krüss has constructed a model of Rousseau's photometer combined with a Bunsen screen, for the use of those who prefer this photometer to that of Rumford. This model is made with great

* *Comptes rendus des travaux de Comité internationale des essais électrique de l'Exposition d'Anvers*, p. 85.

care; it possesses, in particular, an arrangement which allows one to determine quickly whether the voltaic arc is accurately centered with reference to the apparatus or not.

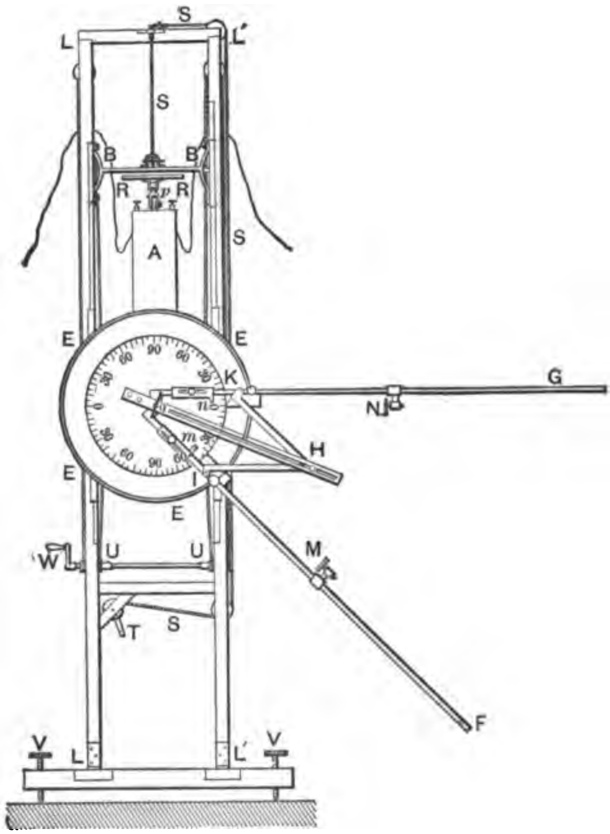


FIG. 62. — Rousseau's Photometer.

Vernon-Harcourt's Holophotometer.

118. This apparatus * is based on the employment of the Bunsen screen combined with a system of mirrors, planned so as to avoid errors due to the movements which the lights compared undergo. What characterizes this apparatus is, that the lamp to be measured and the system of mirrors are not placed on the photometric bench,

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* *Lum. Él.*, Vol. XXIX. p. 286; *Elect. Rev.* (London), July 13, 1888.

but on a table or on independent supports; the screen alone is movable on the photometric bench.

The apparatus is composed of two mirrors, the larger of which (Fig. 63) is clamped at the end of a horizontal axis carried by a support B; the center of the mirror corresponds to the center of the axis, but the mirror may be inclined at any angle. This axis is placed at the height of the Bunsen screen and in the direction of the axis of the photometer. At the other end of this axis (which is not shown in the figure) is hinged a sliding arm carrying at its end a

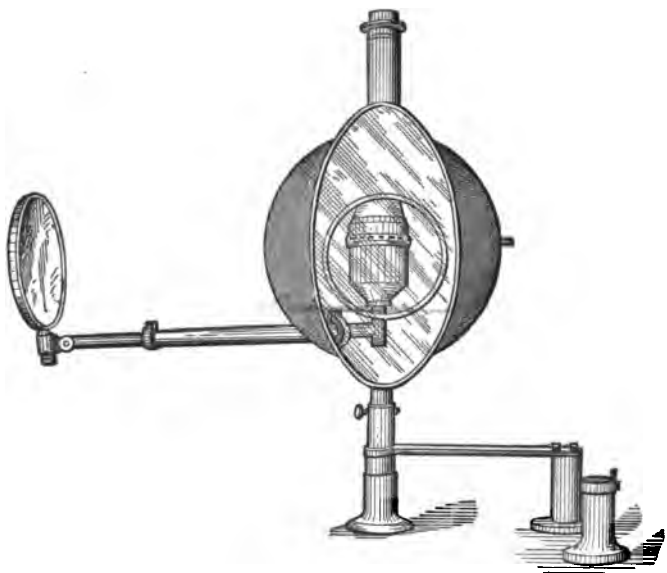


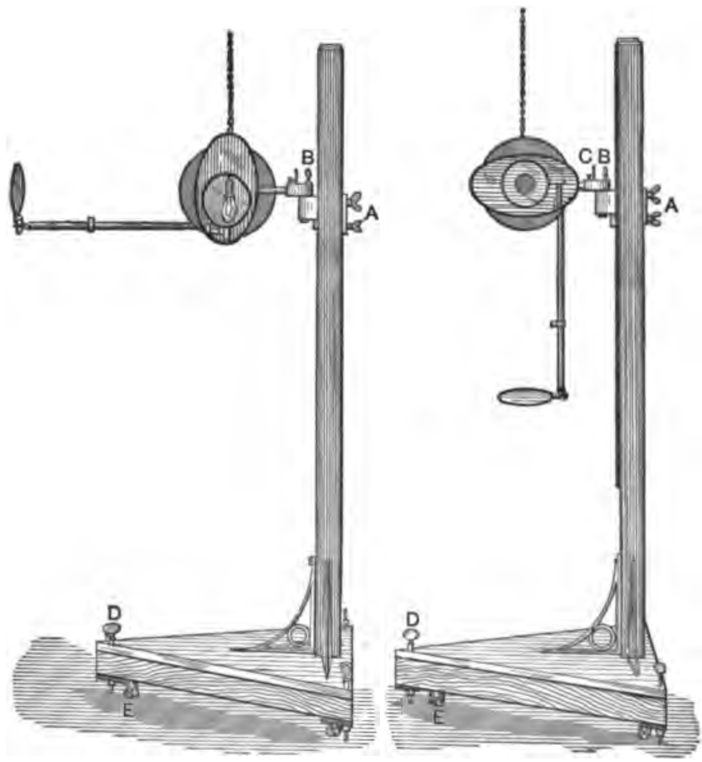
FIG. 63. — Vernon-Harcourt's Holophotometer.

small mirror. By this arrangement it is seen that the two mirrors always turn together about the axis of the photometer, and these rotations are read on the divided disc, which, further, serves as a screen and avoids having the direct rays from the lamp to be studied fall on the Bunsen screen.

Figures 63 and 64 show the apparatus arranged for the measurement of the horizontal rays from the lamp L placed behind and masked by the divided circle. Figure 65 shows the arrangement of the apparatus to measure vertical rays. It is evident from the construction that when once the rays have been adjusted along the axis

of the photometer they will remain there for all positions of the arm *A*, taking with them the horizontal axis of the mirror *M*.

We may thus make all the relative measurements of the luminous intensity at a given angle; to make an absolute measurement, we commence by comparing the horizontal rays emitted by the lamp *L* both with and without the system of two mirrors, very easily removed



FIGS. 64 and 65.

because of the arrangement of the support; there is thus obtained the factor of reduction by which the intensities found must be multiplied to compensate for absorption; naturally, account should be taken, in the measurements, of the increase in the distance due to the various reflections of the rays.

Preliminary measurements showed that the absorption by the two mirrors was only 1.8 per cent. [See Appendix D.]

Millis's Arrangement.

119. Millis replaced the reflecting mirror by a total reflection prism, while using the ordinary photometric bench and the Bunsen screen.

Figure 66 shows the general arrangement of the apparatus in the plan and elevation. The electric lamp is placed on its support at b and b', while at p and p' there is a total reflection prism.

This prism is mounted at the end of a copper tube fixed on a tripod; the tube has a plumb-line. The perpendicular faces of the prism must be large enough (13 sq. cm. at least). The prism may be moved about three axes which intersect at the middle of the principal edge; because of this arrangement the point remains fixed, regardless of the various rotations which may be given to the prism by means of screws. A pointed rod may be so placed that, when put in place of the prism, its point occupies exactly the point where the middle of the principal edge of the latter was found.

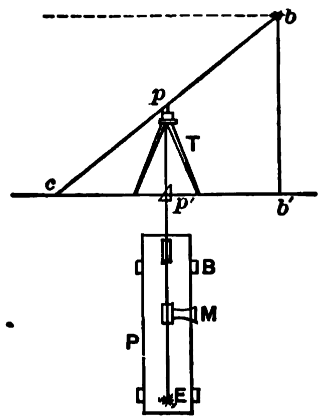


FIG. 66.

To make measurements, we first find the foot b' of the vertical line passing through the lamp, and lay off from this point a line of length $b'c$ determined by the angle at which we wish to measure the luminous intensity of the lamp. Next, a cord is stretched from b to c, and the tripod, with its copper point fixed in the support, is moved until the point touches the cord. From the point p' determined by the plumb-line a perpendicular is drawn to $b'c$, and the photometric bench is so arranged that its axis passes through this perpendicular.

It is necessary to adjust the prism until the rays which come from the source studied are reflected parallel to the photometric bench. We should then determine the correction to be applied to the measurements in order to take account of absorption of light by the prism.

The Employment of Mirrors.

120. The preceding apparatus permits photometric comparisons to be made at any angle. It is, however, not absolutely necessary to have recourse to special apparatus; most frequently the ordinary photometric bench may suffice, care being taken to employ a mirror reflecting the rays from the source studied horizontally upon the screen. It is this method which was adopted by the Committee on Tests at Munich.

The mirror must be movable about a horizontal axis, and its inclination must be easily measured on a divided circle. As the rays coming from the source make an angle α with the normal to the mirror, it is necessary, in order that they may be reflected in a horizontal direction, that this angle α should be exactly half of the angle formed by the rays with the horizontal.

This angle is calculated in advance by measuring the height h of the light above the axis of the mirror and its horizontal distance d to the mirror. It follows that $\tan 2\alpha = \frac{h}{d}$; it is then sufficient to place the mirror at the division α of the divided circle in order that the rays from the source may be reflected horizontally.

In order to vary the angle α, *i.e.* the inclination at which the luminous intensity is measured, we may suspend the source studied by a cord and elevate and lower it at will, or, again, increase or decrease the distance of the mirror from the foot of the perpendicular passing through the source.

This arrangement is satisfactory if the source can be placed vertically above the axis of the photometer. But this is not always the case. The mirror should then be movable about a vertical axis also. The angle of rotation about the vertical axis corresponds to the azimuth A, while the angle of rotation about the horizontal axis corresponds to the height h. We have then, by formulæ of spherical trigonometry, as the condition for the reflected rays being horizontal,

$$\cos 2\alpha = \cos A \cos h.$$

The following arrangement may also be recommended. A horizontal axis R is placed parallel to the axis of the photometer and above it. There is then fixed on this axis a movable arm to which is suspended the radiant, an arc-lamp for instance [the arc being at the same distance below the point of suspension as the photometer axis is below the axis of the arm]. The point of attachment of the

arm is directly above the mirror. Turning this arm, the radiant describes a portion of a circumference of which the mirror occupies the center. The mirror remaining fixed, the distance from the radiant is invariable.

Sautter and Lemonnier also employed advantageously the following arrangement. The arc-lamp is placed on a support movable about a vertical axis, opposite a divided scale on which an index shows exactly the height of the axis.

The mirror is fixed on a divided rod and is movable about a horizontal axis. It has an index which allows the determination of the inclination of the mirror by a simple reading on the rod and an easy calculation.

All the preceding apparatus call for the employment of one or more mirrors. Before using them, the loss of light due to absorption of the mirror should be determined; that is, we should determine the coefficient of reflection at different angles. This measurement is made most easily in the following way.

By means of the ordinary photometer, we compare the luminous intensities of two radiants as constant as possible; for instance, two petroleum lamps or two incandescent lamps. We thus obtain the ratio $\frac{1}{a_1}$. Next, the same determination is made by means of the mirror, and another ratio $\frac{1}{a_2}$ is obtained. The coefficient of reflection of the mirror is then $a = \frac{a_2}{a_1}$, and the loss by absorption is represented by the expression $(1 - a)$. All the results obtained with the mirror should then be multiplied by the factor $\frac{1}{a}$.

Below are some values obtained by Sautter and Lemonnier, using a silvered mirror, and others obtained at the Munich Exposition in the same way.

Angle of Incidence.	Sautter and Lemonnier.	Munich.
	a.	a.
5°	0.68	....
10°	0.74	0.700
15°	0.81	0.690
20°	0.85	0.696
25°	0.85	0.700
30°	0.85	0.695
40°	....	0.696

Recent measurements by Uppenborn confirm these results and show in particular that absorption depends in a sensible manner on the angle of incidence; it is necessary then to determine the coefficient of diminution of a mirror for the various values of the angle of incidence at which it is employed.

Incandescent Lamp-Holders used in Photometry.

121. It is also indispensable to measure the luminous intensity of an incandescent lamp in several directions; however, we usually confine ourselves to making these measurements in the same horizontal

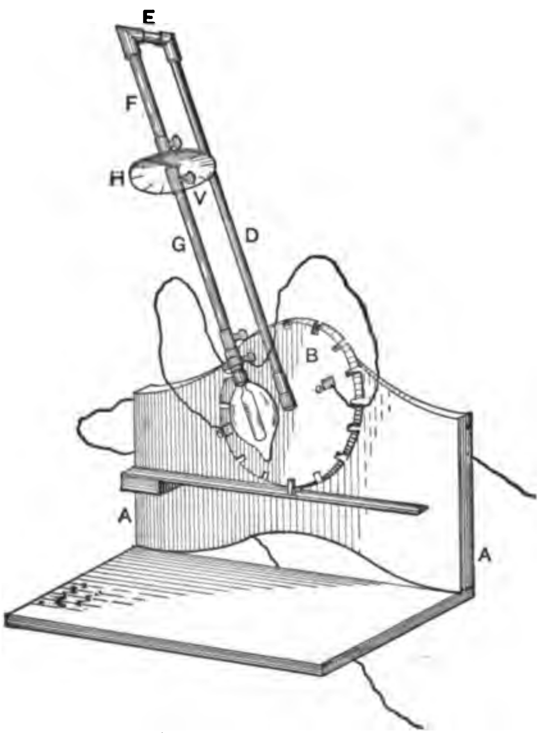


FIG. 67. — Rousseau Support.

plane for various azimuths, the variations of the horizontal intensity being as much as is generally desired.

Many pieces of apparatus have been invented to facilitate these measurements. Below is the description of that which the Com-

mittee on Electrical Tests used with great success at the Antwerp Exposition.

It consists (Fig. 67) of a board fixed vertically at one of the ends of the photometric bench parallel to its axis. Against this board rests a circle divided at intervals of 22°.5 by notches with which a projection on a flexible spring, which is tangent to the circumference of the circle, may engage; this circle is movable about a horizontal axis passing through its center and supported by the board.

At the center of the circle is fixed a tube of three branches D, E, F, the first and the last being parallel to the plane of the circle, the second perpendicular; the support of the lamp is adjusted in the tube G, which may be clamped in the other tube. The lamp may be set and fixed by a screw-clamp, so as to be exactly opposite the center of the graduated circle; the tube G, and consequently the lamp, may be turned about the axis of the filament. The movable tube has a fixed index perpendicular to this axis, serving to measure the angle of rotation on a divided circle; the latter has further a stop which allows the lamp to be fixed during the measurements, in any azimuth.

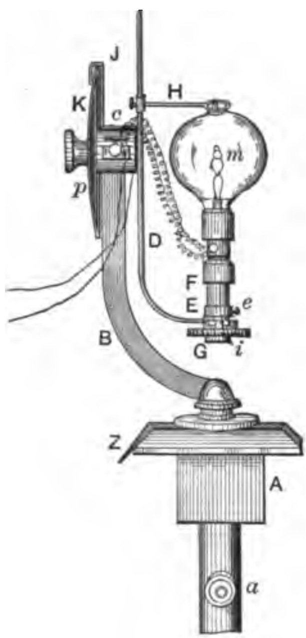


FIG. 68. — Heim-Krüss Support.

Heim also invented a small support which has been slightly modified by Krüss so as to avoid having the support ever placed in the path of the rays. Figure 68 gives a view of this apparatus which is placed vertically on the photometric bench by inserting the block A in the slot of the bench. The index Z serves to indicate the position of the apparatus.

The support B may be moved vertically, but it is not movable about a vertical axis. This arm carries at c the horizontal axis about which the lamp may be turned through an angle read on the circle K by means of the index J. On the axis c is fixed an arm D on which rests the base E to which the lamp is fixed. This base may be turned at will and the angle read on the disc G which turns in front of the fixed index i.

Whatever be the position given the lamp, the point *m* always remains fixed; that is, it is at the same height and at the same distance from the screen.

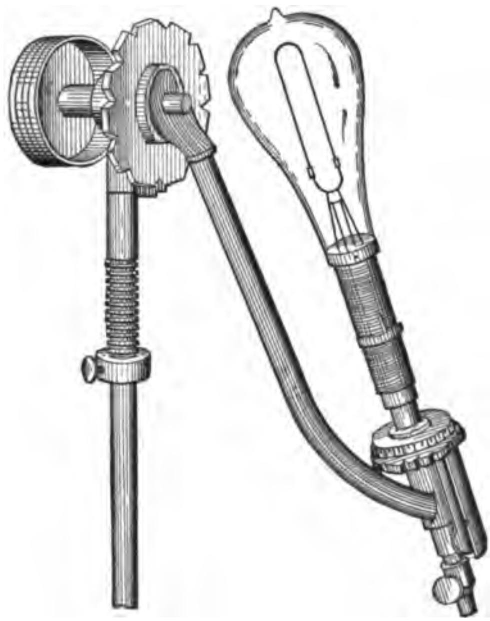


FIG. 69. — Franklin Institute Support.

During the duration tests of incandescent lamps, the Commission of the Franklin Institute employed a support shown in Fig. 69 which resembles in the main that of Rousseau. The notches are fixed and arranged so as to make the measurements at intervals of 22°.5.

CHAPTER V.

ELECTRIC LIGHTS.

A. INCANDESCENT LAMPS.

The Principle of Incandescent Lamps.

122. When a conductor whose electric resistance is R is traversed by a current of intensity I, the quantity of heat developed in this conductor during the time t, according to Joule's law, is equal to

$$\frac{RI^2t}{gE},$$

g being the acceleration due to gravity, and E the mechanical equivalent of heat.

When the constants R and I are sufficiently great, the quantity of heat developed may be sufficient to raise the conductor to incandescence. It is not difficult to deduce the differential equations of the problem in the simplest cases, by taking account of the loss of heat, but this mathematical work is of no use from a photometric point of view.

The incandescent lamp comprises then a conductor consisting of a carbon filament which offers great resistance to the current; it is so arranged as to stand the action of a high temperature without disintegrating. The luminous intensity of the filament depends on its temperature, its surface, and its emissive power; by increasing the last we increase the efficiency of the lamp, that is, the quantity of light which corresponds to a determinate expenditure of energy. It is this increase in emissive power which has above all been realized in the improvements which the incandescent lamp has undergone during the last ten years.

The emissive power of the filament is increased by covering it with a brilliant deposit of carbon, for it has been found that filaments whose surface is a dull black have a smaller efficiency than those whose surface is bright; filaments with a bright surface are

obtained by maintaining them at high temperature in the vapor of a hydrocarbon having a high boiling-point. This process of supplementary carbonizing of the filaments is used nowadays by the majority of manufacturers.

Manufacture of Incandescent Lamps.

123. Vacuum incandescent lamps are the only ones which have come into common use *; there is not space here to discuss systems based on the incandescence of carbon or of platinum in the open air, these systems never having left the experimental stage.

As early as 1841 de Moleyns patented in England an apparatus for the production of light by the incandescence of a platinum wire in a closed glass globe, and in 1845 King received patents relating to an incandescent carbon lamp invented by Starr of Cincinnati. There should next be mentioned the works of de Changy (1858), of Lodyguine (1873), and of Konn, Swan, etc. But it was Edison who constructed industrially the first incandescent lamp (1880) which was really satisfactory and commercial.

Electric lighting by incandescence has then been in existence only twelve years; nevertheless incandescent lamps have already arrived at a satisfactory degree of perfection, owing to the numerous systematic investigations of which they have been the subject. In particular the photometric investigation of incandescent lamps has been pushed very far, which has, moreover, enabled manufacturers to modify with advantage their methods of manufacture.

Every incandescent lamp consists of a carbon filament fixed to two platinum wires, a glass bulb in which a vacuum is formed, and finally a threaded base attached to the bulb and designed to hold the lamp in its socket.

The following is, in a general way, the method by which incandescent lamps are now made.

The bulbs are blown at the glass factory whence the manufacturers obtain them directly; the first manipulation consists of preparing them for the filament.

The nature of the filament varies with different systems; there are three kinds principally employed. Some manufacturers take cotton thread (Swan), others gelatine or vitrified cellulose (Khotinski,

* [There are now, 1894, in the market, several lamps for which it is claimed that the vacuum has been replaced by an atmosphere of gas which keeps the filament in good repair. — *Trans.*]

Lane-Fox); still others use vegetable fibers (Edison, Siemens); finally, some employ a natural fiber submitted to a chemical process (Langhans, Cruto, Seel). Definite form is given to the filament according to its nature, either by means of a die, or between cylinders, or by cutting it out while in a plastic mass.

The fiber thus obtained is transformed into compact carbon by prolonged baking at a high temperature in a crucible, or by heating with the electric current itself. To give the filament homogeneity and the desired resistance, a layer of carbon should be deposited on its surface; this deposit is effected in many different ways, which are peculiar to each manufacturer. A very simple method consists in immersing the filament in petroleum and raising it to a red heat in the liquid.

The filament being cut to the desired length, Edison clamps the carbon with platinum wires, and covers the points of attachment with a layer of electrolytic copper; Lane-Fox and Swan deposit a greater quantity of carbon there, while other manufacturers employ a special cement. Soldering to the carbon tends just now to become more and more employed.

The filaments may be fixed in the bulb in two ways: either the two wires are fused into a piece of glass called the bridge, which is next fused into the neck of the bulb; or else the wires are fixed separately on the edges of a glass socket, which is then fused into the bulb. A small tube is also fused to the top of the bulb, in order to provide for the production of a vacuum. The exhaustion of the lamps takes place by means of mercury pumps. Sprengel pumps are almost exclusively employed.

The vacuum obtained, the lamp is tested; then the luminous intensity and the resistance when cold, are measured. The bulb is not mounted until it is about to be shipped.

The dimensions of the filaments vary naturally with the luminous intensity of the lamp; they should be proportionately greater as the normal luminous intensity of the lamp is higher. These dimensions depend also on the specific resistance of the carbonized substance. As to the form of the section of the filament, the circular one is preferable, because it presents the minimum resistance for a given surface.

In Edison lamps the filaments have a section 0.3 mm. by 0.1 mm.. and a length when straightened out of 125 mm. for 16-candle-power lamps, and 110 mm. for those of 10 candle power. In the Maxim lamp of 16 candle power, the section is 0.5 mm. by 0.1 mm., and the

length straightened out 113 mm. The filaments of the Siemens lamps have circular sections whose diameters are 0.15 mm., 0.20 mm., and 0.27 mm. for lamps of 10, 16, and 25 candle power, respectively. The lengths of the filament for the same lamps are 110, 125, and 145 mm., respectively.

The Luminous Intensity of Incandescent Lamps.

124. It is not possible to establish for all incandescent lamps the general law according to which the luminous intensity varies with the direction of the ray, for this law depends above all on the form of the filament, which varies greatly in different lamps; for instance, the filament of the Edison lamp has the form of an inverted U, that of the Swan lamp, a horizontal buckle; in the Maxim lamp, the filament is in the form of an M, while in the Weston lamp it is wound in a spiral about an arc of the shape of a horseshoe; the filament of the Gérard lamp has the form of an acute angle supported on the base of the bulb; that of the Bernstein lamp of great intensity has also this form; while other types of the same system have a filament in the form of an inverted Δ.

The investigation of the distribution of luminous intensity of an incandescent lamp is very complex. The form of the filament produces a sensible want of symmetry in the distribution of luminous intensity, which varies not only with the inclination of the ray, but also with its azimuth.

Horizontal Intensity.

125. Supposing the lamp to be vertical, it is the horizontal intensity which is usually desired. Variations in the horizontal luminous intensity depend essentially on the form of the filament.

The horizontal intensity is principally characterized by the value of the mean horizontal intensity. It is known that the determination of this element necessitates the measurement of the horizontal luminous intensity at a great number of different angles at equal intervals. The mean horizontal intensity is then the mean of the values thus obtained. In practice it is well to make the measurements at intervals of 22°.5, and then to calculate the mean of the sixteen results. Frequently intervals of 30° or 45° may be sufficient.

126. Even this calculation may be considerably simplified by taking account of the following fact, which is true for the majority of incandescent lamps. For all lamps of a given system, we may

obtain the value of the mean horizontal intensity by multiplying the horizontal intensity, measured with the photometer bar at right angles to the plane of the filament, by a factor of reduction C_0, which is the same for all lamps of this system; this factor C_0 varies between 0.8 and 0.9. The same result is reached by multiplying the horizontal intensity measured in the perpendicular plane by a factor C_1. Finally, the same thing was calculated at the Paris Exposition in 1881, by multiplying the horizontal intensity, measured at an angle of 45° with the plane of the filament, by the factor C_2.

We may calculate, from the fundamental photometric laws, the values of these coefficients C_0, C_1, and C_2, supposing the form and dimensions of the filament to be accurately known.

These values are obtained easily as a particular case of the general problem in which the variations of the intensity with the direction of the ray are determined. To illustrate this let us consider the case of an Edison lamp.

The quantity of light coming from an element ds, falling on an element ds', equals

$$q = i\frac{ds\,ds'\cos\theta\cos\theta'}{r^2},$$

r being the distance between the two elements, θ and θ' the angles made by r with their normals respectively, and i the luminous intensity of the element ds.

The filament of the Edison lamp has the form of an inverted U. Designate by l the length of the vertical branches, by h that of the transverse horizontal branch, and suppose that the filament has a section of rectangular form; designate by a the thickness of this section in the direction of the plane of the filament, and by b the thickness in the perpendicular direction.

Let us suppose that the element ds belongs to a sphere concentric with the lamp and of sufficiently great radius r; we may then assume that $\theta' = 0$, and that $\cos\theta' = 1$.

The quantity of light emitted horizontally in a direction making the angle β with the plane of the filament and received by the element ds' of a very narrow equatorial zone of the concentric sphere r, equals

$$q_{h\beta} = \frac{ids'}{r^2}\cdot[2l(a\sin\beta + b\cos\beta) + ah\sin\beta].$$

The total quantity of light emitted in the horizontal plane, that is received by a narrow zone of height δ for which $ds' = \delta r d\beta$, is then

$$Q_h = \frac{4 i\delta}{r}\int_0^{\frac{\pi}{2}} [2l(a \sin\beta + b\cos\beta) + ah\sin\beta]\,d\beta.$$

Integrating,

$$Q_h = \frac{4 i\delta}{r}[2l(a+b) + ah].$$

The mean horizontal intensity will then be

$$I_{hm} = \frac{Q_h}{2\pi r\delta} = \frac{2i}{r^2}\cdot\left[\frac{2l(a+b)+ah}{\pi}\right]. \tag{1}$$

The horizontal intensity of the angle β is equal to $q_{h\beta}$ divided ds'; that is,

$$I_{h\beta} = \frac{i}{r^2}[2l(a\sin\beta + b\cos\beta + ah\sin\beta]. \tag{2}$$

In the plane of the filament ($\beta = 0$), the intensity becomes

$$I_{h0} = \frac{2lbi}{r^2}. \tag{3}$$

Perpendicularly to the plane of the filament, we have

$$I_{h90^\circ} = \frac{a(2l+h)i}{r^2}. \tag{4}$$

Finally, at an angle of 45° with the plane of the filament, the intensity becomes

$$I_{h45^\circ} = \frac{i}{r^2}\cdot\left[\frac{2l(a+b)+ah}{\sqrt{2}}\right]. \tag{5}$$

The values (1), (3), and (4) show that the ratios between the mean horizontal intensity and the intensities at 0° and 90° are constant for the same type of lamp, but that they depend on the dimensions of the filament. The relation $I_{hm} = C_0 I_{h0}$ or $I_{hm} = C_1 I_{90}$ is then true.

But the ratio between the mean intensity and the intensity at 45° it still simpler, for we find that it is independent of the dimensions of the filament; it is, in fact,

$$\frac{I_{hm}}{I_{h45^\circ}} = \frac{2\sqrt{2}}{\pi} = 0.9003 = C_2.$$

This result has been confirmed by the direct measurements of Hagenbach and by those at the Munich Exposition, the Vienna Exposition, etc.; however, the value of the constant determined experimentally is somewhat higher than 0.9003. This is because the section of the filament is not a perfect rectangle as we have supposed; the factor tends then toward unity as the section approaches the form of a circle.

By means of eight Edison lamps Hagenbach obtained $C = 0.95$; at Vienna 0.94 was obtained for Maxim lamps, and 0.98 for Edison lamps.

In all lamps whose filament has an analogous form to that of the Edison lamp, we may determine the mean horizontal intensity by a single measurement of the horizontal intensity at an angle of 45° with the plane of the filament. We have, then, with satisfactory approximation,

$$I_{hm} = 0.95\, I_{h45^\circ}. \tag{6}$$

Hagenbach has also given the following formula by which to calculate the mean horizontal intensity:

$$I_{hm} = \frac{I_{h0} + 2\, I_{h45^\circ} + I_{h90^\circ}}{4}.$$

This formula agrees, in general, very well with the facts. To illustrate, we give the values obtained at Vienna with two Maxim and two Edison lamps, whose difference in the distribution of horizontal intensity is very considerable*.

Lamp.	I_{h0°	I_{h45°	I_{h90°	I_{hm} Calculated	Observed.
Maxim	0.909	0.793	0.219	0.701	0.716
Maxim	1.021	0.766	0.282	0.709	0.743
Edison	1.020	1.029	1.083	1.040	1.046
Edison	1.018	1.234	1.224	1.178	1.175

The measurements at Munich, Philadelphia, and Antwerp have also shown the exactness of this formula, at least within the limits of precision of the observations.

127. We may also obtain directly the value of the mean horizontal intensity by means of an ingenious method invented by

* *Expériences faites à l'Exposition d'Électricité de Paris*, p. 44.

Crova*, which greatly simplifies measurements and calculations. The lamp is mounted on clock-work which makes it turn about its geometrical axis four or five times a second; the current reaches the lamp by two insulated rings on which brushes rest.

The lamp appears as an immovable luminous spindle whose intensity is exactly the mean horizontal. It would be preferable to mount the lamp on a small electric motor of high resistance in shunt across the lamp terminals. Then the source which illuminates it would also cause it to rotate.

Mean Spherical Intensity.

128. Acquaintance with the form and dimensions of the filament permits the calculation of the distribution of luminous intensity in the various directions; but this somewhat complicated calculation is of no practical interest.

The exact determination of the photometric surface of an incandescent lamp requires, then, precise measurements. From the following considerations it will, however, be seen that they may be considerably abridged. The projection of the filament on a vertical plane varies in a uniform manner when it is turned about its vertical axis. From this it follows that the distribution of luminous intensity is similar at all the horizontal parallels of the concentric unit sphere; that is, the horizontal sections of the photometric surface are curves similar to one another and to that of the horizontal intensity. For this reason it is usually sufficient to determine the variations of the luminous intensity with the inclination, in one vertical plane alone; the variations in the other vertical planes follow the same law. [See Appendix E.]

It is known that the mean horizontal intensity is equal to $C_o I_{h0}$, I_{h0} being the horizontal intensity in the plane of the filament, and C_o the corresponding reduction factor; this factor is the same for all parallels of the unit sphere. Now if $I_{\theta m}$ designates the mean intensity of the parallel corresponding to an inclination θ, the total quantity of light received by the unit sphere is

$$Q = 2\pi \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} I_{\theta m} \cos\theta \cdot d\theta.$$

* *Comptes Rendus des Travaux du Congrès des Électriciens de 1889*, p. 208.

The mean spherical intensity is $\frac{Q}{4\pi}$, or

$$I_{sm} = \frac{1}{2}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} I_{\theta m} \cos\theta \cdot d\theta.$$

Now the intensity $I_{\theta m}$ is a function of the inclination of the form,

$$I_{\theta m} = I_{hm} f(\theta) = C_o I_{h0} f(\theta).$$

Consequently,

$$I_{sm} = \frac{C_o I_{h0^\circ}}{2}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} f(\theta) \cos\theta d\theta.$$

If we put

$$C = \frac{C_o}{2}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} f(\theta) \cos\theta d\theta,$$

we obtain

$$I_{sm} = C \cdot I_{h0}.$$

The constant C, which is called the factor of reduction of horizontal intensity to mean spherical intensity, does not vary sensibly from one lamp to another; to determine it, it is necessary to have recourse to measurements on lamps of various systems.

The mean spherical intensity may finally be calculated by multiplying the mean horizontal intensity by a factor of reduction C', which may be called the factor of reduction of mean horizontal intensity to mean spherical intensity.

We give below the values of the constants C_o and C, calculated by means of the measurements made at the Vienna Electrical Exposition of 1883.

Lamp.	C_o	C
Lodyguine	0.998	0.776
Müller	0.980	0.863
Rawson	0.981	0.758
Maxim	0.735	0.556
Siemens	1.007	0.748
Bernstein	0.973	0.718
Swan	1.150	0.946
Edison	1.175	0.957
Müller	1.011	0.875
Lane-Fox	1.006	0.734

Results of the Franklin Institute Tests.

129. Incandescent lamps have been the subject of numerous photometric measurements with the object of determining the distribution of intensity with the direction of the ray. As early as 1881, at the Electrical Exposition, this question was very carefully studied. The measurements of the committees at the Expositions of Munich, Vienna, and Antwerp have also made documents which are very interesting and very useful to consult. But from the point of view of the importance of the tests, it is the measurements of the Committee of the Franklin Institute which excel. Numerous lamps of each type were studied, so that the values obtained for each of them have the significance of mean values, and on this account a greater importance.

In tables I. to IV. we have given a *résumé* of the principal photometric elements of the lamps studied.

Table I. contains the principal constants of each type obtained by taking the mean of a large number of lamps (10 or 20).

TABLE I.

Name and Number of Lamps.	Volts.	Amperes.	Mean Spherical Candle Power.	Watts per Spherical Candle Power.	Mean Horizontal Intensity.
Edison (20)	97.0	0.709	15.49	4.48	18.83
Stanley (10)	96.4	0.551	13.56	3.92	16.54
Woodhouse & Rawson (10)	55.48	1.026	15.09	3.56	19.11
White (10)	49.99	1.017	12.44	4.08	15.08
Weston (20)	111.4	0.530	16.27	3.63	17.87

Table II. contains the values of the horizontal intensity for different azimuths; in this table as in the following, the origin of the azimuths coincides with the plane perpendicular to the base of the filament.

TABLE II.

Azimuth.	Edison.	Stanley.	Woodhouse and Rawson.	White.	Weston.
0°	16.61	16.65	14.71	14.80	19.96
30°	18.20	16.60	18.23	14.63	14.99
60°	20.45	16.43	20.98	14.97	12.37
90°	20.88	16.36	20.42	15.17	16.67
120°	20.82	16.35	20.02	15.10	21.51
150°	18.86	16.68	18.51	15.13	22.11
180°	16.87	17.03	14.48	14.87	19.79
210°	18.48	16.85	18.71	14.96	14.58
240°	20.74	16.40	21.27	15.00	11.98
270°	21.10	16.20	22.46	15.18	16.51
300°	20.93	16.43	20.63	15.21	21.74
330°	12.12	16.45	18.95	15.08	22.24

In the third table are found the values of the luminous intensity at different inclinations for two azimuths differing by 90°. The origin of the inclinations is in the horizontal plane, and they are measured from 0° to 360°, passing over the top of the lamp.

TABLE III.

Inclination.	Edison.		Stanley.		Woodhouse and Rawson.		Weston.	
	Azimuth.		Azimuth.		Azimuth.		Azimuth.	
	0°	90°	0°	90°	0°	90°	0°	90°
0°	16.70	20.64	16.54	16.23	14.76	20.56	19.82	16.17
30°	15.02	18.31	15.29	14.90	13.48	20.00	19.31	15.40
60°	9.54	11.93	11.04	11.86	9.60	13.32	16.39	13.74
90°	3.57	3.08	6.80	7.00	6.74	5.77	13.39	13.00
120°	8.25	11.54	10.35	11.74	10.71	13.17	16.24	13.41
150°	14.96	18.21	14.99	14.87	14.06	18.72	19.13	15.63
180°	16.82	20.87	16.85	16.81	14.71	21.81	19.76	16.42
210°	14.84	17.85	15.00	14.54	14.34	19.62	18.82	15.76
240°	9.07	11.11	9.57	9.11	11.28	14.11	16.34	15.08
270°	......	......	......	......	......	......	......	......
300°	9.84	11.68	9.26	10.40	9.75	13.52	17.34	13.83
330°	15.06	17.69	14.83	14.33	13.64	18.10	18.78	14.64

Finally, the fourth table contains the constants C_o, C_1, C, and C', which have been calculated from the detailed results published by the Committee of the Franklin Institute.

TABLE IV.

Lamps.	C_0	C_1	C	C'
Edison	1.09	1.26	0.74	0.80
Stanley	1.00	0.98	0.83	0.83
Woodhouse & Rawson	0.88	1.23	0.74	0.83
White	0.99	1.02	0.82	0.83
Weston	1.08	0.90	0.98	0.91

The curves in Figs. 70 to 75 represent the variations of luminous intensity in a horizontal plane, in a vertical plane of azimuth 0°, and in a vertical plane of azimuth 90°. The first three figures relate to the Edison lamp, the other three to the Weston lamp.

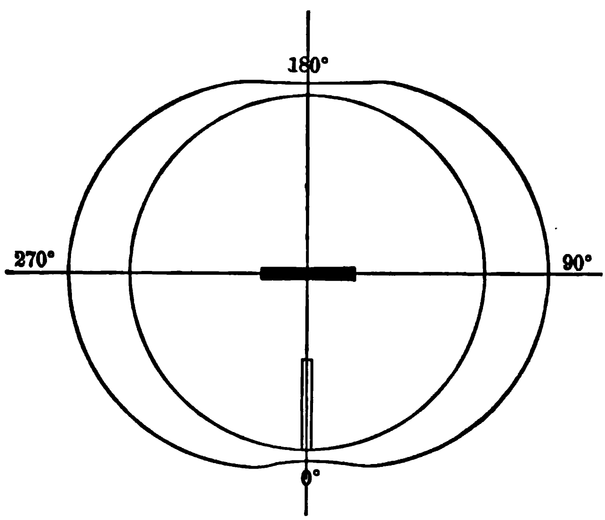


FIG. 70. — Horizontal Distribution of Luminous Intensity in an Edison Lamp.

We have chosen diagrams of these two lamps because of the great difference in the form of their filament. In the first lamp, the filament has the form of an inverted U, while in the second, the

filament is a helix twisted about a horseshoe-shaped axis. The diagrams show well the influence of these differences in form, especially on the variations of intensity in their vertical planes.

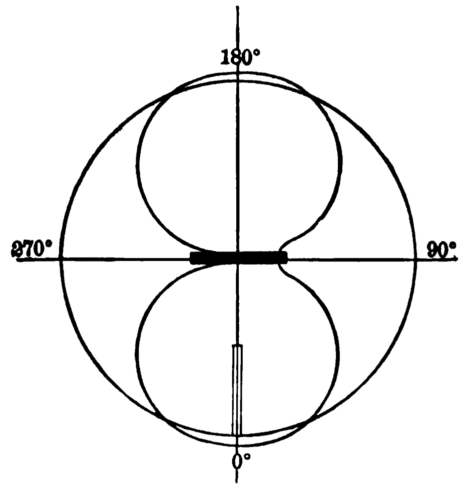


FIG. 71. — Variations in the Luminous Intensity of an Edison Lamp in the Plane of 0° Azimuth.

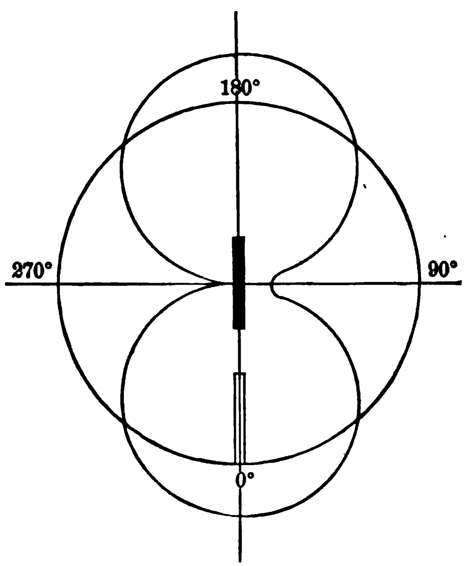


FIG. 72. — Variations in the Luminous Intensity of an Edison Lamp in the Vertical Plane of 90° Azimuth.

We have already seen that the horizontal distribution depends on the section of the filament. If it is circular, as in White and

Stanley lamps, the curve of horizontal intensity practically forms a circle. If it is rectangular, as in the Edison, and Woodhouse

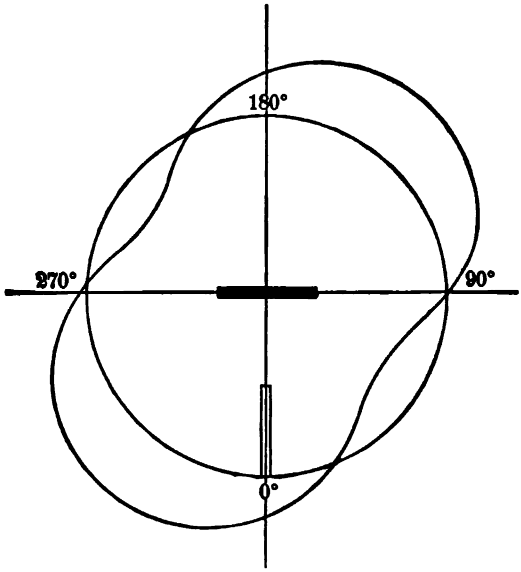


FIG. 73. — Variations in the Horizontal Luminous Intensity of a Weston Lamp.

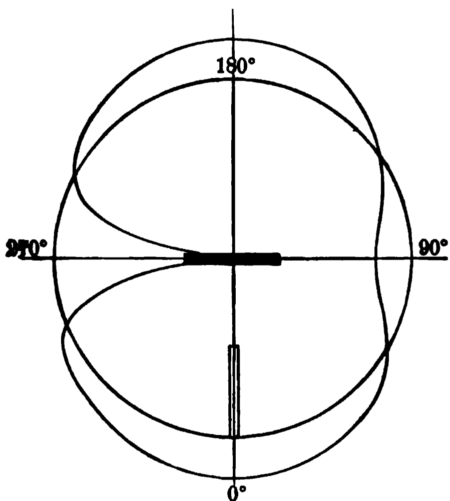


FIG. 74. — Variations in the Luminous Intensity of a Weston Lamp in Plane of 0° Azimuth.

and Rawson lamps, the maximum horizontal intensity corresponds to the largest side of the rectangle.

Variations in Luminous Intensity with the Energy expended in the Filament.

130. The luminous intensity of an incandescent lamp varies with the temperature of the filament, i.e. with the electrical energy spent in it. These variations of luminous intensity play an important *rôle* in the application of incandescent lamps to illumination, for there exists for each lamp a determinate luminous intensity which corresponds to a given life of the lamp.

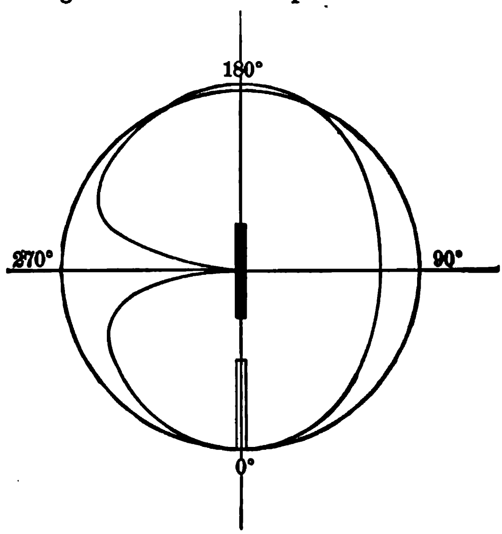


FIG. 75. — Variations in the Luminous Intensity of a Weston Lamp in the Plane of 90° Azimuth.

Jamieson * made the first extended systematic researches on the relation which exists between the luminous intensity of an incandescent lamp and the energy expended in the filament.

He obtained some very interesting diagrams for the ordinary lamps by taking as abscissæ the energy expended, and as ordinates the luminous intensity.

Dr. Higgs attempted to represent this relation analytically by putting

$$I = MW^2,$$

I being the luminous intensity in candles, W the energy expended, and M a constant depending on the nature of the lamp.

* *Lum. Él.*, Vol. VII. p. 137.

Mansel, at Glasgow, gave also the following equations:

$$\log B = \log E + ar,$$

$$\log bB = \frac{1}{B} \log I + Ar,$$

in which E is the difference of potential at the terminals of the lamp, I the luminous intensity, r the resistance of the lamp, a, b, and B constants. Jamieson simplified these equations by putting $a = 2A$, and by supposing, as is shown by experience to be true, that $\log bB$ is a constant; he then obtained the simpler form,

$$\log I = 6 (\log E - \log A),$$

or

$$I = \left(\frac{E}{A}\right)^6,$$

A being a constant.

Voit, in his able report upon the electrical measurements of the Exposition at Munich, endeavored to determine a simple analytical law, giving for all incandescent lamps more concordant results than those furnished by the above equations. He investigated the three following equations:

$$I = a_1 W^2,$$

$$I = a_2 W^3,$$

$$I = a_3 W^4,$$

and found that the last of these equations represents the observations with sufficient accuracy, save for the Cruto lamps.

In 1883, Goetz * of Zurich arrived at the conclusion that the equation

$$I = a W + b W^2$$

gave more concordant results than any of those previously proposed. However, in 1884, an investigation with the Bernstein lamp made by Ganguillet †, showed that the equation of the third degree,

$$I = \alpha W + \beta W^3,$$

was still more exact. This conclusion was also verified by numerous measurements made by Hess in the electro-technical laboratory at Zurich.

* *Lum. Él.*, Vol. XI. p. 207.
† *Lum. Él.*, Vol. XXIII. p. 520.

In the following table are given the results obtained with a 16 candle power Swan lamp. The intensity of the current is represented by i; the horizontal luminous intensity measured at an angle of 45° with the plane of the filament is designated by I, while I_1 represents the luminous intensity calculated by the binomial formula of the third degree, and I_2 the intensity calculated according to the cubical formula of Voit. The mean ΔI_1 of the deviations $I - I_1$ and the mean ΔI_2 of the deviations $I - I_2$, show the relative exactness of the two formulæ.

E	i	W	I	I_1	I_2
33.90	0.90	30.68	1.27	1.00	1.59
40.92	1.10	45.05	4.25	4.51	5.02
44.26	1.18	52.49	7.43	7.67	7.94
45.65	1.22	55.70	9.28	9.26	9.49
48.04	1.29	61.91	13.28	13.26	13.03
49.25	1.32	65.02	13.56	15.54	15.09

$$I_1 = -0.0280\,W + 0.0000632\,W^3,$$
$$I_2 = 0.0000549\,W^3,$$
$$\Delta I_1 = 0.138,$$
$$\Delta I_2 = 0.422.$$

Below are the equations for certain lamps, determined from the observations of Hess of the Committee on Experiments at Munich, etc.; the coefficients are in terms of candles and watts.

Lamp.	$I_2 = \alpha_2 W^3$	$I_1 = \alpha_1 W + \beta_1 W^3$	
	α_2	α_1	β_1
Swan, No. 1	0.0000974	− 0.02778	0.0001164
" " 2	1020	− 0.02434	1196
" " 3	657	− 0.00793	676
" " 4	549	− 0.0280	632
Maxim	247	+ 0.0472	215
Siemens	223	− 0.0156	252
Müller	25	+ 0.0391	211
Cruto, No. 1	320	+ 0.0796	196
" " 2	528	+ 0.0274	4910
Edison	22	+ 0.3173	198

These conclusions have been confirmed by later measurements, made in various countries, of lamps of different systems and makes. It may be assumed that the luminous intensity of an incandescent lamp is given by the sum of two terms, one of which is proportional to the energy, in watts, consumed in the lamp, and the other to the third power of this energy. This formula applies as well to the luminous intensity of the composite light emitted by the lamp as to that of the principal rays; this follows from the photometric measurements of various physicists, Schumann* among others.

Influence of the Degree of Vacuum on the Luminous Intensity.

131. Some exact measurements of the degree of vacuum of common incandescent lamps have been made. It should be remarked at first that the filament has a considerable absorbing property for gases, a property which diminishes at high temperatures; this then explains the increase in the pressure of the gas in lamps when hot. From a practical point of view, it is the degree of vacuum during incandescence which is of interest.

From measurements of Heim† it follows that the pressure in an incandescent lamp when cold is lower than 0.01 mm. of mercury; when hot, this pressure increases rapidly up to a certain value which does not exceed 0.05 mm., and which remains constant during several hours of burning.

There is no advantage in carrying the vacuum too far, to the point, for instance, when there appears about the filament the light blue halo first noticed by Edison; it has been noticed, in fact, that the filaments of lamps exhausted to this point disintegrate and tarnish the bulb more rapidly. The phenomena of electric evaporation lately studied by Crookes then come into play.

From a theoretical point of view, a degree of vacuum as perfect as possible reduces to a minimum the loss of energy produced by the transmission of heat by direct convection of the gaseous particles, a loss which is added to that of radiation. Now, it is known that the luminous intensity varies much more rapidly than the temperature; thus with an imperfect vacuum, to obtain a determined luminous intensity, it is necessary to expend more energy than with a better vacuum.

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* *Lum. Él.*, Vol. XIII. p. 60.
† *Lum. Él.*, Vol. XXIII. p. 415.

The researches of Hess* on the influence of the degree of vacuum in the Swan lamp have proved the accuracy of these conclusions; it follows from this that the tension of the gas in an incandescent lamp should not exceed 0.2 mm. of mercury to obtain a favorable result. Hess represented by an equation the luminous intensity of a lamp for a given expenditure of energy and for a given tension; calling I_1 the luminous intensity in candles corresponding to an absolute vacuum, I_2 the luminous intensity for a considerable tension, a a coefficient, the intensity I at the tension p expressed in millimeters is given by the equation

$$I = \frac{p^2 I_2 - a I_1}{p^2 - a}.$$

For an expenditure of 70 watts, and in a Swan lamp, Hess found

$$I = \frac{4.93\,p^2 + 16.20}{p^2 + 0.96}.$$

It follows that $I = 16.87$ candles if $p = 0$; and $I = 4.95$ if $p = 20$ mm.

The results of Hess were confirmed by those of Higgins, who investigated three lamps, one of which was filled with air at normal pressure (76 cm. of mercury), the other, half exhausted of air (25 cm. of mercury), and the third having the same vacuum as ordinary lamps. The reciprocal of the efficiency of these three lamps, i.e. the number of watts expended per candle, was found to be 7, 5.3, and 3.5, respectively. It was also determined that the transmission of heat to the surrounding air was in the proportion 32, 25, and 3.5.

Variations in the Luminous Intensity with the Life and Rate.

132. As a result of somewhat prolonged use, the filament undergoes quite considerable changes which have an effect on the luminous emission of the lamp. These changes may be summed up as follows:

During the first period, somewhat short, the resistance of the filament diminishes rapidly, which has for its effect an increase in brightness. These conditions remain about stationary during a second period, somewhat longer than the first; they then change, the resistance increases at the same time that the luminous intensity gradually decreases. This phenomenon is produced by modifications in the filament, which becomes little by little wrinkled, while its

* *Lum. Él.*, Vol. XXIII. p. 523.

section diminishes; it then requires more current to obtain the normal luminous intensity, while the increase in resistance gradually diminishes the intensity of the current. There should also be mentioned the diminution in the luminous intensity produced by the deposit of carbon which forms on the bulb, and whose thickness increases with the use of the lamp.

The most complete measurements of the variations of the photometric elements of an incandescent lamp were made at Philadelphia in 1884, under the supervision of the Franklin Institute*.

Following are the results of the measurements made on 20 Edison lamps of 16 candle power, at intervals of 100 hours, for a total lighting of 1000 hours. The filament of lamp No. 16 broke after a run of 300 hours, while all the others survived. Some analogous results were furnished by lamps of other makes.

	Intensity (mean spherical) in Candles.							Watts per Candle.	
	Number of Hours.								
	Beginning.	100	200	400	600	800	1000	Beginning.	End.
	14.57	12.70	12.10	9.20	10.60	9.80	9.20	4.82	6.97
	15.44	14.00	12.50	9.40	11.40	10.80	9.90	4.52	6.65
	16.81	15.40	14.40	10.20	11.30	11.70	9.90	4.24	6.56
	16.09	13.60	12.40	9.20	10.70	10.30	9.10	4.24	6.92
	15.11	14.30	12.10	9.70	11.70	11.00	9.60	4.55	6.55
	14.87	11.00	11.10	9.60	10.60	10.30	9.60	4.58	6.53
	15.60	11.50	11.70	10.00	9.60	9.50	7.10	4.43	7.13
	15.72	14.10	14.50	9.40	10.90	10.80	9.50	4.37	6.52
	16.41	15.10	14.30	9.60	10.90	9.50	10.30	4.27	6.24
	15.62	14.30	12.30	9.50	11.30	10.20	9.10	4.37	6.92
	14.58	13.20	12.10	9.40	11.60	11.00	9.50	4.79	6.55
	16.66	15.20	13.20	9.60	11.20	10.50	9.50	4.21	6.71
	16.02	12.30	10.90	9.10	10.10	10.20	9.60	4.44	6.82
	14.38	12.60	11.60	9.50	10.40	10.30	9.60	4.83	6.57
	13.76	13.70	12.30	9.00	11.30	10.70	10.00	4.92	6.33
	15.65	13.50	12.20	......	......	......	......	4.40	......
	15.82	12.60	11.10	8.40	9.70	9.50	8.90	4.43	7.14
	16.61	14.10	13.90	9.30	10.00	9.80	8.70	4.17	7.15
	15.31	13.50	11.90	9.60	10.30	10.10	9.10	4.45	6.81
	14.67	14.40	14.00	10.10	11.60	10.60	10.10	4.71	6.27
Mean .	15.43	13.55	12.58	9.50	10.87	10.34	9.38	4.48	6.66

* *Lum. Él.*, Vol. XVIII., 1885.

The diminution in luminous intensity is very sensible; it frequently reaches 30 per cent. Greater diminutions are sometimes found. This diminution is also intimately connected with the working of the lamp.

In the first incandescent lamps a mean efficiency of $\frac{1}{8}$ candle per watt was assumed, while now this efficiency has a mean value of $\frac{1}{3}$ candle per watt when the lamp is first lighted. But this increase in efficiency has been obtained at the expense of the life of the lamp, which is as much less as the efficiency is greater. As the luminous intensity varies approximately with the cube of the energy

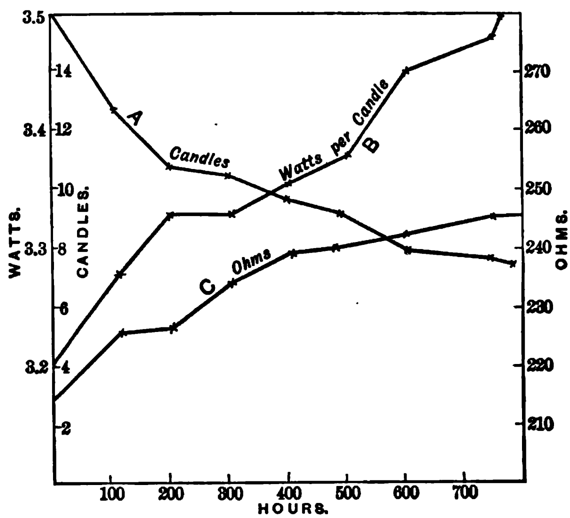

FIG. 76.—Normal Test of a 16-Candle-Power Lamp.

consumed in the filament, it is easy to make a 16-candle-power lamp give 32 candles with an expenditure of about 2 watts per candle, but the lamp will last a much shorter time.

In proportion as the luminous intensity diminishes because of prolonged use of the lamp, the resistance of the filament increases and the efficiency rapidly decreases. The curves of Fig. 76 represent the variations of luminous intensity (A), of watts per candle (B), and of resistance in ohms (C), as functions of the time, these elements being measured every 100 hours. These results were obtained by means of a lamp of 16 candle power (nominal), and the difference of potential was maintained strictly constant at the terminals of the lamp. The results then conform to those obtained in practice.

The diminution in the efficiency of an incandescent lamp with the length of its use is easily explained by means of the physical properties of the filament.

The difference of potential E at the terminals of the lamp remaining constant, the intensity i of the current diminishes as the resistance increases; the energy spent in the filament Ei also diminishes. There results a lowering of the temperature of the filament and, therefore, of the emissive power, since the latter varies about in proportion to the fourth power of the absolute temperature. This fact explains in part the diminution in the efficiency.

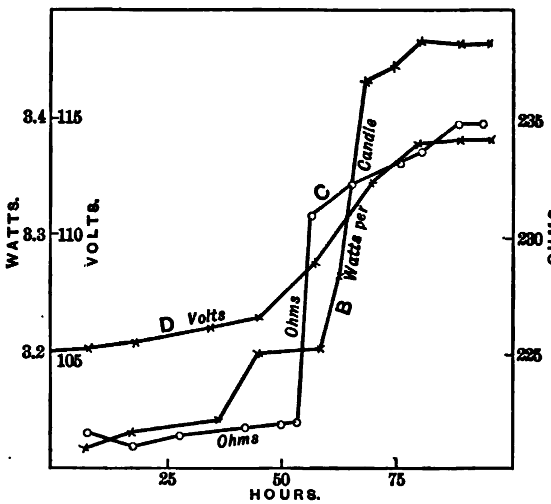

FIG. 77.—Test of a Lamp whose Luminous Intensity was kept at 16 Candle Power.

On the other hand, the resistance of the carbon decreases as the temperature increases. If then the temperature of the filament lowers because of the increase in its resistance, caused by changes in its section, there results a new increase in its resistance and a still more noticeable diminution of the energy expended. These two causes explain the gradual diminution of the lamp's temperature and efficiency.

If it is desired to maintain the luminous intensity constant whatever the length of illumination, it is necessary to gradually increase the difference of potential between the terminals of the lamp in proportion as the resistance of the filament increases; the energy

expended in the filament then increases with the time; the life of the lamp is, however, considerably shortened. Curves *D, B, C* (Fig. 77), represent the variations of the elements of a 16 candle power lamp constantly maintained at this luminous intensity. Curves *B* and *C* represent the same things as in the preceding figure, while curve *D* represents the variations of the difference of potential between the terminals. It is seen that the lamp was not able to stand 100 hours at this rate, the filament failing at the end of about 95 hours.

To show how the elements of an incandescent lamp vary when subjected to an abnormal rate, we give in Figs. 78 and 79 the curves

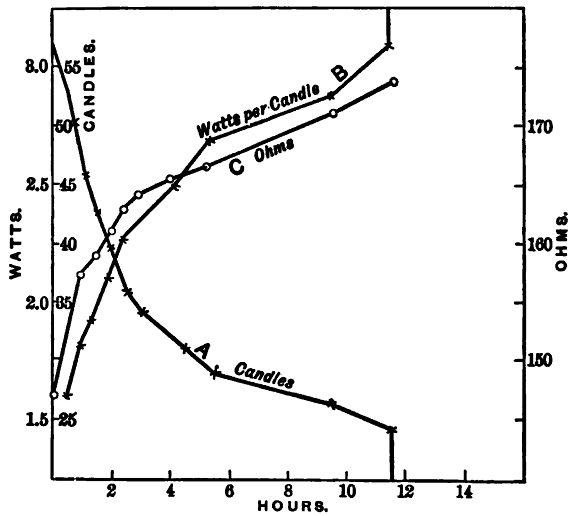

FIG. 78. — Test of a 16-Candle-Power Lamp with an E.M.F. 10 Per Cent too High.

of two 16 candle power lamps analogous to those which the two preceding diagrams furnished. The diagrams in Fig. 78 have reference to a 16 candle power lamp subjected to a difference of potential 10 per cent higher than the normal. Those in Fig. 79 have reference to a 16 candle power lamp maintained at the constant luminous intensity of 64 candles.

These four diagrams given by Nichols * sum up sufficiently the different phases of the life of an incandescent lamp.

* *El. World*, 1890, Vol. XVI. p. 387 ; *Lum. Él.*, Vol. XXXIX. p. 83.

Attention should be called to a fact often kept in the dark when speaking of the efficiency of incandescent lamps. When the merit of a lamp is to be decided, account should be taken of the mean efficiency during the whole life of the lamp, and not merely of its initial efficiency. The latter may be very high for a given lamp, without the mean efficiency being very great, for it frequently happens that the efficiency diminishes very rapidly after a few hours' use.

Peirce* investigated 94 lamps of four different makes, designated by the letters *A, B, C, D*; those which were furnished by the manufacturers themselves are marked with a figure 2; those which were

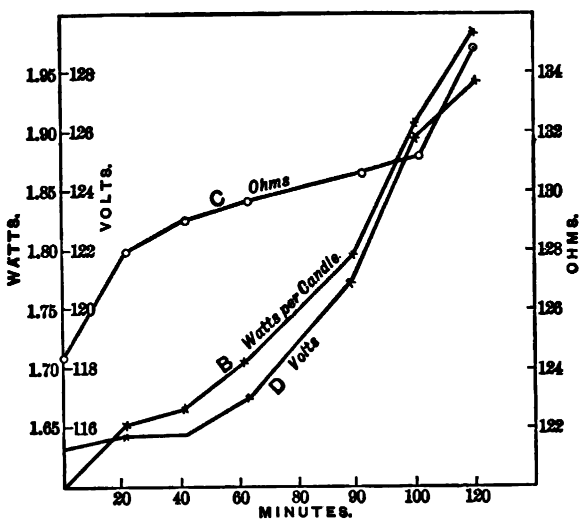

FIG. 79. — Test of a 16-Candle-Power Lamp run at 64 Candle Power.

purchased in the market, by the figure 1. The luminous intensity measured was in every case the mean horizontal intensity.

The curves of Fig. 80 represent the variations in the number of watts per candle required after more or less use; these curves in general fall a little immediately after lighting, to rise slowly afterwards.

The following table contains for each type of lamp, the number

* *El. World*, 1889, Vol. XIII. p. 329; *Lum. Él.*, Vol. XXXIII. p. 257.

of watts per candle required, on the average, during the test (850 hours) and the percentage that the mean efficiency is of the initial efficiency (candle power per watt).

Lamps.	Watts per Candle (mean).	Mean Efficiency in Per Cent of Initial Efficiency.
A_1	5.25	67 %
A_2	4.83	64 %
B	4.58	77 %
C_1	5.80	60 %
C_2	5.36	82 %
D_1	4.92	98 %
D_2	4.92	90 %

Peirce further came to the following conclusions. The higher the initial efficiency of the lamp is raised, that is, the smaller the initial number of watts per candle, the greater the variations of efficiency become. The curve of watts per candle rises quite rapidly, while that of efficiency lowers. If the initial efficiency is small, the variations also are very small.

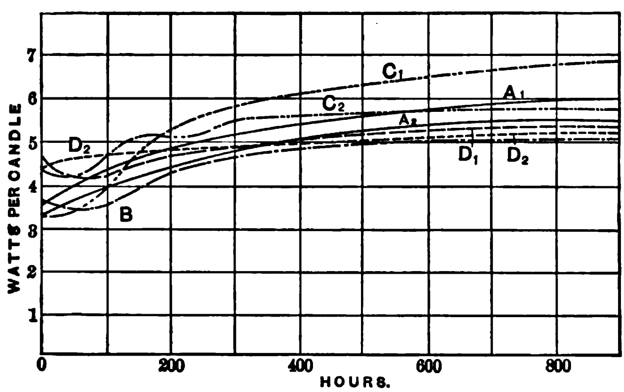

FIG. 80. — Variation in the Efficiency of Lamps with the Time.

There is a striking difference between lamps which at first require from 3.5 to 3.8 watts per candle, and those which require from 4.7 to 5. At the end of the record, the mean efficiency of the

last lamps is superior to that of the first. With reference to this the following figures are significant:

Lamps requiring per Candle:		Watts per Candle.	Mean Efficiency in Per Cent of Initial Efficiency.
From 3.5 to 3.8 watts,	*A*	5.59	61
	B	5.09	71
	C	4.61	82
From 3.8 to 4.1 watts,	*A*	6.07	67
	B	5.25	73
	C	4.54	86
	D	4.57	88
From 4.1 to 4.4 watts,	*A*	5.39	84
	B	4.81	87
	C	4.79	90
From 4.4 to 4.7 watts,	*C*	5.60	82
	D	4.75	98
From 4.7 to 5.0 watts,		4.97	101

The Most Economical Life of an Incandescent Lamp.

133. That which is characteristic of the incandescent lamp is that its efficiency is proportionately higher as its luminous intensity is greater. But this increase in efficiency is obtained at the expense of its life.

Now, in an electric light plant the running expenses include, aside from general expenses, two items, viz.:

1°. The cost of the energy W expended in the filament.

2°. The expense of replacing old lamps.

When the cost of the energy decreases, the expenditure for replacement increases, and *vice versa*. There is evidently a determinate life of the lamp which corresponds to a minimum total expenditure; in place of determining the life, we may also calculate the efficiency which makes the total cost a minimum.

These calculations are of real interest from the point of view of the economy of a plant; but it is difficult to treat the subject theoretically; for the elements of a lamp depend on one another in a way which is too complicated for it to be possible to establish formulæ which are simple and really practical. This work has been under-

taken many times, in different ways and by various men, in particular by Dietrich *, Picou †, Ayrton and Perry ‡, Desroziers §, Grassi ‖, and lately by Simon ¶. But these theoretical calculations have not, up to the present time, given results which are really useful.

To study this question in a practical way, we should start with a type of lamp for which the duration corresponding to various efficiencies is known. We then calculate the total expenditure for energy used and for lamps renewed. By making this calculation for the various values of the efficiency comprised, for instance, between 1.5 and 4.5 watts per candle, we obtain that which gives the greatest economy. This calculation should be repeated for each particular case, for the result depends essentially on the cost of energy.

Calculation of the Tint of an Incandescent Lamp.

134. From a study of the incandescent lamp, we may conclude that the filaments of two lamps are at the same temperature when they emit light of the same composition. This conclusion has been confirmed by all the researches, theoretical and experimental, to which the incandescent lamp has been submitted. The composition of the light emitted is shown by the tint; this is somewhat red when the lamp is below its proper candle power; it becomes whiter as the rate of the lamp becomes more forced. A simple examination of the tint of an incandescent lamp is enough to tell the rate to which it is subjected.

The calculation of the tint of an incandescent lamp may be made quite easily by using Crova's photometric method (§ 51), which consists, as we know, in comparing the intensities of the luminous radiations in the neighborhood of 0.582μ.

We know that the luminous intensities of two ordinary sources are equal when the intensities of the radiations in the neighborhood of 0.582μ are equal. We determine, then, the intensity of the incandescent lamp in terms of the carcel lamp, for the radiations indicated above. The intensity thus obtained is equal to the total intensity.

* *Electrotech. Zeitschr.*, August, 1884.

† *Bull. de la Soc. int. des Él.*, Vol. I. p. 315, 1884.

‡ *Phil. Mag.*, Vol. XIX., 1885, p. 304; *Lum. Él.*, Vol. XVII. pp. 10 and 60.

§ *Bull. de la Soc. int. des Él.*, Vol. II., 1885; *Lum. Él.*, Vol. XVIII. p. 603.

‖ *Atti del R. Istituto d'incorag. di Napoli*, Vol. II., 1889.

¶ *Électricien* of July 4, 1891.

Next, a second determination I' is made for the red rays corresponding to 0.657μ; it is sufficient for this to make the observation by means of red glass, chosen with the spectroscope. This glass should transmit rays of wave-lengths included between 0.726μ and 0.589μ, with a well-defined maximum at 0.657μ. Crova then defined the red tint of an incandescent lamp by the ratio $\frac{I'}{I}$.

Below are the values obtained by Crova* for the tints of various lights:

Sun.	Voltaic Arc.	Drummond Light.	Carcel.
0.50	0.59	0.94 and 0.69	1

This is the same thing as saying, that, with the same intensity of light, sunlight contains only one-half as much red light as is contained in the normal carcel light, etc.

Using many incandescent lamps nominally of 50 volts, Crova found that, when the rate varied between the extremes of 30 and 90 watts, the red tint varied from 1.33 to 0.88; in the first case the lamp is very red, in the second it is overtaxed and of a dazzling white.

Comparing different 50-volt lamps, Crova obtained the same tint as that of the carcel, by operating them at a rate varying from 56 to 57.2 watts, according to the lamp.

A normal rate which might characterize each lamp would be for instance the rate which gives the lamp the same tint as the carcel lamp or a determined fraction of this tint. The voltage of a lamp might be defined by the number of volts required to give a tint equal to that of a carcel or a determined fraction of this tint. Thus with the tint 1, lamps are too dim; but they give good light without being overtaxed with the tint 0.9.

These considerations would enable makers to furnish lamps of an exactly known voltage.

As an example, we may cite three 50-watt lamps, *A*, *B*, *C*, which gave the following results, the watts varying from 30 to 90:

	A	*B*	*C*
The red tints varied from .	1.33 to 0.88	1.18 to 0.79	1.12 to 0.87
The tint is that of the carcel using	57.2 watts	56.5 watts	56 watts
The luminous intensities varied from	0.13 to 3.68 carcel	0.12 to 3.50 carcel	0.15 to 3.25 carcel

* *Comptes Rendus du Congrès des Électriciens de 1889*, p. 207.

Instead of defining the red tint of a light by the ratio $\frac{I'}{I}$, its degree of incandescence might be defined by the ratio inverted, $\frac{I}{I'}$. The adoption of this method of measuring the rate of lamps would only necessitate the addition of two glasses or colored baths for which $\lambda = 0.582\mu$ and $\lambda = 0.657\mu$.

Different Values of the Efficiency of Incandescent Lamps.

135. To complete the study of incandescent lamps, and especially as historical data, we give the principal results relative to the efficiency of the incandescent lamps investigated at the Expositions of Paris and Munich. We might also complete this table by data relative to present lamps; but these data, taken in great part from the advertisements of makers, have not sufficient scientific value; besides, they may be found in all the annuals and formularies of the electrician. Further, it should not be forgotten that the initial efficiency of a lamp is far from describing it; its mean efficiency is more important, and this element is very rarely given.

EXPOSITION OF PARIS (1881).

Lamps.	Mean Intensity in Candles.	Amperes.	Volts.	Watts.	Efficiency.		
					Watts per Candle.	Candles per Horse Power.	Lamps per Horse Power.
Edison *A*, 16 c. p.	15.38	0.651	89.11	59.11	3.7	196	12.7
Edison *A*, 32 c. p.	31.11	0.7585	98.39	76.04	2.4	307	9.9
Swan *A*	16.61	1.471	47.30	79.59	4.15	178	10.7
Swan *B*	33.21	1.758	54.21	96.70	2.86	262	8.0
Lane-Fox *A*	16.36	1.593	43.63	70.89	3.64	173	10.6
Lane-Fox *B*	32.71	1.815	48.22	89.36	2.76	276	8.5
Maxim *A*	15.96	1.380	56.49	79.39	4.09	151	9.5
Maxim *B*	31.93	1.578	62.27	10.03	3.07	239	7.5

EXPOSITION OF MUNICH (1882).

Lamp.	Mean Intensity in Candles.	Amperes.	Volts.	Watts.	Efficiency.		
					Watts per Candle.	Candles per Horse Power.	Lamps per Horse Power.
Edison *A*, No. 1. .	18.473	0.678	107.6	74.4	4.0	184	10.0
" *A*, No. 2. .	21.159	0.809	110.1	90.6	4.2	175	8.2
" *B*, No. 1. .	15.576	0.896	59.22	54.1	3.4	216	14.0
Maxim *L*, No. 1. .	17.975	1.344	65.14	89.3	4.86	151	8.4
" " No. 2. .	21.571	1.353	63.05	86.7	3.9	188	8.6
Small Swan *L*, No. 1,	14.909	1.318	39.46	53.0	3.46	212	15.0
Large Swan *L*, No. 5,	18.334	1.161	97.65	114.9	6.15	120	6.5
Siemens, No. 1 . .	17.157	0.946	95.60	92.3	5.2	141	8.1
" No. 2 . .	17.170	0.956	100.10	97.6	6.15	120	7.6
" No. 3 . .	17.742	0.906	96.99	89.6	5.0	147	8.3
Small Müller, No. 5,	17.661	1.189	70.95	86.0	4.77	154	8.7
" " No. 6,	22.385	1.305	75.29	99.3	4.38	168	7.5
Medium " No. 1,	21.338	1.573	89.16	143.1	6.5	113	5.2
" " No. 2,	24.529	1.423	97.75	141.9	5.67	130	5.3
Large " No. 1,	32.643	1.710	121.20	281.1	8.7	85	3.5
Cruto.	19.687	3.257	26.62	88.4	4.45	165	8.9

B. ARC-LAMPS.

The Voltaic Arc.

136. The voltaic arc is produced by the passage of a current between two carbon electrodes in air raised to a high temperature. The first experiments relative to the voltaic arc were made by Humphry Davy, beginning in 1800*, but those in which this illustrious English chemist was able to obtain the arc in a continuous manner date from 1808 and 1809.

Davy having taken two small, sharpened rods of carbon, put them in contact, and passed between them the current of a voltaic battery of 2000 elements. On separating the two points a very small distance, he saw produced between them a slightly convex flame, which remained until the distance between the carbons reached 10 cm. The arc disappeared when this distance became greater, and the points rapidly became cold. The carbons when slowly

* *Lum. Él.*, Vol. IX. p. 248.

brought toward one another did not give place to any luminous or calorific phenomenon while they were not in contact; but as soon as they touched, the points became hot, and, at the moment of their separation, the flame burst out instantaneously. Davy gave this arched flame, whose brilliancy was comparable with that of the sun, the name of *voltaic arc*, which it has kept up to this time.

Nature and Appearance of the Voltaic Arc.

137. The voltaic arc results from the incandescence of a jet of particles detached from the electrodes and projected in all directions. This projection, when continuous currents are used, takes place principally from one electrode to the other, and more particularly from the positive to the negative. The positive electrode has a very much higher temperature than the other; while the negative carbon is scarcely a dark red, the positive carbon, at the same distance from the arc, is a reddish white over a considerable length. The consumption of the positive electrode for a given time is double that of the negative. It is this difference in the consumption and temperature, observed from the beginning by physicists, which first led them to explain the phenomenon of the luminous arc as a simple transportation of particles from the positive to the negative electrode. It is now well demonstrated that although the transference from the positive to the negative electrode predominates in the arc, there exists nevertheless a very active transference from the negative to the positive.

The arc resembles, in fact, a trembling flame, whose form is ovoidal between the points of carbon. From time to time a brilliant particle may be seen to leap from one electrode to the other, producing a luminous flame. On each of the carbons there appear here and there liquid and incandescent globules, due to mineral substances, which become displaced, glide to the point, and leap forth to gain the other electrode. These liquid globules do not appear when the carbons are chemically pure.

When the voltaic arc is produced in air, the two carbon rods diminish rapidly in volume because both of them burn; but *in vacuo*, this combustion does not take place, and the positive point is seen to become hollow and diminish in weight, while the negative point elongates and increases in volume. The consumption is almost nothing, and only results from particles being projected by the two carbons outside their reciprocal action.

The voltaic arc is a portion of the electric circuit having all the properties of the other parts of the circuit. The detached particles constitute between the two points a movable chain more or less conductive and more or less heated according to the intensity of the current on the one hand, and the nature and separation of the electrodes on the other. Things go on exactly as if the electrodes were united by a metallic wire or a carbon rod of small section, which amounts to saying that the light produced by the voltaic arc and that obtained by incandescence are due to the same cause, which is the heating of a resisting body interposed in the circuit. Matteucci showed this similarity between the voltaic arc and the other parts of the electric circuit, by slowly separating two iron bars previously put in contact: a thread of liquid metal appeared at first, became luminous, then broke to give place to the voltaic arc.

By examining attentively the voltaic arc produced between two carbon points by continuous currents, we clearly distinguish the arc and the flame. The arc is blue; it connects the bright parts of the two carbons. The flame is reddish; it envelops the arc and often gives it a violet appearance because of its interposition. This flame sometimes becomes very long and starts to lick the sides of the positive carbon to a considerable distance; it is changeable and mobile; it contributes to the variations of luminous intensity, and in certain cases disappears completely. It is then especially that the arc appears with the very slightly luminous blue color which is peculiar to it.

With currents of from 50 to 70 amperes, the most intense practically in use to supply a single arc, the appearance is modified and presents some peculiarities less known.

At first the flame has a purple color around the negative, then, when the carbons are normally shaped, a narrow blue band is distinguished on the bright surface of the positive, and a red halo about the negative; the intermediate region of the arc is white.

Whatever may be at first the form of the carbon rods, they shape themselves in due course; the positive point takes on the appearance of a truncated cone terminated by a concave surface; the negative point takes the form of a cone terminated by a blunt point. These forms are more regular when the points are accurately opposite one another and where the carbons are carefully made and free from foreign substances.

The arc is produced not only between carbon points, but between all sorts of substances sufficiently conductive. Its brilliancy depends

on the intensity of the current, the nature of the electrodes, and the medium in which it is produced. With potassium or sodium, for instance, the light is more brilliant than with platinum or gold; there is more light in air than in mercurial vapors.

The color of the arc varies with the substance of the electrodes: it is yellow with sodium, white with zinc, green with silver, etc.

The appearance of the arc depends also on the form of the electrodes: between a positive point of coke and a negative plate of platinum it presents the form of a cone; between two carbon points it has the form of an egg or more often that of a truncated cone, etc.

138. The preceding information, taken from Fontaine's treatise on electric lighting, relates to the voltaic arc produced by continuous currents. The conditions of working of the alternate current arc are not the same.

In the latter there is no difference between the two carbons which are the seat of similar phenomena. They both become pointed. However, the upper carbon is consumed a little more rapidly than the lower, because of the ascending current of air which tends to consume it at the edge. There results a sensible difference in the distribution of the luminous intensity.

The characteristic feature of the alternate current arc is the very disagreeable humming which it produces; this is very intense and cannot be suppressed. Its intensity may be diminished by surrounding the lamp by a closed globe.

The cause of this humming is not surely known; it is thought, however, that it is produced in great part by the rapid extinction and relighting of the arc and by the rapid variations of temperature which result*.

The Difference of Potential between the Electrodes of the Voltaic Arc.

139. The voltaic arc is characterized by a considerable fall of potential between the electrodes, a fall which shows that the production of the arc requires a considerable expenditure of energy.

This difference of potential depends on the nature and diameter of the carbons as well as on the length of the arc; it is given, according to Edlund, by the equation

$$V = a + bl,$$

* Nichols, *Electr. World.* 1891, Vol. XVII. p. 399.

a and b being constants, and l the length of the arc. Physicists differ greatly as to the significance of the constant a.

In the opinion of certain physicists, this constant corresponds to a resistance to passage whose seat is found on the positive electrode or in its immediate vicinity; other physicists, on the contrary, think with Edlund that there is no appreciable resistance to passage, and that the constant a represents an electromotive force opposed in direction to that of the current. [See Appendix F.]

Beside these two opinions there should be mentioned other hypotheses, among them that of G. Wiedemann, who assumes that the voltaic arc is a discontinuous discharge, and that of Lecker, who thinks that the electric current broadens out in passing from one carbon to the other, and is not confined to the neighborhood of their common axis.

Among explanations so different it is best to assume that the actions which they contemplate all contribute to the production of the phenomena which accompany the voltaic arc.

It seems, however, that resistance to passage plays a principal part in the production of the voltaic arc; at least this follows from precise measurements made by Uppenborn and Lecher in particular.

Uppenborn found from many photographs that the voltaic arc has the form of a truncated cone whose very large base rests on the positive carbon, which indicates a very great resistance to passage at this place; the fall of potential then takes place principally at the passage of the current from the positive carbon into the air. This fact is, moreover, confirmed by direct measurements of potential.

With 12 mm. carbons and with lengths of arc varying from 6 to 16 mm., Uppenborn found a mean value for a of 38 volts, formed in part by a fall of potential of 32.5 volts on going from the positive carbon into the air. Lecher proved, moreover, that the potential remains constant in the layer of air included between the two electrodes even at a considerable distance from the axis.

This discontinuous variation of potential only takes place with carbon electrodes, and is not found with electrodes of iron, platinum, silver, or copper.

The constants a and b of Edlund's equation vary with the intensity of the current, as well as with the nature and dimensions of the carbons. In this way Nebel found the following values of the difference of potential observed with carbons of different diam-

eters and with currents of different intensities; l is expressed in millimeters.

Intensity of Current.	Volts with Carbons of		
	10 mm.	12 mm.	14 mm.
12 amperes	$39.3 + 2.2 l$	$35.2 + 2.6 l$	$32.4 + 2.8 l$
16 "	$39.2 + 2.0 l$	$36.1 + 1.4 l$	$34.1 + 2.8 l$
20 "	....	$38.0 + 1.9 l$	$34.4 + 2.1 l$
24 "	....	$38.6 + 2.1 l$	$34.9 + 1.9 l$

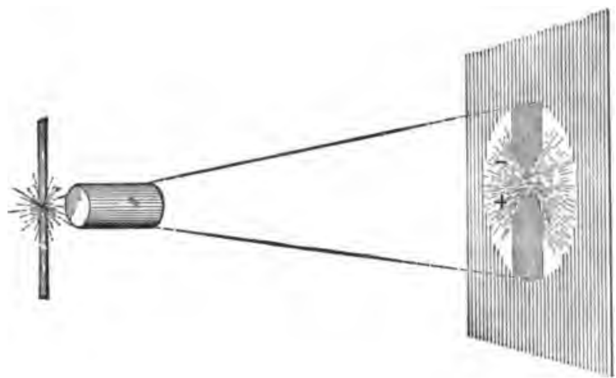

FIG. 81.

With carbons of different composition, but of the same diameter, using currents of the same intensity (12 amperes), Uppenborn obtained the following values for the difference of potential:

Carbon No. 1	$35.4 + 2.10 l$
" " 2	$39.0 + 1.74 l$
" " 3	$40.0 + 2.20 l$
" " 4	$41.0 + 2.16 l$
" " 5	$45.4 + 1.99 l$
Mean . .	$40.1 + 2.24 l$

From the figures which precede and from still other results it follows that the difference of potential between the electrodes diminishes as the diameter of the carbons increases, i.e. as their resistance diminishes.

This fact is moreover used in low-tension arc-lamps, of which the Weston, much used in the United States, is an example. This dif-

ference of potential varies also with the nature of the carbons and may vary by 10 or even 15 volts for carbons of the same diameter but of different sources.

The study of the voltaic arc — the form and length of the arc, the appearance of the carbons — is made very easy by projecting the whole on a white screen by means of a magnifying lens. Figure 81 represents the arrangement of the apparatus with so much precision that it is needless to enter into its details.

The Nature and Manufacture of Carbons.

140. The composition of carbons having considerable influence on the photometric properties of the voltaic arc, we give certain details of the manufacture of the pencils employed.

It is known that Davy employed pencils of charcoal extinguished in water or mercury; these burned quite regularly, but too rapidly to be employed commercially. It was Foucault who first replaced the charcoal by the deposits made on the sides of gas retorts. Gas-retort carbon is much denser than charcoal and resists much longer the destructive action of the voltaic arc; but its composition is not uniform, which produces considerable variation in the luminous intensity; this variation is due to the presence of foreign matter, alkaline salts or earths and silica, which evaporate and help to form the flame which surrounds the arc. Attempts have been made to purify gas-carbon by various processes, but without complete success.

The process of manufacture invented by Jacquelaine consists in producing artificially gas carbon free from all impurity by bringing into contact with the incandescent sides of the retort very dense hydrocarbureted material, of which a part volatilizes, and the remainder decomposes, leaving as a residue a layer of carbon; by employing well-purified tars, perfectly pure gas carbon is obtained. The carbon pencils of Jacquelaine gave a very white, steady light about 25 per cent more intense than that given by ordinary carbons with equal current. These carbons had the inconvenience of being very hard, which required considerable work in sawing them and occasioned considerable waste.

The methods used at present for making carbon pencils are due to Carré and Gauduin; they consist in preparing a suitable paste, triturating and compressing it, then passing it through a die so as to obtain cylindrical pencils which are cut the desired length and baked in a furnace like those used for pottery.

The composition of the paste varies according to the make. Carré employed at first a mixture of 50 parts very pure powdered coke, 20 parts calcined lampblack, and 30 parts of a syrup made of cane sugar and gum. Pulverized gas carbon mixed with gas tar is now generally used. The quality of the product depends on the pulverizing of the material, and on the baking; the latter is many times repeated, and between times the carbons are plunged into a boiling concentrated syrup of cane or caramel.

In order to maintain the arc at the middle of the carbon, a thing which increases the stability and regularity of the axis, a central hole is made in the carbon, while in the die, and filled with a substance called a core, a little better conductor of electricity than the carbon. The cored carbons give excellent results. They are only employed for the positive pole, because of their relatively high price.

In order to prolong the life and increase the conductivity of the carbons, they are often electroplated with copper. Coppered carbons are very much employed, especially in America. In Europe, bare carbons are more used, because the light is more regular. As to nickeled carbons, they have not yet come into the market.

Besides carbons passed through a die, as those of Carré, moulded carbons are also made. In general, carbons made with a die are more dense than those moulded, and are more suitable for low tensions, which require a more intense current and should possess greater conductivity.

For fuller details concerning the manufacture of arc-light carbons we refer to special works.

Regulators and Candles.

141. The continuity of the voltaic arc is obtained by means of a regulating mechanism which maintains the separation of the carbons at a constant distance for a determined intensity of current and difference of potential. There are a great number of different regulators, a great majority of which depend on the following principle:

When the length of the arc varies, the intensity of the current varies also, so that the intensity diminishes as the carbons are consumed. This diminution of intensity is profitably used to maintain the carbon points at a constant distance, within very narrow limits. For this, there is introduced into the circuit an electro-magnet, whose armature is drawn in one direction by magnetic action and

in the other by a spring. When the intensity of the current diminishes, the magnetic action diminishes, and the armature moves under the preponderating influence of the opposing spring. This movement is used to unclamp a mechanism which allows the carbon-holders to approach.

In electric candles, the regulation of the voltaic arc is obtained without a regulator. The candles are composed of two parallel rods of carbon with an insulating substance placed between them, and the whole made into one bundle. The carbons burn under the action of the current, and need no mechanism for lighting or regulating them. The length of the arc depends solely on the distance of the pencils, and lighting is brought about by means of conductive priming placed at the top of the points at the time of manufacture.

We refer to special treatises for all that concerns the various mechanisms employed in arc-lamps and the study of the best conditions of operating them.

History of the Photometry of Arc-Lamps.

142. From 1808, when Davy first made the arc-light burst forth, up to the time when the electric light entered into commercial use, the photometric study of arc-lights had scarcely been undertaken. We cannot consider as a complete investigation the researches of Fizeau and Foucault in 1843 and 1844, in the course of which these physicists made a series of comparisons between the chemical action of sunlight, electric light, and oxy-hydrogen light. Although the results obtained have passed into all the treatises on physics, they have only a very restricted value, for they refer to the action of light on a plate of iodide of silver and not to photometric action properly so called. Below are the results:

Intensity of sunlight, Apr. 11, 1844, at 11.15 A.M., with a very clear sky	1000
Intensity of sunlight, Sept. 20, 1843, at 2 P.M., with a pale blue sky	751
Intensity of the voltaic arc produced by 3 series of 46 Bunsen elements	385
Intensity of the oxy-hydrogen light	6.85

The feeble chemical action of the oxy-hydrogen light led Fizeau and Foucault to determine directly by an optical method the photometric action of the voltaic arc and the oxy-hydrogen light. The result of this comparison was that the light emitted by the voltaic arc was to the oxy-hydrogen light as 32.6 is to 1.6, while the same ratio determined by chemical action had been found to be as 34.3 is to 1.

The similarity of the two results led the two physicists to conclude that, from a practical point of view, the chemical and photogenic actions of two luminous sources are equivalent.

Passing rapidly over the measurements of Casselmann (1843), which were the first measurements made with the Bunsen photometer, over those of Becquerel made on the occasion of the electric lighting tests of Lecassagnac and Thiers at Lyons and in the *Place de l'Étoile* at Paris (1855 to 1857), and over those occasioned by the introduction into light-houses of the electric light produced by means of the Alliance machines, we come to the introduction of the Gramme machine.

From this time, the daily increasing development of electric lighting has occasioned a great number of photometric measurements.

One of the first complete photometric studies of the voltaic arc is that of H. Fontaine, made by means of a Serrin regulator provided with current from a Gramme machine. Mention should also be made of the measurements of Allard and of Sautter and Lemonnier. But photometric measurements have become more numerous and more complete since the time of the Electrical Exposition of 1881, when the more perfect regulation of the machines and lamps, as well as the good quality of the carbons, permitted a more fixed light to be obtained. It is sufficient to mention the photometric tests of the Exposition of Paris (1881), at Munich (1882), at Vienna (1883), at Philadelphia (1884), at Antwerp (1885), etc., to which should be added a number of wholly unofficial tests.

Theoretical Examination of the Variations of Luminous Intensity of an Arc-Lamp.

143. From a photometric point of view, that which characterizes the voltaic arc is the great variations in luminous intensity according to the direction of the ray, variations which are greater than in any other common source of light. When the voltaic arc is supplied by continuous currents, these variations are greater than when it is supplied by alternate currents. The cause of this difference should be attributed to the preponderating part taken by the positive carbon in furnishing the light emitted by the continuous-current arc-lamp.

The greatest part of the light emitted by the voltaic arc is due to the electrodes which are raised to incandescence. Thus it has

been estimated that in the continuous-current arc-lamp 5 per cent only of the total light emitted is due to the arc, a proportion of 10 per cent is furnished by the negative carbon, and the rest, or 85 per cent, by the positive carbon.

The intensity of the lamp is then the resultant of the intensities produced by the arc and the carbons. Now the quality of the light emitted by an incandescent body is proportional to the emissive power of the latter; a quality of carbon should then be employed whose emissive power at high temperatures is as great as possible. We know that the emissive power of carbon is very great, greater for instance than that of the majority of metals: it is this that explains the particular brilliancy of the arc obtained with carbon electrodes.

In the continuous-current arc, the greater part of the light emitted comes from the end of the positive carbon, which is generally placed above the negative so that the rays may be thrown upon objects below which it is desired to illuminate. A simple examination of the calorific conditions of the arc permits one to deduce with sufficient exactness the variations in luminous intensity with the direction of the rays.

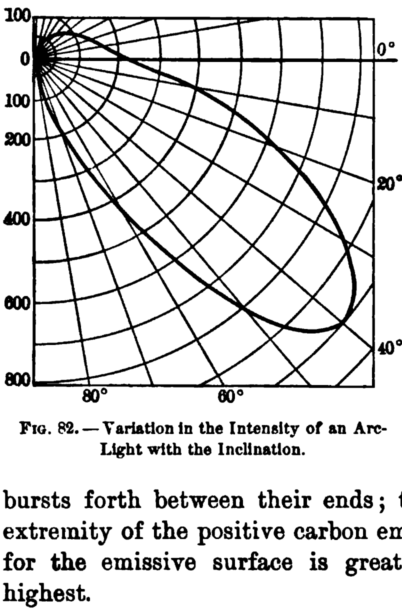

FIG. 82.—Variation in the Intensity of an Arc-Light with the Inclination.

Suppose that the carbons terminate theoretically in two equal truncated cones; the arc bursts forth between their ends; the crater which is formed at the extremity of the positive carbon emits the greater quantity of light, for the emissive surface is greatest there and the temperature highest.

It is practically only the arc and the sides of the carbons near their ends which emit light horizontally. Further, the shadows thrown by the positive and negative carbons limit the emission of light more rapidly as the arc becomes shorter; the opening of the luminous cone is greater as the arc is longer. But as the length of the arc increases, the temperature of the carbons falls, and the intensity of the light emitted diminishes.

This summary examination shows, then, that the luminous intensity must be zero in a vertical direction, must pass through a maximum in a particular direction below the horizontal, and through a minimum in the horizontal, to increase and also decrease rapidly above the horizontal. This last conclusion alone was not entirely confirmed by the photometric observations because of various things which modify the regularity of the radiation of light from the negative carbon, and principally because of the fact that the light emitted by the arc in a horizontal direction is more intense than that which the negative carbon emits upward. Thus the minimum in the horizontal plane does not always exist, and the diminution continues regularly above it.

The diagram which represents the variations in luminous intensity with the inclination of the rays is well known; that in Fig. 82 gives the result of all the photometric measurements made on arc-lamps at the Antwerp Exposition.

Variations of the Luminous Intensity with the Azimuth.

144. Since everything in the voltaic arc is in general symmetrical with reference to the axis of the carbons, the luminous intensity should be independent of the azimuth of the ray; i.e. the photometric surface should be a surface of revolution. This would be so, in fact, if the carbons were homogeneous and well centered and if the regulator always performed its functions. Unfortunately these conditions are not always fulfilled, so that it is not exact to assume that the intensity is independent of the azimuth, although this hypothesis is generally accepted in practice. Centering the carbons, in particular, offers the greatest difficulty; whatever be the care with which the carbons are made, they always undergo a slight bending after burning some moments, which produces a lateral displacement of the points and a disturbance in the distribution of the luminous intensity.

This disturbance is much more sensible than is generally supposed. It has been noticed by all who have made photometric measurements of the arc-lamp. It has been recently studied with care by Wedding*. This engineer measured the luminous intensities for various inclinations in two azimuths differing by 180°. He used in all nine pairs of carbons, working regularly with a current of 14 amperes. He further photographed the carbon points in the course of each of these tests to determine the cause of the irregularities.

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* *Elektr. Zeitschrift*, 1889, p. 337.

The following table contains the measurements of Wedding; they confirm a conclusion arrived at from the measurements at Paris and Antwerp, namely, that the variations of the maximum luminous intensity are much less than those of the horizontal intensity. In this table the luminous intensities are expressed in candles.

Carbons.	Horizontal Intensity.		Maximum Intensity in Candles.				Mean Spherical Intensity.
	Left.	Right.	Left.	θ	Right.	θ	
1	109	136	1720	43°	1860	40°	1246
2	147	350	2000	43	2110	39	1246
3	121	157	1790	46	1890	42	1114
4	152	199	1670	42	2310	46	1260
5	155	228	2050	45	2500	41	1355
6	124	373	1860	42	2180	41	1239
7	143	408	2020	42	2480	38	1359
8	183	224	2010	43	2990	43	1179
9	136	171	1710	44	2000	43	1056
Mean. .	141	250	1870	43	2158	41	1228

The curves which represent the detailed results of these measurements show that the real distribution of the luminous intensity of the voltaic arc is most irregular; the photometric surface is far from being a surface of revolution. In calculations based on the luminous intensities of electric lights, one must be content with a rough approximation.

Variations of the Luminous Intensity with the Inclination.

145. This photometric element of arc-lamps is the best known; the curve which represents these variations differs relatively very little from lamp to lamp.

Wybauw has combined all the photometric measurements of arc-lamps made at the Antwerp Exposition in 1885, and has obtained from them a mean curve representing the variations of luminous intensity with the inclination; this diagram is reproduced in Fig. 82.

The curve obtained* is much like an ellipse whose axis is the radius vector at 40°; it decreases about symmetrically above and

* See Appendix G.

below, but this decrease is very small in the neighborhood of 40°. The mean values deduced from 26 lamps differing in their construction and rate, deviate very little from the particular values of each lamp, the mean intensity being represented by 1000.

The following table gives the values of the intensity for the inclinations included between 60° above and 70° below the horizontal:

Inclination θ.	Luminous Intensity.	
	Observed.	Calculated.
60° Above the horizontal	48	28
30 " " "	110	104
0	208	208
10 Below the horizontal	401	421
20 " " "	612	629
30 " " "	871	824
40 " " "	1000	1000
50 " " "	807	800
60 " " "	457	457
70 " " "	188	206

146. These results have been confirmed by all measurements since those at Antwerp. An exhaustive study of them permits the following conclusions to be made:

In the majority of continuous-current arc-lamps, we may distinguish on the unit sphere four well-defined zones relating to the distribution of luminous intensity about the light-center.

In the upper hemisphere, which receives only a small part of the total light, the luminous intensity varies about proportionally to $\sin\theta$; it may then be represented by

$$i' = H(1 - \sin\theta),$$

H being the horizontal intensity and θ being reckoned positive above the horizontal.

In the lower hemisphere, there is distinguished: first, a zone (between 0° and 40°) in which the luminous intensity is of the form

$$i'' = H + a\sin\theta,$$

θ being counted positive here also; then a zone more or less narrow (between 40° and 45°), in which the variations of luminous intensity are very slight, for which

$$i''' = M,$$

M being the maximum intensity; the width of this zone depends on the nature of the crater of the positive carbon; finally a zone in which the intensity decreases according to the law

$$i^{iv} = b - c \sin \theta.$$

The luminous intensity I is then represented by the complex formula

$$I = [H(1 - \sin \theta)]_{0°}^{90°} + [H + a \sin \theta]_{0°}^{\theta_1} + [M]_{\theta_1}^{\theta_2} + [b - c \sin \theta]_{\theta_2}^{90°},$$

using that part only whose limits include the inclination for which the luminous intensity is to be calculated.

When the variations of luminous intensity are sensibly the same above and below the maximum, at least in its immediate vicinity, the constant term may be suppressed, which is the same as supposing $\theta_1 = \theta_2$. We then have

$$I = [H(1 - \sin \theta)]_{0°}^{90°} + [H + a \sin \theta]_{0°}^{\theta_1} + [b - c \sin \theta]_{\theta_1}^{90°}.$$

The constants a, b, c may easily be expressed in terms of H, M, θ_1, and θ_2.

For $\theta = \theta_1$ we should have

$$I = M;$$

whence, $$M = H + a \sin \theta_1,$$

or $$a = \frac{M - H}{\sin \theta_1}$$

Similarly, we may find

$$b = c = \frac{M}{1 - \sin \theta_2}.$$

The complete equation then becomes

$$I = [H(1 - \sin \theta)]_{0°}^{90°} + \left[H + \frac{M - H \sin \theta}{\sin \theta_1}\right]_{0°}^{\theta_1} + [M]_{\theta_1}^{\theta_2}$$
$$+ \left[\frac{M}{1 - \sin \theta_2}(1 - \sin \theta)\right]_{\theta_2}^{90°}.$$

No considerable error is made by assuming in practice that the luminous intensity is a maximum, and sensibly constant between 40° and 45°; the equation may then be written

$$I = [H(1 - \sin \theta)]_{0°}^{90°} + [H + 1.5557\,(M - H) \sin \theta]_{0°}^{40°}$$
$$+ [M]_{40°}^{45°} + [3.413\,M(1 - \sin \theta)]_{45°}^{90°}.$$

By substituting for H and M the mean values deduced from the Antwerp experiments, we obtain

$$I = 208\,[1-\sin\theta]_{0^\circ}^{90^\circ} + [208 + 1232\sin\theta]_{0^\circ}^{40^\circ} + [1000]_{40^\circ}^{45^\circ} + 3413\,[1-\sin\theta]_{45^\circ}^{90^\circ}.$$

By means of this equation, the values included in the third column of the preceding table (p. 237) were calculated. Consulting the table, we find the agreement as great as possible.

By means of an analogous equation the luminous intensity of arc-lamps of any system may be represented with sufficient exactness provided that continuous currents are used.

It should always be remembered that individual results may show considerable differences if the measurements are limited to a single azimuth; to obtain sufficient agreement, the measurements should be made in at least four different azimuths, so as to eliminate irregularities in the working of the lamp or in the consumption of the carbons. Rousseau, moreover, dwells on this point with considerable force in his report on the Antwerp tests, and the measurements of Wedding, mentioned above, confirm this conclusion.

Mean Spherical Intensity.

147. In what precedes no attention has been paid to mean spherical intensity. Up to the present too great value has been given to this element, which only plays a very small part in problems of illumination by bare arc-lamps.

The most important element in the arc-lamp is the maximum intensity; a knowledge of the horizontal and maximum intensities easily permits the calculation of the intensity for a given inclination, and it is the latter which should be employed in calculating the illumination at a given point. The mean spherical intensity should not be employed in calculating illumination, instead of the effective intensity for the inclination considered.

The mean spherical intensity gives a basis for calculating the coefficient of transformation of the lamp, or its efficiency if desired, but that is all. Of two lights of the same mean spherical intensity, that will be the better which has the higher maximum intensity, since it is this maximum which determines the limit of the lighting power of the lamp, the illumination always being sufficient below the lamp, especially if a reflector is employed.

However, it is sometimes of interest to know the mean spherical intensity of a given lamp, especially as this quantity is easy to calculate by an approximate formula which is precise enough.

The exact calculation of the mean spherical intensity is very laborious, since it requires precise measurements at a great number of inclinations close together. However, if we designate by H the horizontal intensity and by M the maximum intensity, we may represent the mean spherical intensity S by the equation already proposed at the Electrical Exposition at Paris in 1881:

$$S = \frac{H}{2} + \frac{M}{4}.$$

The following table made from the official report of Rousseau represents the result of the Antwerp photometric tests on lamps of various systems with carbons of different kinds (Siemens, Schmelzer, etc.).

	Intensity (in Carcels).				Deviations in Per Cent O–C.
	Horizontal.	Maximum.	Mean Spherical.		
			Observed.	Calculated.	
1. Jasper	102	557	198	188	+ 5
2. Brush	102	522	192	182	+ 5
3. Gramme	72.5	471	166	154	+ 7
4. Piette and Krizik	92	446	161.5	158	+ 2
5. Crompton	60	373	132	123	+ 7
6. De Puydt	61	362	120	121	− 1
7. Dulait	73	423	137	142	− 4
8. Gramme	95.5	265	119	114	+ 4
9. "	54	265	96	93	+ 3
10. Piette and Krizik	56	276	100	97	+ 3
11. Cramer and Dornfelt	57	185	68	75	− 10
12. Piette and Krizik	36	190	66	66	0
13. Pieper	25	120	43	43	0
14. Brush	42	209	70	73	− 4
15. Piette and Krizik	34	206	59	68	− 15
16. " " "	26.5	177	58	58	0
17. Gulcher	44.5	207	61	74	− 21
18. Pieper	21	100	35	35	0
19. "	17	94	31.5	32	− 2
20. "	19	72	28	27.5	+ 2
21. Brush	32	102	35	41.5	− 12
22. Gramme	22	145	48	47	+ 2
23. Pieper	10	52	18	18	0
24. "	9.4	60.6	16.3	19.8	− 21

The last column of this table gives the percentage deviation between the results furnished by direct observation and those calculated by means of the preceding equation.

The mean of the deviations is 5.7 per cent; that of the positive deviations (9 in number), 5.2 per cent; and that of the negative deviations (10 in number), 9 per cent.

Uppenborn also applied this rule to the photometric measurements made on seven different lamps; the results of this comparison are given in the table below.

Lamps. Authority: Uppenborn.	Candle Power.				Deviation in Per Cent O–C.
	Horizontal.	Maximum.	Mean Spherical.		
			Observed.	Calculated.	
1	250	1464	470	491	− 4
2	456	3250	1145	1040	+ 9
3	560	3071	1221	1048	+ 14
4	744	1227	692	679	+ 2
5	122	840	274	271	+ 1
6	586	2100	802	818	− 2
7	935	1150	767	755	+ 2

Finally, Marks also made use of this formula to discuss his own measurements; he further took a certain number of different observations made in America by many electricians. All these results of such varied origin are included in the following table (p. 242); they agree almost perfectly with the equation.

The agreement between observation and calculation is close enough, although somewhat less so than in the preceding measurements; the mean of the deviations is 9 per cent in place of 5.7 per cent.

All these results then give to the equation

$$S = \frac{H}{2} + \frac{M}{4}$$

a really practical value; it may be applied in all cases with surety greater in proportion as the values of H and M have been deduced from measurements made in several azimuths.

This equation might also be simplified by noticing that, according to Rousseau's measurements with respect to 24 different lamps,

Lamps. Authority: Marks.	Candle Power.				Deviation in Per Cent O–C.
	Horizontal.	Maximum.	Mean Spherical.		
			Observed.	Calculated.	
Weston	344	609	348	324	+ 7
"	299	576	295	293	+ 1
"	576	1235	640	597	+ 7
Brush	313	1395	609	504	+ 17
Ball.	233	534	240	249	− 4
Brush (1200 c. p.). . .	180	617	263	244	+ 7
Brush (2000 c. p.) . . .	389	1380	653	539	+ 17
Van De Poele	451	1377	574	569	+ 1
" " "	333	1155	470	455	+ 3
Weston Electric . . .	263	355	186	220	− 18
Thomson-Houston . . .	227	1080	425	383	+ 10
" " . . .	222	626	288	267	+ 7
" " . . .	382	1131	525	474	+ 9
Weston	594	1183	514	593	− 16
"	475	871	400	436	− 9

the horizontal intensity is equal to 0.208 of the maximum intensity; we have then approximately $H = 0.2\ M$, whence

$$S = 0.35\ M.$$

However, this equation is not absolutely reliable, for the horizontal intensity is an element which varies considerably from lamp to lamp; it would be better to keep to the complete formula.

The Employment of Opal Globes and Reflectors.

148. Bare arc-lamps are quite rarely employed, for the light emitted is too harsh in the neighborhood of the lamp, and the total effect of illumination is less advantageous. Bare lamps are employed in the illumination of work-yards and large open spaces.

In street lighting, the use of opalescent globes has prevailed, although they absorb a quite large proportion of the light, estimated by von Hefner-Alteneck * at

* *Lum. Él.*, Vol. X. p. 498.

15 per cent for alabaster glass,
20 per cent for opal glass,
30 to 60 per cent for milky glass.

These globes are almost always surmounted by reflectors planned to throw the light from the upper hemisphere down toward the ground. The presence of the globe and the reflector modifies totally the distribution of the light.

Uppenborn found that spherical globes give better results, from a photometric point of view, than elliptical ones. There is less absorption, and the distribution is more advantageous.

The opalescent globe which surrounds the arc does not absorb the light uniformly; the luminous intensity of the parts of the globe which correspond to the maximum intensity is sensibly decreased, while that part which corresponds to the minimum is increased. The globe itself becomes luminous and plays the part of a radiant.

The nature of the globe, its dimensions, and its form are not without importance. It is generally assumed that the globe should be small, like that of the Cance lamp for instance, so as to replace the point which is too brilliant for the eyes and casts too dense shadows, by a disc whose brightness seems about uniform and which appears as the sun, subtending a small angle. If too large a globe is taken, this effect is completely wanting. In the middle is seen a brilliant point, then the surface of the globe whose brightness is divided into more or less luminous zones.

The glass of which the globe is made should be opaque enough to obtain sufficient diffusion, while absorbing as little light as possible. This result is obtained by taking an uncolored glass, having on one of its faces a very thin layer of milky glass sufficiently transparent so that in the daytime, when the lamp is extinguished, the form of the carbons may be clearly distinguished through the globe. With this glass as good results are obtained as with the Cance globes, which are very thick, and much less light is lost.

The most exact data which we possess concerning the employment of globes and reflectors with arc-lamps, are those obtained by Wedding* in the course of an examination of the arc lighting of the streets of Berlin.

The addition of different globes (I, II, III) to the 14-ampere arc-lamps mentioned in § 144 gave the following results:

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* *Elektr. Zeitschrift*, 1889, p. 338.

Number of Globe.	Horizontal Intensity.	Maximum Intensity.	Inclination of Maximum.	Mean Hemispherical Intensity.	Weakening in Per Cent.
I	419	970	35°	710	41
II	519	1093	37°	777	40
III	497	715	35°	590	53

The curve which represents the variations of luminous intensity of the lamp with a globe and a polished tin reflector has the same form as the curve of the bare lamp. The dimensions are slightly larger than those above, which naturally corresponds to an increase in the mean hemispherical intensity. In fact, there was obtained with the reflector and globe II:

Horizontal Intensity	548
Maximum Intensity (at 38°)	1207
Mean Hemispherical Intensity . . .	865

The mean hemispherical intensity without either globe or reflector was 1278. Consequently the diminution was 32 per cent in place of the 40 per cent found without the reflector.

The employment of the opalescent globe and the reflector prevents taking into the calculation the real intensity instead of the mean hemispherical intensity, for it is not possible to express the luminous intensity simply in terms of the inclination. In this case everything points to taking the mean hemispherical intensity of the lower hemisphere and treating the light as uniform.

Variations of the Luminous Intensity with the Intensity of the Current.

149. To establish a relation between the luminous intensity of an arc-lamp and the energy expended in the lamp it is necessary to consider that photometric element of the lamp which is the most constant, namely, the maximum luminous intensity. The most varied photometric measurements have, in fact, always proved that the maximum luminous intensity is least liable to the sudden variations which are, on the contrary, so noticeable in the horizontal intensity.

The energy expended in the lamp may be divided into two parts, viz. that which is employed in regulating the lamp, and that which supports the voltaic arc; it is known that the loss of potential due to the regulation of the lamp amounts, on the average, to 15 volts in

60, or 25 per cent. A lamp gets from 60 to 65 volts, while the arc itself gets only from 45 to 50. From the point of view of luminous intensity, it is only the part of the energy expended in the arc which should be taken into account: calling V the difference of potential between the two carbons of the lamp and i the intensity of the current, the energy expended in the voltaic arc is equal to Vi.

The problem consists then in finding a relation between the maximum luminous intensity I_m and the work Vi.

It may, however, be simplified. In practice, lamps always work with a difference of potential which is sensibly constant, even when the regulating is done by constant current. The great majority of lamps use a difference of potential of from 45 to 50 volts. There are, however, lamps which use from 37 to 40 volts, and others, called low tension, for which a difference of potential of about 30 volts is sufficient. We may then assume a constant value for V, so that the problem is reduced simply to finding a relation between the maximum luminous intensity I_m and the current i. But for this, it is necessary to determine the constants of this relation for each particular group; we may distinguish three principal groups in which we shall classify arc-lamps: high-tension lamps (50 volts on the average), medium-tension lamps (40 volts), and low-tension lamps (30 to 35 volts).

Even a summary examination of the results of the principal photometric measurements to which arc-lamps have been submitted permits the conclusion that the establishment of an *equation of output* is very difficult, because of the influence on the luminous intensity of the particular qualities of each lamp, and because of the nature of the carbons; further, account should be taken of the fact that the photometric unit in terms of which the results are given is frequently badly defined; it is said simply that the luminous intensity is expressed in candles, without indicating either the kind of candle used or the conditions under which it is employed; now there are differences between the values of different candles which may reach 20 per cent. Hence a new source of difficulty arises in establishing an equation giving even an approximate relation between the luminous intensity and the intensity of the current.

We shall only mention, in order to call attention to them, two empirical rules given about 1880 by Maxim and Gravier for calculating the luminous intensity of the arc by measuring simply the surface of the crater of the positive carbon or the hourly consumption of carbon. According to Maxim, to obtain the luminous intensity in

candles, it is sufficient to multiply the surface of the crater, expressed in hundredths of an inch and squared, by the coefficient 10. Gravier calculated in carcels the horizontal luminous intensity of an arc-lamp by multiplying the volume of carbon consumed per hour by a coefficient which is the same for carbons of the same quality. It is needless to say that these two methods have no practical or scientific value.

Tischendoerfer* has given a formula which, according to him, represents exactly the variations of luminous intensity of an arc-lamp with the intensity of the current. In this formula, although the author does not expressly say so, he deals with the maximum intensity expressed in candles, the intensity of current being expressed in amperes. This formula is

$$I_m = 100\left[i + \left(\frac{i}{4}\right)^2\right] - 200;$$

the author does not say on what measurements he has based his calculation, nor between what limits it is valuable.

At first sight, it may be said that this formula represents with sufficient exactness the values given by direct observation, since it includes two arbitrary constants and three independent terms. But as the author has not said on what measurements he has based his calculations or whether this formula is a simple empirical rule, it should be compared with direct observation before forming a definite opinion of its value; this has been done in the table on page 249.

150. We have made a formula of photometric output deduced from experiment alone†. For this we have chosen the most precise photometric observations, made under the same conditions and using the same photometric standard in order to avoid as far as possible errors due to reductions to the same unit, and personal errors. Now, in our opinion, the observations made by Rousseau at the Antwerp Exposition best fulfill the preceding conditions.

These observations are twenty-four in number, no account being taken of some which are more or less uncertain; the intensity of the current varied between 4 and 20.7 amperes, the maximum luminous intensity between 52 and 557 carcels. The observations are reproduced in the following table; the order of lamps is the same as in the table on page 240.

* *Elektr. Zeitschrift*, 1890, p. 304.
† *Lum. Él.*, Vol. XXXVII. p. 408.

Lamp.	Intensity of Current, i.	Differences of Potential, V.	Maximum Luminous Intensity I_m (Carcels). Observed.	Calculated.	Deviations O–C.	Watts per Spherical Carcel.
1	20.7	47.5	557	583	−26	4.96
2	19.0	50.6	522	519	+ 3	5.00
3	15.9	46.2	471	417	+54	4.43
4	15.6	46.4	446	407	+39	4.48
5	14.9	47.7	373	386	−13	5.38
6	14.8	44.9	362	382	−20	5.53
7	14.6	47.0	423	376	+47	5.00
8	12.9	45.5	265	325	−60	4.92
9	12.5	47.3	265	313	−48	6.13
10	10.8	45.5	276	265	+11	4.91
11	8.6	47.6	185	205	−20	5.99
12	8.2	47.5	190	195	− 5	5.93
13	8.0	38.5	120	...	..	7.22
14	8.0	46.3	209	190	+19	5.28
15	7.9	48.0	206	187	+19	5.43
16	7.6	44.9	177	179	− 2	5.91
17	7.6	46.0	207	179	+28	5.78
18	7.0	38.4	100	...	..	7.70
19	6.1	38.2	94	...	..	7.37
20	6.0	38.5	72	...	..	8.11
21	6.0	47.1	102	140	−38	8.12
22	5.6	46.2	145	130	+15	5.38
23	4.2	37.0	52	...	..	8.66
24	4.0	38.4	60	...	..	9.45

The Pieper lamps require a difference of potential of from 35 to 40 volts only, while others require from 40 to 50 volts; the results which the former furnish should be excluded, since these are lamps of mean tension, while the others are of high tension.

We have put

$$I = x + yi + zi^2,$$

x, y, z being coefficients to be so determined that the above equation may represent as faithfully as possible the experimental results.

Each observation furnishes one equation; we thus obtain as many equations as observations, viz. eighteen. Applying the method of least squares to this set of equations, we obtain

$$z = 0.3815, \qquad y = 19.666, \qquad x = 7.93;$$

and for the equation desired,

$$I_m = 7.93 + 19.666\,i + 0.3815\,i^2. \qquad \text{(I.)}$$

By this equation the maximum luminous intensity I_m was calculated for each of the eighteen observations of the table, and the values in the fifth column were obtained. The numbers in the next to the last column are resulting errors, i.e. the deviation between the calculated and the observed values. In the last column we have written the values of the mechanical equivalent of the carcel for each lamp.

Therefore the above equation represents as faithfully as possible the variations of luminous intensity with intensity of current, at least as found at Antwerp. Attention should be called to the fact that these experiments were made on lamps of different systems, using carbons of different natures; from this then results a much greater generality for the equation, although the residual errors would have been much less had the work been done on lamps of the same make, employing carbons of the same kind.

The equation applies to arc-lamps under normal conditions having a difference of potential of about 48 volts; it may be applied to intensities of current varying from 4 to 30 amperes, although the measurements on which it depends were not carried above 20.7 amperes; we may, nevertheless, assume without difficulty this extrapolation.

We have simplified equation (I.) so as to give it a practical value by putting

$$I_m = 20\,i + 0.4\,i^2. \qquad \text{(II.)}$$

The loss of the constant term is compensated by the increase of the coefficient of the term of the second degree. The agreement of the results of the simplified formula with those of the original formula is as close as could be desired. In the following table will be found the values obtained by means of the two formulæ for different values of the intensity of the current.

VALUE OF I_m (IN CARCELS) ACCORDING TO FORMULÆ.

i	I	II	Of Tischendoerfer.
4 amperes	92.7	86	37
6 "	139.6	134	78
8 "	190.0	186	125
10 "	242.7	240	178
12 "	298.8	298	237
14 "	358.0	358	303
16 "	420 2	422	375
18 "	484.5	490	453
20 "	553.8	560	537
30 "	941.2	960	603

To compare the values given by Tischendoerfer's formula with those given by formulæ (I.) and (II.) it is necessary to transform candles into carcels; assuming that Tischendoerfer had in view the German candle, which is approximately $\frac{1}{8}$ carcel, we obtain the values in the last column of the table. It is seen that this formula gives results which are too small for small intensities of current.

The formula which we have obtained makes no pretension to being perfectly exact; it is simply an approximate formula allowing the deduction, within 10 or 20 per cent, of the luminous intensity of a normally working arc-lamp, by the simple reading of an amperemeter.

This exactness is quite sufficient for arc-lamps, where the variations are frequently so great.

We have seen above that it may be assumed without harm that the horizontal intensity is equal to one-fifth of the maximum. The formula in § 146 then permits the intensity to be calculated for a given inclination, so that the photometric problem of arc-lamps may be completely solved by a simple galvanometer-reading.

In recent years, the habit has grown more and more of designating arc-lamps not by their luminous intensity, but by the intensity of the current. This use has spread because of the considerable difference which generally exists between the nominal intensity and the real intensity. The preceding formula may give some useful information on the maximum luminous intensity of these lamps, and by taking account of the relation $S_m = 0.35 I_m$, on the mean spherical intensity.

151. Of the value of the mechanical equivalent of the unit of light obtained with continuous current arc-lamps, equation (II.) permits an exact enough determination in terms of the current; we obtain, in fact,

$$S_m = 0.35 I_m = 0.35\ (20i + 0.4i^2).$$

The energy expended is equal to $W = Vi$; the mechanical equivalent of the carcel is then equal to the quotient of W divided by S_m; i.e.

$$\epsilon = \frac{Vi}{7i + 0.14i^2} = \frac{V}{7 + 0.14i}.$$

But V is equal to about 50 volts; we have then

$$\epsilon = \frac{50}{7 + 0.14i}.$$

This formula gives the following values for ϵ:

$i =$ 4 amperes	$\epsilon =$ 6.61 watts
6	6.38
10	5.95
15	5.49
20	5.10
30	4.46

Below are the values given by Fontaine for constant current regulating arc-lamps:

Amperes.	Mean Luminous Intensity.		Mechanical Equivalent.	
	Carcels.	Candles.	Watts per Carcel.	Watts per Candle.
50	600	4980	4.8	0.60
30	525	3500	5.7	0.72
20	240	1990	6.0	0.75
14	175	1450	6.2	0.78
10	100	830	6.5	0.82
8	75	620	7.2	0.90
6	50	410	8.0	1.00
4	26	210	9.0	1.13

[The relation between carcels and candles is not exactly the same in columns 4 and 5 (8 : 1), as in columns 2 and 3 (8.3 : 1). *Trans.*]

Alternate-Current Arc-Lamps.

152. The preceding results cannot be applied to alternate-current arc-lamps, for the law of variation of luminous intensity with inclination of the rays is completely different. A simple examination of the physical state of alternate-current carbons shows the reason for this.

In continuous-current arc-lamps there is a fundamental difference between the positive and negative carbons, the former emitting much more light than the latter. In the alternate-current arc, this difference does not exist, and the two carbons have absolutely the same appearance, so that the emission of light from the upper carbon is the same as that from the lower. The distribution of luminous intensity should be symmetrical with respect to the horizontal plane passing through the arc; moreover, measurements have shown this. Further, in a horizontal direction it is the arc especially which emits the greatest quantity of light, the two carbons emitting their light in an oblique direction. Now we know that the luminous power of the arc itself is relatively low; it follows then that the luminous intensity should have its minimum value in the horizontal direction.

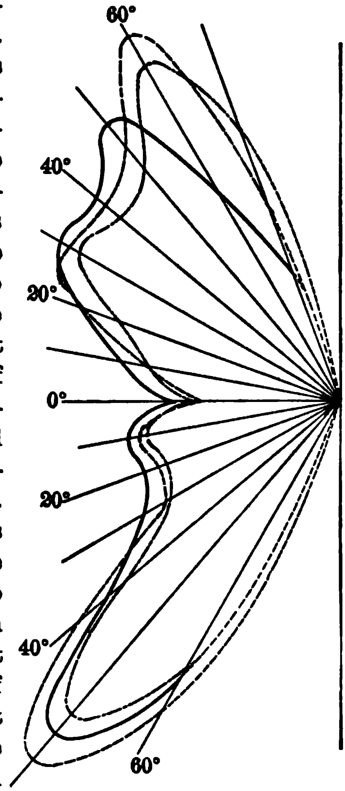

FIG. 83.

The measurements of Fontaine, made at the introduction of the Gramme lamps, had given a result contrary to the preceding conclusions in this particular, that the horizontal luminous intensity had a value about equal to the maximum value. This result was accepted for a long time, although contrary to the conclusion arrived at from a theoretical examination of the light emitted by the lamp. Some recent measurements by Uppenborn, made with great pains with eight different carbons, gave for the meridianal curve of the photo-

metric surface some quite regular curves, but agreeing with the preceding conclusions. The three curves of Fig. 83 relate to three different lengths of arc, 2, 3, and 4 mm., obtained with a constant intensity of current of 8 amperes, and with the same carbon; the curve with a continuous line was obtained from a length of arc of 2 mm.; that with dotted lines from a length of 3 mm., and that with alternate dots and lines from a length of 4 mm.

From these diagrams the conclusion follows that the distribution of light is not entirely symmetrical with respect to the horizontal plane, and that the minimum is produced in a direction differing slightly from the horizontal.

To complete these results, we give the values of the mean spherical intensity obtained with three sorts of carbons (*S*, *M*, *K*) for lengths of arc of 2, 3, and 4 mm.

	2 mm.	3 mm.	4 mm.
S	180	188	331
M	210	265	264
K	292	290	280

These figures show then that the most favorable length of arc may vary considerably with the kind of carbon employed.

Below are some data, according to Fontaine, relating to alternate-current arc-lamps.

ALTERNATE-CURRENT ARC-LAMPS.

Intensity of the Current in Amperes.	Mean Luminous Intensity.		Mechanical Equivalent.	
	Carcels.	Candles.	Watts per Carcel.	Watts per Candle.
35	175	1450	9.0	1.13
20	110	910	10.8	1.35
12	46	380	14.4	1.80

153. The distribution of luminous intensity of arc-candles, *inter alias* Jablochkoff candles, differs sensibly from that of lamps in which the carbons are situated in the same axis. The two pencils being placed side by side, the intensity varies greatly with the azimuth, no account being taken of variations with the inclination.

The horizontal intensity is maximum in the direction normal to the plane of the candles, and minimum in this plane. The curve which gives the distribution of luminous intensity in the horizontal plane has the form of a lemniscate whose minor axis corresponds to the plane of the pencils and whose major axis is perpendicular to the same.

The efficiency of the candles is somewhat less than that of regulating arc-lamps, as the following table shows:

Intensity of Current in Amperes.	Mean Luminous Intensity.		Mechanical Equivalent.	
	Carcels.	Candles.	Watts per Carcel.	Watts per Candle.
10	62	510	14.5	1.81
8	45	370	17.6	2.20
6	32	260	19.5	2.44
5	24	200	22.4	2.80
3	10	63	28.8	3.60

C. MISCELLANEOUS INFORMATION CONCERNING THE USUAL SOURCES OF LIGHT.

Rate of Consumption of the Principal Sources of Light.

154. In the chapter devoted to photometric standards certain information was given as to the luminous intensity and consumption of the common sources of light.

This information is sufficient for candles and oil lamps. It is well to complete it for the various systems of gas-burners and for the most common petroleum lamps. Of magnesium lamps, which give a very intense light with very easy manipulation, it is interesting to reproduce the principal elements in order to facilitate their comparison with other lights.

Following are the results obtained by Heim* in the laboratory of the Polytechnic School at Hanover, from a careful study of the principal lights. By means of these results, it is easy to deduce the net cost of the carcel-hour obtained by means of one or another of the lamps enumerated: for this it is sufficient to know the price per gram of the combustible employed.

* *Lum. Él.*, Vol. XXVI. p. 219.

PETROLEUM LAMPS.

Designation.	Diameter of Burner, mm.	Inclination with the Horizontal.	Luminous Intensity in German Candles.	Consumption of Petroleum.	
				In Grams per Hour.	In Grams per Hour per Candle.
Ordinary Argand burner, with round wick . . .	25	0°	16.1	54.2	3.37
		45°	12.3	53.6	4.36
Central disc, round burner, small size	30	0°	19.2	63.4	5.30
		45°	11.1	61.1	5.51
Central disc, round burner, large size	62	0°	67.3	229.0	3.40
		45°	33.9	228.0	6.72
Kosmos burner	30	0°	22.9	84.9	3.70
		45°	17.8	85.5	4.80

MAGNESIUM LAMPS.*

Number of Magnesium Ribbons.	Luminous Intensity in German Candles.		Luminous Intensity per Ribbon.	Without Reflector.	
	Without Reflector.	With Reflector.		Consumption of Magnesium per Hour per Ribbon in Grams.	Quantity of Magnesium per Hour per 100 Candles in Grams.
1	150	3200	150	16.7	11.14
2	237	5880	118.7	16.7	14.10
4	450	8000	112.5	16.7	14.80
6	700	11300	117	16.7	14.15
8	950	17000	119	16.7	14.03

* These magnesium lamps burn magnesium ribbons 2.5 mm. wide by 0.13 mm. thick. The variations of luminous intensity are quite large. The reflector of these lamps increases considerably the luminous intensity in the desired direction.

GAS-BURNERS.

Designation.	Inclination with the Horizontal in Degrees.	Luminous Intensity in German Candles.	Consumption.	
			Of Gas per Hour in Cubic Meters.	Of Gas per Hour per Candle in Liters.
Butterfly burner	0	16.9	0.251	14.8
" "	45	17.2	0.256	14.9
Argand burner	0	21.9	0.239	10.9
" "	45	19.4	0.241	12.4
Auer incandescence burner . . .	0	14.4	0.095	9.60
" " " . . .	45	10.5	0.104	9.88
Siemens intensive regenerative burner	0	65.3	0.460	7.05
" " " "	45	46.9	0.456	9.75
" " " "	0	222	1.621	7.30
" " " "	30	162	1.614	9.96
" " " "	45	132	1.604	12.2
Wenham burner	0	28.4	0.249	8.77
" "	45	44.5	0.257	5.77
" "	90	45.8	0.256	5.58
" "	0	99	0.685	6.92
" "	25	152	0.686	4.51
" "	45	170	0.677	3.98
" "	65	200	0.685	3.42
" "	90	202	0.671	3.33

155. The measurements of Baille and Féry * are also very interesting. These physicists determined the price per unit of the light furnished by ordinary lighting apparatus; they adopted the carcel-hour as the unit. The following results were obtained from direct measurements of luminous intensity and consumption of combustible or of energy. The net price is relatively of interest to consult, but it should be calculated anew according to the present conditions, for it was determined by means of the prices quoted at Paris for the combustibles employed.

Questions of efficiency are very complex; it is necessary to state precisely whether the net cost of the carcel-hour applies to the intensity measured in a single direction or whether it applies to the mean spherical intensity. For incandescent and arc lamps, and for "butterfly" gas-burners as well as for flat-wick lamps, the differences are apt to be quite large.

* *L'Électricien*, 1889.

Designation of Source of Light.	Intensity in Carcels.	Rate.	Price of the Carcel-Hour at Retail.	Observations.
Candles.		Grams per hour.	Centimes.	
1. Paraffine candle . .	0.14	8	18.5	Yellowish flame.
2. Perforated candle . .	0.14	10	17.1	
3. Star candle	0.14	9	12.0	
4. Ordinary full candle .	0.15	9	12.0	
Oil.				
5. Moderator lamp . .	1.04	36	5.6	Double current of air; rape-seed oil purified and filtered.
6. Ordinary moderator lamp	1.06	42	6.5	Rape-seed oil purified and filtered.
7. Ordinary moderator lamp	0.94	46	6.8	Ordinary oil.
8. Carcel standard lamp.	1.00	42	9.6	
Petroleum.				
9. Flat wick lamp. . .	0.81	20	2.2	13 mm. wick.
10. " " " . . .	2.13	62	2.6	50 " "
11. Lamp with two flat wicks	2.07	63	2.7	25 " "
12. American lamp without chimney . . .	1.82	52	2.5	
13. Round burner lamp .	1.06	28	2.4	Diam. of the burner 23 mm., ordinary burner, constricted chimney.
14. " " " .	1.49	51	3.0	Diam. of burner 25 mm. mushroom burner.
15. " " " .	0.94	30	2.9	Diam. of burner 19 mm. calotte burner, chimney with knee.
Gas.		Liters per hour.		
16. Ordinary butterfly burner	0.64	132	6.1	
17. Bengel burner . . .	1.10	134	3.6	Height of flame 6.5 mm.
18. Zircon gauze burner .	1.39	62	1.3	Much green radiation.
19. Magnesia gauze burner	1.61	191	3.5	" blue "
20. Albo-carbon burner .	3.35	135	——	
Incandescent Lamps.		Watts.		
21. Edison lamp. . . .	0.65	29.44	6.8	Below normal rate.
22. Gérard "	0.72	36.74	7.5	

Brightness of Radiants.

156. We have not yet dealt with the brightness of radiants. According to definition the brightness or the intrinsic intensity of a light whose luminous intensity is uniform is equal to the quotient of the total quantity of light emitted, divided by the surface of the illuminating part of the source; we then have

$$i = \frac{Q}{S}.$$

Brightness has the same dimensions as intensity of illumination.

Two lamps of the same luminous intensity, an incandescent lamp and a carcel lamp for instance, are far from having the same brightness. The quantity of light which the first gives is emitted by the small surface S' of the filament, while that from the second is emitted by the much larger surface S'' of the flame of the carcel lamp; we have then

$$i' = \frac{Q}{S'}, \text{ and } i'' = \frac{Q}{S''}.$$

To determine the brightness of a source requires the simultaneous measurement of the quantity of light emitted and the area of the illuminating surface. In his investigations on the lighting of lighthouses, Allard was the first to study this element. He determined the brightness of the oil lamps of from one to six wicks employed in the lighthouse service, and obtained the following values expressed in carcels per square centimeter of flame:

Number of wicks	1	2	3	4	5	6
Brightness	0.197	0.288	0.360	0.415	0.460	0.493

These results allowed Allard to determine the brightness of the sun in the following manner.

The intensity of an arc lamp being 200 carcels, he considered this radiant as having the surface of a sphere 1 cm. in diameter. The apparent surface of this sphere being 0.7854 sq. cm. (the area of a disc of the same radius), it followed that the brightness of the lamp per square centimeter was 255 carcels, that is, 500 times as great as that of the flame of the six wick lamps. Now Bouguer found that the sun at noon on a clear day was 11,664 times as intense as a candle at a distance of 16 French inches,

[nearly 17 English inches]; this result corresponds to a luminous intensity 62,280 times as great as that of a candle at a distance of 1 m. Wollaston having found the value 59,850, Allard assumed the mean value 8200 carcels at a distance of 1 m. as producing an illumination equal to that of the sun; in this number account is taken of atmospheric absorption.

Let us consider, at a distance of 1 m., a sphere subtending a visual angle of 32′, equal to that of the sun; the apparent surface of this sphere is 0.6085 sq. cm., and, in order that it may be as brilliant as that of the sun, it should have a brightness of 12,050 carcels per square centimeter.

The brightness of the sun is then 47 times as great as that of the voltaic arc and 25,000 times as great as that of the six-wick oil lamp.

The following results were obtained by Voit at the time of the photometric measurements of the Munich Exposition:

	Brightness (in candles per square centimeter).
Giroud candle-burner (*bec-bougie*)	0.06
Argand burner	0.30
Small intensive regenerative Siemens burner	0.38
Large " " " "	0.60
Incandescent lamps	40.00
Arc-lamps	484.00

The number relative to the arc-lamp does not agree with that of Allard; the lamp investigated by the latter was without doubt one of greater intensity whose carbons were carried thus to a much higher temperature.

We conclude this brief study of the brightness of radiants with the following considerations, which are of real interest not only in the study of radiants, but also in that of lighting in general.

Unit of Brightness.

157. There is a fundamental difference between the platinum photometric standard and the candle; the first gives the luminous intensity represented by the quantity of light which is emitted through an opening 1 cm. square in the diaphragm above the platinum bath, and at the same time the unit of brightness represented by melting platinum. The candle gives only the unit of intensity, and the unit of brightness which should be deduced is in general totally different from the real brightness of the flame. Thus, the surface of a principal section of the Hefner lamp is

about 2.27 sq. cm. The mean brightness of the flame is then $\frac{1}{2.27}$ as large as that which it represents; to form an idea of this unit of brightness, the flame should be represented as keeping the same luminous intensity, but reduced to a section $\frac{1}{2.27}$ as large.

We pass, by all degrees of the scale, from the brightness of a surface illuminated at the extreme limit of visibility up to that of the solar disc; these two illuminations are in the ratio of 1 to 10^9. Between the two all possible degrees of illumination are found. But their mensuration is still in its infancy; photographs are not yet able to express numerically the brightness of objects which they photograph. They show simply the length of exposure.

To express in figures all possible illuminations, either very large or very small numbers are required, according to the unit adopted. If, for instance, we take as the unit of brightness the total intensity of the Hefner lamp, the brightness of melting platinum would be expressed by the number 20, that of the solar disc by 160,000, that of the flame of a candle by 0.4, of the sky by from 0.1 to 1, that of white paper, on which it is possible to read the print, by 0.0006, and that of a white surface at the limit of visibility by a still smaller fraction.

The following values were obtained by L. Weber, for various types of brightness, determined for red rays ($\lambda = 0.6306\,\mu$) and for green rays ($\lambda = 0.5415\,\mu$):

	$\lambda = 0.6306\,\mu$	$\lambda = 0.5415\,\mu$
I. Brightness of the absolute platinum standard	1	1
II. Brightness of the Hefner flame concentrated into a surface of 1 sq. cm.	0.0635	0.049
III. Brightness of an absolutely white plain surface illuminated normally by the platinum standard at a distance of 1 m.	0.0000318	0.0000318
IV. Brightness of a white plain surface illuminated normally by the Hefner standard at a distance of 1 m.	0.00000202	0.00000516

The following table gives the brightness of a certain number of well-defined objects; this brightness is represented in terms of the units I. and IV. The first of each bracketed pair of numbers applies to red rays, the second to green.

	I.	IV.
1. Solar disc, outside the atmosphere . .	8417 4092	5394000000 2025000000
2. Sky, near the solar disc.	1 1	640900 494800
3. Flat carburetted burner, seen at the side	0.509 0.615	326200 304500
4. Horizontal white card, at bright noon-day	0.295 0.138	189100 68310
5. White card, illuminated normally by the sun 60° high	0.144 0.069	92410 34200
6. White cloud, illuminated by the sun .	0.089 0.021	57040 10390
7. Flat carburetted gas-burner, seen edge-wise	0.073 0.088	46790 43680
8. Argand burner	0.044 0.057	28150 28150
9. Bright sky, the sun at azimuths of 60° and 90°.	0.05 0.008	33000 3800
10. Horizontal white card, on a dark winter day	0.0030 0.0010	1945 508
11. Black velvet, on a bright summer day, same as No. 4	0.00059 0.00028	378 137
12. White card, on which one may read without difficulty	0.000020 0.000015	10 10

The Mechanical Equivalent of Light.

158. The vibratory motion of the ether produced by a radiant, which is felt on the retina as a luminous sensation, requires for its support an expenditure of energy which it is easy to calculate. The radiant plays the part of a simple transformer of energy. In lights supported by combustion, for instance, the energy is taken from the combustible, resin, oil, or gas; in electric lamps, it is furnished by the current. Now every transformation is accompanied by losses; the radiant cannot fail to obey this general law, the losses being proportionately greater as the mode of transformation employed is more imperfect. Every source of light has, then, a determinate

mechanical efficiency which indicates what fraction of the total energy expended and transformed into a vibratory movement of the ether is capable of producing a luminous impression.

This mechanical efficiency is easy to calculate. Following are the details of the calculation for a petroleum lamp and a gas-burner.

According to the measurements of Heim, at Hanover, an ordinary petroleum lamp, with a round burner 25 mm. in diameter having an intensity of 16 candles, consumes 3.37 grams of oil per hour per candle. Assuming that the heat of combustion of petroleum is 11,000 calories per kilogram, the lamp consumes then the equivalent of 37 calories per hour per candle. Now a calorie corresponds to 41,700 megergs. The energy consumed in the lamp is then

$$37 \times 41{,}700 = 1{,}542{,}900 \text{ megergs},$$

which corresponds to a power of

$$\tfrac{1542900}{3600} = 428.6 \text{ megergs per second},$$

or 42.86 watts. Such is the mechanical equivalent of the luminous intensity equal to one candle power, obtained by means of an ordinary petroleum lamp. The efficiency increases, i.e. the equivalent energy diminishes, if improved burners of great intensity are used.

To calculate the mechanical equivalent of gas-light, we use as a basis also the results obtained by Heim, who found that an Argand burner of 22 candle power consumes 11 liters of gas per hour per candle. The heat of combustion of gas is about 5400 calories per cubic meter. The expenditure of gas corresponds, then, to 59.4 calories per hour per candle, or to 2,476,980 megergs per hour, produced by a power of 68.8 watts. The mechanical equivalent of a gas flame is then 68.8 watts per candle; it is correspondingly less for intensive burners.

The mechanical equivalent of an incandescent lamp is, on the average, 3.5 watts per candle. The mechanical equivalent of the arc-lamp is still lower; it amounts to about 0.8 watts per candle.

The great difference between the efficiency of the two lights just mentioned and that of the incandescent lamp is due to the enormous losses undergone in combustion. The emission of light in these lamps is due to the incandescence of carbon; in the first two the luminous particles are particles of carbon not yet burned, whose incandescence is maintained by the combustion of the gas. There is a loss because of more or less incomplete combustion, and especially because of the convection of heat due to the surrounding air.

In the incandescent lamp the filament is maintained incandescent by the electric current, and, as it is in a vacuum, the loss by convection is null, and the only loss is that due to radiation.

The following table contains the values of the mechanical equivalent of the candle power obtained by means of the commonest lights; these values are not absolute, because the consumption in a lamp diminishes considerably when its luminous intensity increases.

Candle	86.0	watts	per	candle.
Oil lamp	57.0	"	"	"
Petroleum lamp	42.8	"	"	"
Butterfly gas-burner	93.2	"	"	"
Argand " "	68.8	"	"	"
Siemens intensive burner, 230 candle power	45.6	"	"	"
Incandescent lamp, 16 candle power	3.5	"	"	"
Arc-lamp	0.8	"	"	"

Optical Efficiency of Sources of Light.

159. The production of light by a given source is obtained by means of the expenditure of a quantity of energy W which is employed to produce the vibratory movement of the ether. The quantity of energy W radiated by a source of light is composed of two parts; the one W_1 represents the energy of the luminous radiations, the other W_2 that of the obscure radiations. Among these three quantities there exists the relation,

$$W = W_1 + W_2.$$

The ratio $\frac{W_1}{W}$ of the energy of the luminous radiations to that of the totality of the radiations is called the optical efficiency of the source. The efficiency is zero when the temperature of the source is below 400° C., since W_1 is then equal to 0. It increases rapidly with the temperature.

We may employ two methods in measuring this efficiency. The former consists in passing the rays, emitted by the source investigated, successively through a layer of bisulphide of carbon which freely allows all radiations to pass, and through an equal layer of an alum solution, which allows only luminous rays to pass. In these two cases the intensity of the radiations is measured by a thermo-electric pile. The second method is more complicated, but is susceptible of much greater exactness; it can only be employed for incandescent lamps. The lamp is placed in a calorimeter with thin blackened

copper sides, filled with water. The whole of the energy radiated by the lamp in the form of heat is absorbed by the water and the metallic sides of the calorimeter. It is sufficient then to measure the elevation of the temperature of the water in the calorimeter in order to deduce the total heat emitted during a unit of time. The calorimeter is then replaced by a similar one of thin glass; in this case the obscure rays alone are absorbed by the water and by the glass of the calorimeter, while the luminous radiations undergo only a negligible absorption. The elevation of the temperature of the calorimeter is due then solely to the action of the obscure rays. By this method exactness within 0.3 per cent is easily attained.

These two methods have been employed by many physicists to determine the optical efficiency of common lights. Tyndall was the first to make researches of this nature. Recently new measurements have been made by Blattner* of Zurich (1885) and Merritt and Marks† in the United States (1890).

The measurements of Blattner and Merritt concerning the efficiency of incandescent lamps have a real interest. The incandescent lamp is in fact the only one whose temperature may be varied at will, since it is sufficient for this purpose to increase the intensity of the current which heats the carbon. We may then vary at will the nature of the light emitted and investigate the light furnished by incandescence passing successively from a dull red color to brilliant white. All these experimental results confirm the conclusion furnished by the preceding theoretical deductions, viz. that the luminous efficiency should increase with the temperature.

This conclusion has received still another confirmation by the measurements of Nakano and Marks upon the luminous efficiency of the voltaic arc. These electricians proved that, for an arc working with a definite difference of potential and intensity of current, the luminous efficiency varies with the inclination of the rays. This fact is very easily explained by the very nature of the voltaic arc. In this lamp the greatest part of the light emitted is due to the upper (positive) carbon, whose temperature is much higher than that of the lower (negative) carbon. The light emitted is due principally to the positive or to the negative carbon, according to the direction of the ray; that is, it is due to incandescent bodies whose temperature is different. The quality of the light is then different

* *Lum. Él.*, Vol. XXIII. p. 519.
† *Lum. Él.*, Vol. XXXIII. p. 255.

according to the inclination of the luminous rays, and consequently the optical efficiency should also be different.

In the following table we have collected the most interesting results of the measurements of efficiency which have been made on light sources; among the numerous values relating to arc-lamps, we have chosen the highest values obtained with carbons 6 mm. in diameter; the values really obtained in practice are appreciably less.

Designation of Light Source.	Per Cent Optical Efficiency.	Authority.
Hydrogen flame	0.0	Tyndall.
Oil lamp	3.0	"
Ordinary gas-burner	4.0	"
Swan lamp, 16 candle power, run at 2.6 c.p.	2.3	Blattner.
" " " " " " " 9.2 "	2.8	"
" " " " " " " 13.2 "	3.6	"
" " " " " " " 20.6 "	5.2	"
Edison " " " " " " 4.0 "	3.6	"
" " " " " " " 8.3 "	4.5	"
" " " " " " " 17.0 "	6.2	"
" " " " " " " 28.6 "	8.5	"
Bernstein 32 " " " " 15 "	4 2	"
" " " " " " 30 "	6.5	"
" " " " " " 50 "	7.3	"
" " " " " " 90 "	9.9	"
Arc-lamp, inclination 0°	8.4	Nakano.
" " " 10°	12.4	"
" " " 20°	17.4	"
" " " 30°	18.0	"
" " " 40°	18.2	"
" " " 50°	19.8	"
" " " 60°	5.5	"
" " spherical efficiency	16.6	"
Magnesium lamp	15.0	Nichols.
Geissler tube	32.7	Staub.

These results show that the optical efficiency of the usual sources of light does not exceed 10 per cent, and that it is generally about 5 or 6 per cent. In other words, in our ordinary sources of light 95 per cent of the energy spent is devoted to producing radiations of the ether which do not affect our eye, that is, radiations whose wave-length is greater than $0.810\,\mu$. From the point of view of the production of light this energy is lost.

To produce vibrations of the ether whose wave-length is comprised between 0.810μ and 0.360μ, we are forced to produce the totality of the vibrations whose wave-length is greater than 0.810μ. We find ourselves then, according to the happy comparison of Lodge, in the position of an organist who in order to sound certain high notes of his instrument would be obliged to sound all those of the key-board.

The low value of the optical efficiency of common radiants is explained by the fact that they are based on the incandescence of carbon and that the temperature of the latter is about the same in all. From the point of view of a physicist we have made no progress in this domain since the beginning of civilization, and the resin torch, of which the savage made use, is, with respect to luminous efficiency, about as perfect as the arc-lamp which spreads the light with profusion in large cities. At most we have succeeded in obtaining a mean efficiency of about 6 per cent instead of 3 or 4 per cent. There is still some room then for future progress.

Up to the present no attention has been paid to this special point in artificial illumination. All research has aimed at producing electricity as economically as possible; none has tended to diminish the expenditure of energy in the lamp by a process allowing the production of obscure rays to be done away with, or at least diminished, without injuring that of the luminous rays.

A simple numerical example shows how extravagant are our present methods of illumination.

Let us assume that the power necessary for the production of the electric current is furnished by a steam engine whose efficiency does not exceed 10 per cent, under the best conditions. The efficiency of the dynamo-electric machine being 90 per cent, we see that 9 per cent only of the energy accumulated in the coal is transformed into electric energy. If we assume a loss of 10 per cent in the conductors, etc., there remains to be expended in the lamp energy equal to 0.081 of the original energy. But of this energy expended in the lamp 90 per cent is expended in the production of heat; the remaining 10 per cent alone serves for the production of light. The final efficiency is then 0.0081, or 1 per cent only. It is this result which is characterized as brilliant and marvellous.

This showing is made by the process of electric lighting. But we easily console ourselves by reflecting that the result is still less satisfactory if we consider other sources of light.

The Artificial Light of the Future.

160. To improve the optical efficiency of light sources, there should only be produced such vibrations of the ether as are susceptible of affecting the retina; that is, vibrations whose wave-lengths are included between 0.810μ and 0.360μ. Nature has solved this problem in the most perfect manner in the luminous organ of glow-worms and of other luminous insects.

This light of peculiar nature has been specially studied by R. Dubois* and by Langley†; their researches bore on the light emitted by pyrophori, coleopterous insects of the tropics whose photogenic function is well developed.

The nature of this photogenic function is still little known; Dubois thinks that it corresponds to a simple physico-chemical phenonemon whose activity the insect supports, and which might, for instance, offer some analogy to that which transforms glycogen into sugar in the liver.

The light emitted by pyrophori is very remarkable; it is composed almost solely of green and yellow radiations, and its spectrum is continuous, without showing bands or lines. The radiations emitted have a wave-length included between 0.450μ and 0.650μ, the maximum being at 0.550μ.

We know that the eye is much more sensitive to green and yellow radiations than to the rest of the spectrum. The insect emits, then, precisely the luminous radiations which correspond to this maximum sensibility, which is still another advantageous property of this light from a photometric point of view.

Langley determined for four different lights the distribution in the various parts of their spectra of a quantity of energy equal to unity.

The following results were reached: In gas-light and that of the voltaic arc, the maximum energy is found at wave-lengths of 1.6μ and 1.16μ; i.e. at wave-lengths which affect the retina no longer. We see that almost the whole energy of the spectrum of each of these lights is expended in the infra-red. In solar light and that of the pyrophorus, the maximum energy is in the visible part of the spectrum, at 0.62μ for the former and at 0.57μ for the latter.

This coincidence shows that these two lights of such different

* *Séances de la Société de Physique*, 1886, p. 138.

† *Amer. Journal of Science*, Vol. XL., 1890, p. 97.

nature are the best qualified for illuminating. But the second is still better than the first, as the energy expended outside of luminous radiations is absolutely null, while in sunlight the energy of the infra-red spectrum is not at all negligible in comparison with that of the visible spectrum.

If the perfect light *par excellence*, sunlight, is composed of vibrations which are outside of the limits perceptible to the eye, it is because lighting us is not its only object. The energy which the sun sends us by means of the vibratory movement of the ether has also for its object the maintenance of the temperature of the earth between determined limits. In sunlight all the vibrations are useful, while in artificial light it is desired to produce only vibrations which affect the retina, and not calorific vibrations; cold light should then be produced.

In conclusion we give a brief *résumé* of the researches which have been made with a view to the direct production of this light of the future.

All our sources of light, resin torches, candles, gas-burners, arc-lamps, etc., are identical, as we have already remarked. The brightness of all these lights is produced by the incandescence of carbon; they only differ in the temperature to which these particles of carbon are raised.

We must, then, find a substance other than carbon, emitting at the same temperature a much greater quantity of light; i.e. such that the vibrations of its molecules supported by the high temperature to which it is raised would be able to impress on the surrounding ether a much more rapid vibratory movement. This substance once found, the question would have taken a great step forward.

It seems that this is not impossible. Nichols has found, for instance, that incandescent magnesium emits light under conditions different from those of carbon. First, this light is much more like that of the sun than that of other sources. With equal luminous intensities the magnesium flame is nearly ten times as brilliant in the violet as the gas flame, and one-half less in the red; it surpasses that of the arc-lamp also up to the yellow. Some approximate measurements permit the conclusion that the magnesium flame has an efficiency of about 15 per cent, or three times as much as that of the incandescent lamp at its normal rate.

If the luminous substance were carbon, the brightness of the magnesium light would correspond to a temperature much above that of the voltaic arc, while it appears from direct measurement

that its temperature scarcely exceeds 1400° C.; that is, the temperature of burning gas. Nichols assumes that the law of radiation of magnesium differs essentially from that which governs ordinary cases of incandescence.

The luminous vibrations from magnesium oxide are out of proportion to the temperature of incandescence, the radiations of short wave-length being very strongly represented. Perhaps there should be considered in the luminous emission of magnesium the phenomena that E. Wiedemann designates by the general name of luminescence. This word is applicable to all the phenomena known as phosphorescence, fluorescence, etc.

It is assumed that luminescence is due to a particular class of molecular vibrations distinct from those which cause ordinary incandescence; this mode of vibration has a particular tendency to produce a selection of wave-lengths, one of them always having a tendency to predominate.

The energy expended in luminescent bodies has then for effect the supporting of molecular vibrations of definite period, these vibrations producing in the surrounding ether a vibratory movement of the same period. The whole thing is to find bodies such that molecular energy is easily supported in them and which produce waves which correspond exactly to the vibrations of the ether of short wave-length.

It seems to us that there is room for research in this direction, the phenomena of luminescence being necessarily at the base of the light-producing power of glow-worms and other luminous insects.

In the enumeration of sources of light whose thorough investigation might lead to important new results, no mention has been made of the luminous phenomena produced directly by electric discharges. Among these luminous phenomena we should place in the first rank those which are produced in Geissler's induction tubes.

The optical efficiency of this source of light has been recently measured by Staub* at Zurich, by means of Bunsen's ice calorimeter. The Geissler tube, carefully blackened with lampblack was placed in the ice calorimeter; the quantity of ice melted during a determined time measured the total quantity of heat produced in the tube by electric discharges; a second measurement made with the tube unblackened, thus allowing luminous rays to pass, permitted the measurement of the quantity of energy corresponding to the obscure radiations. Proceeding in this way, Staub obtained 32.7

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* *Beiblätter*, Weid. Ann., Vol. XIV. (1890), p. 538.

per cent as the optical efficiency of the Geissler tube. This efficiency is the highest obtained up to the present with artificial sources of light. Unfortunately, the quantity of light thus produced is too small to be used in lighting.

The recent work of Tesla gives an exceptional importance to this mode of light production. It seems that this engineer succeeded in making lamps based on the principal of Geissler tubes practical. The electric discharges were obtained by means of high tension alternate currents of very great frequency (20,000 alternations per second). In this way a quite great luminous intensity was obtained. The precise details of this new apparatus and its efficiency are still wanting.

Finally, a few words remain to be said concerning a theoretical process of light-production, although this process cannot lead to practical results. The theoretical works of Maxwell have shown that electric phenomena are transmitted by undulations in the surrounding ether, and that these undulations coincide with luminous vibrations when their wave-length is sufficiently short.

According to this theory light would only be a particular case of electric undulations. The recent experiments of Hertz have confirmed experimentally this view and have given methods for the production of electric vibrations of a determined wave-length. These vibrations are produced by the discharge of a condenser in a circuit characterized by its capacity C and its self-induction L. The wave-length of the electric vibrations is then expressed by the equation

$$\lambda = 2\pi\sqrt{LC},$$

L being expressed in electro-magnetic units, and C in electrostatic units.

This equation allows us to calculate the dimensions of the circuit which would give undulations with wave-lengths $\lambda = 0.6\,\mu$. We find, thus, that the circuit should be of such dimensions that the geometrical mean of its capacity and its self-induction is less than $0.1\,\mu$. This amounts to saying that the dimensions of the circuit should be of the order of molecular dimensions. The electro-magnetic theory of light leads then to a result identical with the preceding. To obtain cold artificial light there must be impressed on the molecules vibrations whose period is equal to that of the luminous undulations, without passing through intermediate vibrations of longer period. But the maintenance of the molecular vibrations could perhaps be obtained by means of electric undulations produced directly.

CHAPTER VI.

THE DISTRIBUTION AND MEASUREMENT OF ILLUMINATION.

161. To determine the value of a system of lighting, men have been content for a long time to multiply the number of lamps by the luminous intensity of each, then to divide the total number of light units by the area of the surface lighted. It is needless to say that this method of procedure can only give very imperfect results, for the variations of luminous intensity with the direction of the rays are quite different, according to the light employed; it is, for instance, inadmissible to compare, in this manner, illumination by gas with that by arc or incandescent lamps. The variations of luminous intensity with the direction of the rays which are not comparable for two gas-burners of different systems, are still less so for a gas-light and an arc-light. This procedure would be about correct if the luminous sources employed emitted the same quantity of light in all directions, or at least if the law of variation of luminous intensity with the inclination of the rays were the same in both.

From a practical point of view, what a system of illumination requires, is that the surface to be illuminated should receive throughout its whole extent a minimum quantity of light per unit of surface; i.e. that its illumination should not fall below the given limit. To compare, for instance, the value of two systems of illumination, we must compare the illuminations produced by each of them and examine their variations; the system for which the mean illumination will be the highest, while having the least variations, should be considered the best.

The photometric elements of a light being known, we may theoretically determine the value of illumination at each point of a given surface, provided the position and height of each light is given; we may also determine by this calculation what distribution of these lights gives the most favorable illumination. However, this problem is not as simple as it appears; for account must be taken, in practice, not only of the variations more or less regular in the luminous inten-

sity with the direction of the rays, but also of the phenomena of absorption and reflection of light. The influence of reflection is almost insensible in the illumination of a large plane surface, while it plays an important part in the illumination of enclosed places. The absorption of light by the surrounding medium plays no part except in public lighting. We may, moreover, neglect it, for this absorption only takes place, in an appreciable manner, in the case of a fog. Now this case should be considered as an exception whose exigencies systems of lighting cannot satisfy.

Calculations of the distribution of illumination on a surface are relatively simple when we consider only one luminous source of uniform intensity, i.e. one having the same illuminating power in all directions. But if we consider the general case of many lights having different illuminating powers according to the direction of the ray, the study of the distribution of illumination is very complicated.

Before entering upon calculations of illumination, we should define the unit employed.

Intensity of Illumination.

162. The quantity of light dq received by an element of surface dS, whose normal makes an angle i with the direction of the ray, is proportional to the cosine of the angle i (law of obliquity) and inversely proportional to the square of the distance d from the source. We have then the relation

$$dq = \frac{IdS \cos i}{d^2}. \tag{3}$$

We mean by *intensity of illumination* at a given point of a surface the quotient of the quantity of light dq received by the element dS of this surface divided by the area of this element dS. We have, then,

$$e = \frac{dq}{dS} = \frac{I \cos i}{d^2}. \tag{4}$$

We may thus consider intensity of illumination as the quantity of light received per unit of surface ($dS = 1$).

The distinction between intensity of illumination and illumination, introduced by Wybauw is very useful, for it allows us to speak of the intensity of illumination at a given point, while we may only speak of the illumination of a surface. Intensity of illumination is

a quantity mathematically defined, $e = \frac{dq}{dS}$, while illumination is a physical or even a physiological notion.

It is well to define the unit adopted for intensity of illumination. We must take as the unit of intensity of illumination ($e = 1$) the intensity of illumination produced with a normal incidence ($i = 0°$) by the unit of luminous intensity ($I = 1$) placed at a unit distance ($d = 1$).

The unit of intensity of illumination is then connected with the unit of luminous intensity. If the carcel lamp is adopted as the photometric standard, the unit of intensity of illumination will be the *carcel-meter;* i.e. the intensity of illumination produced by the carcel lamp at a point situated at a distance of a meter in a horizontal plane passing through the middle of the flame. If, on the contrary, we choose the candle as the standard, the unit of intensity of illumination will be the candle-meter, etc.

Hospitalier has proposed to express the intensity of illumination in candles per square meter and not in candles-meter, because the intensity of illumination is inversely proportional to the square of the distance, which the expression candle-meter does not indicate. This criticism does not seem to us well founded; for the intensity of illumination at a point is independent of the surface considered, since this intensity is the limit of the quotient of the quantity of light received by the element of surface normal to the luminous ray passing through this point, divided by the area of the element as the latter approaches zero.

We shall adopt, then, the terms carcel-meter, candle-meter, etc. Let us recall, however, that it has been proposed to give the name of *lux* to the unit of intensity of illumination, but this name has not yet been sanctioned by usage, and, in particular, it was not adopted by the International Congress of Electricians in 1889. We may, however, employ it, but by applying it to the decimal candle equal to one-twentieth of the absolute platinum standard, which was adopted as the practical unit of intensity by the Congress of 1889. Instead of saying a decimal-candle-meter, we would, then, simply say a *lux.*

Calculation of the Illumination of a Horizontal Plane.

163. Let I be the luminous intensity of a radiant placed at a distance h above the horizontal plane to be lighted. The intensity of the illumination at any point P (Fig. 84), situated at a distance x

from the foot of the vertical line passing through the source, is given by the formula,

$$e = \frac{I \sin \theta}{h^2 + x^2}, \tag{1}$$

θ being the angle formed with the horizontal by the line joining the source and the point P.

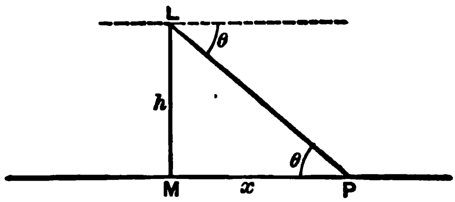

FIG. 84.

Some very simple transformations allow us to give this formula any of the three following forms:

$$e = \frac{I \cos^2 \theta \sin \theta}{x^2}, \tag{2}$$

$$e = \frac{I \sin^3 \theta}{h^2}, \tag{3}$$

$$e = \frac{Ih}{(h^2 + x^2)^{\frac{3}{2}}}. \tag{4}$$

A simple inspection of these formulæ allows some very interesting conclusions to be drawn.

Formula (3), among other things, shows that the variations of illumination in the horizontal plane is proportional to $\sin^3 \theta$, the light being uniform and at a constant height. This illumination is then a maximum at the foot of the perpendicular, i.e. when $\theta = 90°$.

164. By means of formula (2) we easily determine the height at which the radiant (supposed uniform) should be placed to obtain the maximum intensity of illumination at a distance x; it is sufficient for this to equate to zero the derivative of the second member of this equation; we thus obtain the equation,

$$\frac{I \cos \theta}{x^2} \cdot (\cos^2 \theta - 2 \sin^2 \theta) = 0,$$

which gives $$\text{Tang}\, \theta = \sqrt{\tfrac{1}{2}}.$$

The angle θ corresponds, then, to 35° 16′; the corresponding height h is given by the formula,

$$h = x \tan\theta = 0.707\,x. \tag{5}$$

To obtain the maximum intensity of illumination at a given point situated at a distance x from the foot of the perpendicular, the light should be placed at a height equal to $0.707\,x$.

Formula (4) allows the solution of an analogous problem: to calculate the radius x of the circumference which receives a determinate intensity of illumination e, the height of the light being given. We obtain

$$x^2 = \left(\frac{Ih}{e}\right)^{\frac{2}{3}} - h^2. \tag{6}$$

We may also suppose h variable, and determine the greatest value of the radius x corresponding to a given intensity of illumination e. It is sufficient for this to equate to 0 the derivative of the value of x^2 obtained by means of equation (3). We thus obtain for θ the condition,

$$\sin^2\theta = \frac{1}{3}; \quad \cos^2\theta = \frac{2}{3}; \quad \text{or } \tan\theta = \frac{1}{\sqrt{2}}.$$

This angle is the same as in the first problem; it is then equal to 35° 16′, and corresponds to $h = \frac{x}{\sqrt{2}}$. The distance x then becomes, replacing $\sin\theta$ and $\cos\theta$ by their values,

$$x^2 = \frac{2}{3\sqrt{3}} \cdot \frac{I}{e} = 0.385 \cdot \frac{I}{e};$$

therefore

$$x = 0.62\sqrt{\frac{I}{e}}. \tag{7}$$

Mean Illumination.

165. Let us consider a part S of the horizontal plane and lay off as an ordinate vertically at each point of the plane the value of the intensity of illumination at that point. The locus of the extremities of the ordinates is a surface which represents exactly the variations of illumination on the surface S. The volume comprised between the latter and the surface, the locus of the extremities of the ordinates, represents the total quantity of illumination of S; this

characteristic volume is called the *volume of illumination* of the surface S (Wybauw).

The formula which gives this volume of illumination V is simply

$$V = \int e dS,$$

e representing the intensity of illumination of the element dS of the surface S, the integration being extended over the whole surface.

Taking account of the dimensions of e and dS, we see that the volume of illumination is a quantity of light; it is then a new expression to designate a thing already defined. From this point of view we may protest against the introduction of this new term; but, taking account of the considerations which have led to the expression "volume of illumination," we cannot but approve of this term, which in many cases advantageously replaces the expression "quantity of light."

A surface S being given as well as its volume of illumination, we may calculate what the intensity of illumination, supposed uniform, should be in order that the volume of illumination (limited then by two parallel planes) may be equal to the original volume of illumination. This uniform intensity of illumination is called the *mean intensity of illumination* or *mean illumination*, e_m; we have, then,

$$e_m = \frac{V}{S}.$$

The mean illumination of a given surface S is then the quotient of the volume of illumination V of this surface divided by its area S, or the quotient of the total quantity of light which falls on this surface divided by its area S.

The calculation of mean illumination has many interesting features.

166. We may, for instance, calculate in the following manner the mean illumination of the circular space of radius x having its center at the foot of the perpendicular passing through the light.

Let us first consider a circular ring comprised between circles of radii x_1 and x_2, corresponding to rays of light making with the ground the angles θ_1 and θ_2 (Fig. 85).

The quantity of light received by this circular ring is the same as that which is received by the zone of the sphere of unit radius limited by the cones of which the circles of the ring are the bases.

For an elementary ring corresponding to an angle $d\theta$, this quantity of light is represented by the luminous intensity of the source multiplied by the surface of the zone of width $d\theta$; it is then equal

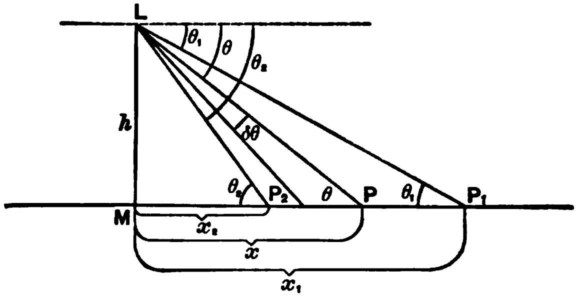

FIG. 85.

to $I 2\pi \cos\theta d\theta$. The total quantity of light received by the ring $\theta_1\theta_2$, or its volume of illumination, is thus

$$V = \int_{\theta_1}^{\theta_2} 2\pi I \cos\theta d\theta.$$

The mean illumination of the ring is equal to V divided by its surface; now this surface is equal to

$$\pi\left[\frac{h^2}{\tan^2\theta_1} - \frac{h^2}{\tan^2\theta_2}\right].$$

The mean illumination is then

$$e_m = \frac{2}{h^2}\frac{\int_{\theta_1}^{\theta_2} I \cos\theta d\theta}{\left[\frac{1}{\tan^2\theta_1} - \frac{1}{\tan^2\theta_2}\right]}. \tag{8}$$

Making $\theta_2 = 90°$, we obtain the mean illumination of the circle of radius x_1, having its center at the foot of the perpendicular passing through the source; we have, thus,

$$e_m = \frac{2\int_{\theta_1}^{\frac{\pi}{2}} I \cos\theta d\theta}{x_1^2}. \tag{9}$$

This formula is general. If we suppose that the intensity of the luminous source is the same in all directions, *i.e.* if I is constant, the formula becomes

$$e_m = \frac{2I(1 - \sin\theta_1)}{x_1^2}. \tag{10}$$

Useful Effect of Illumination.

167. Should we, when we calculate the illumination produced by a source of light, measure this illumination on the horizontal plane or on a plane normal to the rays? The question is very much debated.

In the calculations which precede we have considered the illumination on the horizontal plane; that is, we have multiplied by the sine of the inclination of the rays the illumination produced on a plane normal to those rays. The presence of this factor $\sin\theta$ in the formulæ constitutes the whole difference between these two methods.

Certain very competent specialists in matters of illumination, among others Weissenbruch *, estimate that it is the illumination of the horizontal plane alone which is the element to be considered; others, on the contrary, among them Wybauw ‡, are of the opinion that the illuminations of the plane normal to the rays should play the principal part in the distribution of illumination. "It is not, in general, the horizontal geometrical plane, properly so called, which is to be lighted, but the objects on this plane. On public streets it is the passers-by, carriages, the ups and downs of the pavement; and it may be said in general that the bodies which are to be illuminated present faces and forms most frequently not in the horizontal plane. . . . Although we find ourselves on a material horizontal plane, we have no motive for considering a horizontal element rather than any other, and what interests us most, and with most reason, is the maximum illumination which a light can give, at the point where we are, on a surface normal to the direction of the rays."

It may be objected to Wybauw's views of the subject that the horizontal earth has a preponderating influence because all objects to be illuminated are found there either stationary or moving about on its surface; it determines the distance of these various objects from the source of light as well as the inclination of the ray with reference to any surface which might move from one point to another of the ground.

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* Comparaison de plusieures projets d'éclairage d'un espace découvert par grands et par petits foyers, *Bull. de la Soc. belge des électr.*, 1889.

‡ Measure et répartition de l'éclairment, *Bull. de la Soc. belge des électr.*, 1885.

Suppose, for instance, the ground to be in the form of concentric rings perpendicular to the luminous rays, each receiving a maximum of illumination ($\theta = 90°$); it is evident that the connecting surfaces are then in the shade, and the horizontal ground shows a certain number of luminous rings becoming narrower and narrower, separated by dark rings becoming wider and wider (Fig. 86).

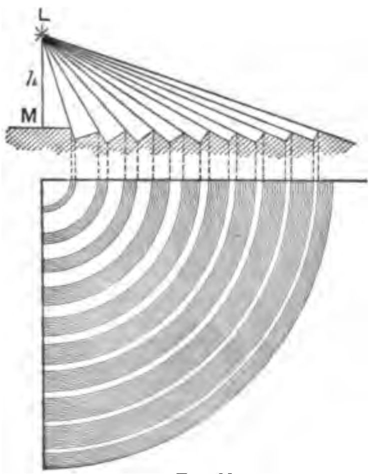

FIG. 86.

Assuming that the illumination should be measured on the plane normal to the luminous rays, we assume implicitly that a ray oblique to the horizontal plane is as valuable as regards useful effect as a vertical ray, provided that this obliquity does not exceed a certain practical limit. This limit is confounded, moreover, with the limit of the distances beyond which no account is taken of the illumination, which has become too feeble.

Illumination on the Horizontal Plane and on the Normal Plane.

168. We may then define the *useful effect of illumination* ϵ produced by the light of intensity I, placed at the height h above and at the horizontal distance x from the element dS of the horizontal plane, by the formula

$$\epsilon = \frac{I}{h^2 + x^2} dS, \tag{11}$$

to which we may also give one of the following forms:

$$\epsilon = \frac{I \cos^2 \theta}{x^2} dS, \tag{12}$$

$$\epsilon = \frac{I \sin^2 \theta}{h^2} dS. \tag{13}$$

The useful effect of illumination, introduced by Wybauw, being thus simply defined, we may study the importance of this concept from a practical point of view.

The preceding formulæ show that the useful effect at a given point of the horizontal plane is to the intensity of illumination at this point as 1 to $\sin\theta$; i.e

$$\frac{\epsilon}{e}=\frac{1}{\sin\theta};$$

consequently the useful effect is always greater than the intensity of illumination.

If, then, we calculate the illumination produced by a given source, basing it on the useful effect, we obtain too favorable a result. The following considerations, based on precise experiment, show that the results furnished by calculations based on the intensity of illumination are somewhat too small. In fact, the calculations relative to intensity of illumination are based on the law of the cosine, which is not exact, giving only a poor approximation in the majority of cases. We should remember, further, that the horizontal plane appears proportionately more illuminated as its reflecting power is greater; a black board appears less illuminated than a white one, and objects are distinguished less clearly from the former than from the latter. The illumination obtained on a given surface will be proportionately more advantageous as the quantity of light emitted by the surface after reflection is greater. It has been generally supposed that the law of photometric emission was strictly exact, or, at least, as exact as possible. But recent investigations by Seeliger, at Munich, have proved that the majority of the substances employed in buildings do not follow, even remotely, this theoretical law. There are often found errors of 20 per cent, with inclinations of from 20° to 25°. It follows from this that calculations of illumination, based on the law of the cosine, cannot give results strictly exact; we are forced to be content with results which are more or less approximate.

It seems to us useful, then, to take account practically of the two methods, that of illumination properly so called, and that of useful effect. We shall consider the intensity of illumination deduced from the fundamental photometric laws as the lower limit of illumination, and the intensity of useful effect defined by Wybauw as the upper limit of this same quantity. This point of view is purely empirical, but it seems sufficiently justifiable since it allows account to be taken of the arguments which militate in favor of one system or the other.

169. We may calculate the total quantity of useful effect received by a circle of radius x, whose center is directly below the light, in the same way that the total quantity of illumination is calculated.

The quantity of useful effect received by a circular ring of radius x and of width dx is equal to

$$\frac{2\pi I x dx}{h^2+x^2}, \quad \text{or} \quad 2\pi I \frac{\cos\theta \cdot d\theta}{\sin\theta}.$$

Consequently, the total quantity of useful effect received by the circle of radius x, corresponding to the obliquity θ_1, is

$$\Sigma_{x_1} = 2\pi \int_0^{x_1} \frac{I x dx}{h^2+x^2},$$

or

$$\Sigma_{\theta_1} = 2\pi \int_{\theta_1}^{\frac{\pi}{2}} \frac{I\cos\theta \cdot d\theta}{\sin\theta}. \qquad (14)$$

Integrating, we obtain immediately, supposing I constant,

$$\Sigma_{x_1} = \pi I \log_e \left[\frac{h^2+x_1^2}{h^2}\right],$$

or, in common logarithms,

$$\Sigma_{x_1} = 7.234\, I \log \frac{h^2+x_1^2}{h^2}.$$

In the same way,

$$\Sigma_{\theta_1} = 7.234\, I \log\left[1+\cot^2\theta_1\right].$$

These values of Σ become infinitely large for $x_1=\infty$, or for $\theta_1=0$. This result was easy to foresee, for it is supposed that the intensity of useful effect diminishes proportionately to x^2, while the surface illuminated increases proportionately to πx^2.

Because of this result the introduction of the notion of useful effect of a light source has been criticised; these criticisms are not well founded, for Wybauw made the express reservation that the intensity for useful effect may only be substituted for intensity of illumination up to a certain limiting incidence; it is then this value θ_0 of the limiting incidence, corresponding to the radius x_0, which should be taken as one limit of the integration when it is desired to calculate the total useful effect of a light.

Following are some particular values deduced from the formulæ: $\Sigma = I$, when $x = 0.6124\,h$; i.e. when $\theta = 58° \, 31'$. In the same way $\Sigma = \pi I$, when $x = 1.3115\,h$; i.e. when $\theta = 52° \, 40'$.

When $x = h$, we have $\Sigma' = 2.177\,I$, and when $x = h\sqrt{3} = 1.73\,h$, we have double the preceding value; i.e. $\Sigma = 4.354$. The expression Σ_θ or Σ_x represents a *volume of useful effect* assuming for this volume the definition corresponding to that of volume of illumination; the limiting surface of this volume may also be called the *surface of useful effect.*

We may also define the *mean useful effect* ϵ_m, in the same way as the mean intensity of illumination, as the quotient of the volume of useful effect divided by the surface illuminated; consequently, the mean useful effect of a circle of radius x whose center is directly below the light is

$$\epsilon_m = \frac{7.234\, I \log\,(1 + \cot^2\theta)}{\pi x^2}.$$

Introduction of the Real Luminous Intensity into the Calculation.

170. It was supposed in the preceding calculations that the luminous intensity of the source considered was uniform. Now this hypothesis is not realized by any luminous source, as has been seen in the preceding chapter; we may, however, assume that it is true for some of the usual lights, for alternate-current arc-lamps, and even for continuous-current arc-lamps with opalescent globes.

However, with bare continuous-current arc-lamps, it is impossible to assume uniformity of luminous intensity. Calculations made on this hypothesis would lead to completely erroneous results.

If it is desired to calculate the intensity of illumination or the useful effect at a point, we must take the value of the luminous intensity corresponding to the inclination at which the rays of light striking the point considered are emitted. Thus, for a point at a distance from the foot of the perpendicular equal to the height, the maximum intensity ($\theta = 45°$) should be taken. For a point at a distance $2\,h$, the luminous intensity corresponding to $\theta = 28°$ should be taken, etc.

171. Knowing the meridianal curve of the photometric surface of the radiant, we may determine by graphical methods the intensity of illumination at any given point of the horizontal plane. The following method is given by Loppé* (Fig. 87).

If the luminous intensity in the direction LA is equal to $K \cdot LA$, and if the height h is proportional to LM, i.e. if h equals $K' \cdot LM$,

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* *Électricien*, 1890, p. 936

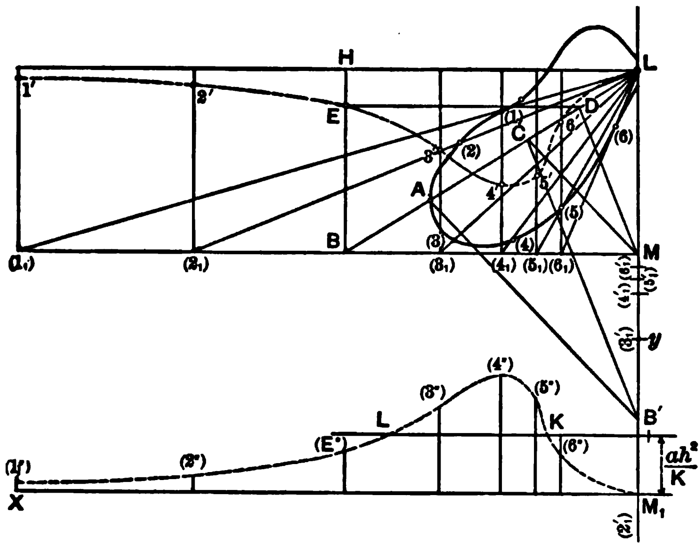

FIG. 87.

the intensity of illumination at the point B of the horizontal plane passing through M will be

$$e = \frac{K \cdot LA}{(K' \cdot LB)^2} \cos HBL.$$

Lay off $LB' = LB$, join A and B', and through M draw a parallel to AB'; we obtain a point C on LA; join C and B', and through M draw a parallel to CB'; we obtain a point D, and we have

$$LD = LA \left(\frac{LM}{LB}\right)^2.$$

If through D we draw a horizontal until it meets at E the vertical through B, we shall have

$$HE = LD \cos HBL = LA \left(\frac{LM}{LB}\right)^2 \cos HBL.$$

From the preceding relations it follows that

$$e = \frac{K}{(K' \cdot LM)^2} HE = \frac{K}{h^2} HE.$$

Proceeding in the same way for directions L (1), L (2), etc., we obtain a curve $1'$ $2'$ E $3'$, etc., whose ordinates, multiplied by $\frac{K}{h^2}$,

give the quantity of light received by the horizontal plane, and whose abscissæ, multiplied by $K'\left(=\frac{h}{LM}\right)$, give the horizontal projection of the distance from the point considered to the light. The construction then allows us to find, by a simple reading of the scale, the illumination at a point of the horizontal plane situated at any distance below the light.

By means of this curve shown at $1''\ 2''\ E''\ 3''$, etc., we may easily find the radii of the circles on whose circumferences are the points of the plane whose intensity of illumination is equal to a given quantity e.

To do this we lay off from M_1 on M_1B' a length M_1I so as to have

$$e_1 = \frac{K}{h^2} M_1I.$$

Through I we draw a parallel to the axis of x; this cuts the curve at L and K. The points of the plane situated at distances $K' \cdot IK$ and $K' \cdot IL$ from the foot of the perpendicular passing through the light will receive from the latter a quantity of light equal to e_1.

We may then draw on tracing paper circles corresponding to illuminations 1, 2, 3, etc., and find the illumination produced by any number of lights.

In case of a light of uniform intensity, to each value of the intensity of illumination on the horizontal plane there corresponds only a single circle, while in the case of a bare continuous-current arc-lamp, two circles correspond to each intensity.

This graphical method offers, further, the following great advantage. It has been seen in the preceding chapter that the photometric surface of continuous-current arc-lamps is of nearly the same shape whatever be the intensity of the light. The preceding figure may then serve in all cases if the coefficients K and K' are changed every time, so that the maximum luminous intensity may be represented by the expression $K \cdot LA$ and the height of the light by $K' \cdot LM$. It is sufficient then to determine each time the particular values of the coefficients K and K'.

We cannot insist too strongly on the necessity of employing, in calculations of illumination by continuous-current arc-lamps, the real luminous intensity and not the mean spherical or the maximum intensity. The mean spherical intensity gives too small values for the illumination, while the maximum intensity gives too great values.

172. The curves in Fig. 88 represent these different cases. There were taken as abscissæ horizontal distances and as ordinates intensities of illumination expressed in *luxes* (decimal-candles meter), the light being assumed at a height of 1 m., and supposing

1°. The luminous intensity uniform and equal to the maximum intensity 1000 decimal candles (curve A);

2°. The real luminous intensity as given by Fig. 82 (curve B);

3°. The luminous intensity uniform and equal to the mean spherical intensity 352 decimal candles (curve C).

These curves permit an important conclusion to be drawn. Below a certain value of the intensity of illumination corresponding, for instance, to a distance equal to five times the height of the lamp, the diminution of the intensity of illumination is very small for a considerable increase in the distance. But these illuminations

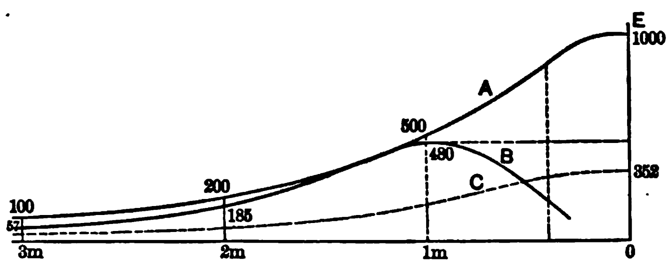

FIG. 88.

are too small to take account of. In all calculations of illumination, then, we should not exceed a certain limit, and especially is it necessary, as Wybauw says, "to avoid long and minute calculations which the result does not warrant."

Using as the basis the normal diagram of the luminous intensity of the continuous-current arc-lamp, Wybauw made a diagram (Fig. 89) which allows the easy calculation of the quantity of light received (volume of illumination) or of the useful effect of illumination, corresponding to a circle of radius x whose center is directly below the light. In this case direct formulæ become very complicated and graphical methods alone allow the practical solution of the problem. The two upper curves A and A' represent the calculations of useful effect, while the lower curves B and B' refer to intensity of illumination. The curves A and B are those of a uniform light of 1000 candles; the curves A' and B' those of an arc-lamp whose maximum intensity is 1000 candles.

A mere inspection of the drawing is enough to show its use. Let us suppose, for instance, that there is required the volume of illumination furnished on the ground, in a circle of 30 m. radius, by a light placed at a height of 10 m. whose maximum intensity is 2000 candles. The distance being equal to 3, we have for the abscissa $x = 3$; the corresponding ordinate y is 5.877. Then the volume sought is $5.877 \times 2000 = 11{,}754$.

If in place of an electric lamp, one had a light of uniform intensity of the same power, the diagram would give 13,232.

This number is not exactly that which would be obtained by applying the formulæ. The reason of this is that the diagram was drawn by Wybauw, who considered as uniform the illumination in

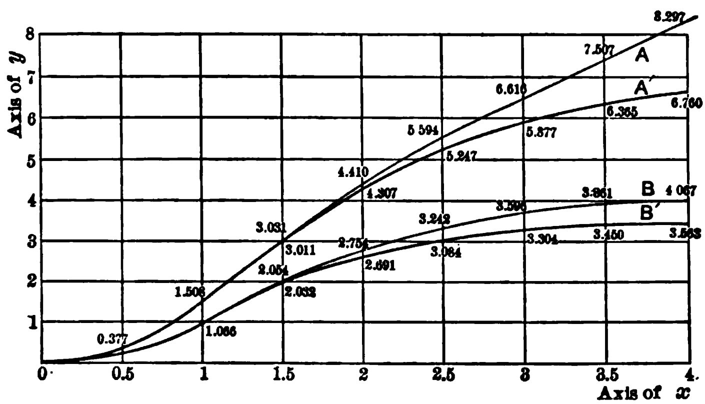

FIG. 89.

the circle of radius h about the foot of the perpendicular passing through the light, the intensity of this uniform illumination being equal to the intensity at the foot of the oblique line of 45° inclination. In fact, in practice, the illumination on the base of the 45° cone is always more than sufficient. If we have to do with lights of uniform intensity, the superabundance of illumination in the circle is of no use and need not be taken into account; it is an inevitable and forced excess. With electric lights, the nearness, the effect of the reflector and of diffused light, render this illumination more than satisfactory. This way of calculating is unfavorable for lights of uniform intensity. Moreover, it is not allowed by all specialists, and the elimination of the superabundant illumination in the calculation of the mean intensity is not to be recommended, as we shall see later.

It is well understood that these data have to do only with bare lights. Account should be taken of the loss occasioned by electric light globes. It may be assumed as 30 per cent, so that a 200 carcel light with a globe becomes in calculations a 140 carcel light with a bare flame. It is to be remarked, further, that the distribution of light about a lamp with a globe is not entirely similar to that of a bare lamp. The ellipse of the diagram becomes less elongated; the difference may even be quite considerable, as has been seen in the preceding chapter (§ 148).

To calculate the volume of illumination furnished by a light on a ground of irregular limits, we shall proceed by calculating first the volume for the greatest circle of radius r described about the foot of the perpendicular passing through the light wholly within the polygonal ground; then the volume for the fraction $\frac{x}{360}$ of the ring rr', x being the angle at the center subtended by the part of this ring included in the ground; then for the second ring $r'r''$, etc. The value for each whole ring is given immediately by the diagram. Each of them has, in fact, for a volume of illumination the difference between the two extreme ordinates corresponding to the abscissæ $\frac{r}{h}$, $\frac{r'}{h}$, etc., multiplied by I, the maximum intensity of the light expressed in candles. We may evidently, by suitably spacing the circles, obtain such degree of exactness as is desired.

These calculations are very simple, and their exactness is sufficient for all practical purposes.

173. We could not better conclude this study of the distribution of illumination than by giving a *résumé* of the solution which Loppé has given of certain particular problems in which he supposes it to be true that the luminous intensity is uniform. The formulæ which he arrived at, notwithstanding this restriction, are interesting enough to find place here.

Illumination of a Surface, the Lights being placed at the Angles of Equal Squares.

174. Suppose four lights of the same uniform intensity, placed at equal heights above a horizontal plane, at the angles of a square; we are to find the point of the plane within the square where the intensity of illumination is minimum.

If $ABCD$ are the projections of these lights (Fig. 90) placed at

a distance $2a$ from one another, choosing the axes of co-ordinates as indicated in the figure, the total intensity of illumination e at a point is given by the expression

$$e = Ih\,\Sigma\,\frac{1}{(h^2+d^2)^{\frac{3}{2}}},$$

in which

$$d_1 = [(a-x)^2 + (a-y)^2]^{\frac{1}{2}},$$
$$d_2 = [(a-x)^2 + (a+y)^2]^{\frac{1}{2}},$$
$$d_3 = [(a+x)^2 + (a+y)^2]^{\frac{1}{2}},$$
$$d_4 = [(a+x)^2 + (a-y)^2]^{\frac{1}{2}}.$$

Solving for maxima and minima of this expression, we find that the point of intersection of the diagonals of the square receives the minimum quantity of light when $h < a\sqrt{3}$. This point, on the contrary, has a maximum illumination when $h \geq a\sqrt{3}$.

The first case, $h < a\sqrt{3}$ or $h < 1.732a$, is that which ordinarily presents iself in practice. We may also find on the line which joins

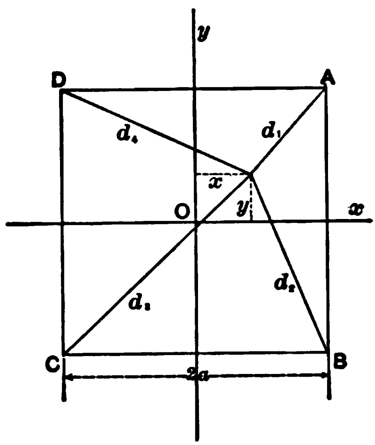

FIG. 90.

the projections of two lights the point where the illumination is maximum. If $2a$ is the distance between the two lights, we find that the illumination of the middle point of the line is minimum when

$$h < 2a,$$

and this illumination on the contrary is maximum for

$$h \geq 2a.$$

Illumination of a Given Surface corresponding to a Minimum of Expenditure.

175. The solution of the following problem leads to some interesting conclusions which we will enumerate. The problem may be stated as follows:

To illuminate a horizontal plane by means of lights of uniform intensity, in such a manner that the intensity of illumination at any point may be at least equal to a given quantity e_0, employing for this the least energy possible.

There are two cases to consider.

1°. A surface is to be illuminated. In this case, the lights will be placed at the corners of squares.

2°. A street is to be illuminated. In this case, most usually the lights are placed alternately, on one side and the other of the street.

176. *To illuminate a surface.*—The majority of the light which falls at a point within one of the squares comes evidently from the four lights which are in projection in its corners.

If $2a$ is the side of one of the squares, we have seen that when the condition $h < 1.732a$ is realized (which occurs ordinarily in practice), the point of minimum illumination is at the point of intersection of the diagonals.

The spacing of the lights is to be calculated so that at this point the intensity of illumination produced by the four lights shall equal e_0; then if the influence of the other lights is not negligible, they may be placed a little farther apart.

In the latter case, the following conclusions are none the less valid, for the greatest part of the light is due to the lights placed at the four corners.

If h is the common height of the lights above the plane, $2a$ the distance between the lights, or the side of the square, and I their uniform luminous intensity, we have for the intensity of illumination at the intersection of the diagonals the formula

$$e_0 = \frac{4Ih}{(h^2 + 2a^2)^{\frac{3}{2}}}. \tag{15}$$

This formula affords a solution of the following problem.

177. The height h of the lights above the plane being given, to choose a lamp of such luminous intensity I that the energy expended may be minimum.

The number N of lights necessary to illuminate a surface S being

$$N = \frac{S}{4a^2},$$

the expenditure in watts is given by the formula

$$D = \frac{BIS}{4a^2}.$$

In the case of arc-lamps of medium intensity B may be taken as a constant; in the case of lamps of great intensity, B is a function of I.

To solve the problem in the case where B is constant, it is sufficient to find the minimum of $\frac{I}{4a^2}$; we thus obtain

$$I = \frac{(3)^{\frac{3}{2}} h^3 e_0}{4h} = 1.30\, e_0 h^2. \tag{16}$$

We take for I the practical value nearest the value found, and we find $2a$, by equation (15). We shall increase the distance between the lights a little, as needed, if the influence of the other lights is not negligible. If, after having calculated I and $2a$, we find that the condition $h < 1.732\,a$ is not fulfilled, we must see whether the illumination at the various points reaches a given value e_0, for in this case we know that the illumination due to the four lights is maximum at the point of intersection of the diagonals.

178. The same formula gives also the solution of the following question.

I is given, to find such a value for h that the energy expended, that is the number of lights employed, may be a minimum.

The number of lights is given by the relation

$$N = \frac{S}{4a^2}.$$

From (15) we obtain, making $\left(\frac{4I}{e_0}\right)^{\frac{2}{3}} = B$,

$$2a^2 = Bh^{\frac{2}{3}} - h^2,$$

whence

$$N = \frac{S}{2(Bh^{\frac{2}{3}} - h^2)}.$$

The minimum of N corresponds to the maximum of

$$Bh^{\frac{2}{3}} - h^2.$$

Equating the derivative to 0, we have

$$h = \left(\frac{1}{3}\right)^{\frac{3}{4}}\left(\frac{4\,I}{e_0}\right)^{\frac{1}{2}} = 0.877\sqrt{\frac{I}{e_0}}.$$

If, after having calculated a by equation (16), we find $h \geq 1.732\,a$, we should proceed to the verifications indicated above.

179. *To illuminate a street.* — The lights (Fig. 91) are placed at ABC. The points D and E on the perpendicular passing through the middle of AB receive from A and B the minimum of light, if $h < 2\,a$.

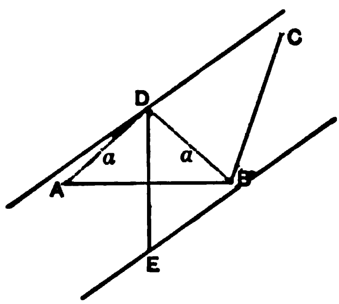

FIG. 91.

Let $AD = a$ as before, and we write the fundamental condition

$$e_0 = \frac{2\,Ih}{(h^2 + a^2)^{\frac{3}{2}}}. \qquad (17)$$

We may then solve the same problem as in the case of illuminating a surface.

For instance, h being given, to determine I so that the energy to be expended may be minimum.

The number of lights is inversely proportional to a; the expenditure in watts will be proportional to $\frac{BI}{a}$ or to $\frac{I}{a}$. In the case where B is constant, we obtain from (17) the value

$$a = (C^{\frac{2}{3}}I^{\frac{2}{3}} - h^2)^{\frac{1}{2}}, \text{ putting } \frac{2\,h}{e_0} = C.$$

We should then obtain the minimum of the expression

$$\frac{I}{(C^{\frac{2}{3}}I^{\frac{2}{3}}-h^2)^{\frac{1}{2}}};$$

which gives

$$I=\frac{(\frac{3}{2})^{\frac{3}{2}}h^3\cdot e_0}{2\,h}=0.92\,e_0h^2. \tag{18}$$

We take for I the practical value which is nearest the calculated value and deduce a from equation (17). In case that the influence of more distant lights is not negligible, a may be slightly increased.

180. To solve the second problem, I is given to find h.

It is noticed that the number of lights is inversely proportional to a; in order to have the minimum of energy expended, a or a^2 must then be made maximum.

From (17) we obtain

$$a^2=D^{\frac{2}{3}}h^{\frac{2}{3}}-h^2, \text{ putting } D=\frac{2\,I}{e_0}.$$

Equating to 0 the derivative with respect to h, we obtain

$$h=\left(\frac{1}{3}\right)^{\frac{3}{4}}\sqrt{2}\sqrt{\frac{I}{e_0}}=0.62\sqrt{\frac{I}{e_0}}. \tag{19}$$

Practical Points.

181. The photometric calculations relative to the lighting of uncovered places enable us to solve a considerable number of particular problems. We have studied a number sufficiently large to show the method of procedure in each special case.

It remains to compare the results of the preceding formulæ with those obtained in practice.

The height of the lights is an important element in all systems of lighting. The distance between lights being equal to $2\,a$, the illumination at a distance a from the light is maximum when the height is given by the formula

$$h=\frac{a}{\sqrt{2}}=0.707\,a.$$

This height is rarely adopted in practice. With lights 100 m. apart, we should have towers 35 m. high. Æsthetic considerations relative to the decoration of streets forbid the adoption of such high lights. In certain cities in the United States, where these considerations have not as much weight as in Europe, very powerful lights are frequently employed placed on the top of latticed towers 30 to 40 m. high, or even higher. In Europe the height of the lights is generally between 6 and 15 m.

We must distinguish, with a view to the disposition of the lights, between the illumination of streets and that of open spaces.

Lighting of Streets.

182. The lamps may be placed in two ways: either alternately on the two sides of the street, along the edges of the sidewalk, or in a line with the middle of the driveway. The former arrangement is used in gas lighting; the latter is that which has been used for the illumination of the great boulevards of Paris.

To begin with, we give some data as to the intensity of the mean illumination by gas, in the principal streets of Paris in 1889:

Rue Royale	1.6	lux.
Rue de la Paix	1.5	"
Place de l'Opéra	0.7	"
Avenue de l'Opéra	0.43	"
Rue du Quatre-Septembre	0.43	"
All Paris	0.05	"

Using arc-lights, illumination along the middle of the street is the most rational. The other system of illumination, along the edge of the sidewalk, is only allowable with lights whose luminous intensity does not exceed 100 candles.

The intensity of illumination at a horizontal distance x from the foot of the lamp-post is expressed by the formula

$$e = \frac{Ih}{(h^2 + x^2)^{\frac{3}{2}}}.$$

The height h being given, as well as the value of the intensity of illumination below which we should not go, it is easy to calculate the distance between the lights. We thus obtain the following table, which gives the distances between two lights at heights of 6, 10, and 14 m. and of different uniform intensities corresponding to different minima of intensity of illumination.

DISTANCE BETWEEN TWO LIGHTS IN METERS.

Minimum Intensity of Horizontal Illumination in Luxes.	$I=500$ Dec. Candles. h			$I=600$ Dec. Candles. h			$I=700$ Dec. Candles. h			$I=800$ Dec. Candles. h			$I=900$ Dec. Candles. h			$I=1000$ Dec. Candles. h		
	6	10	14	6	10	14	6	10	14	6	10	14	6	10	14	6	10	14
0.5	44	50	54	47	54	58	50	57	62	52	60	67	54	64	68	56	65	71
1	34	38	39	37	41	43	39	44	46	41	46	49	43	48	52	44	50	54
2	26	28	26	28	30	30	30	33	32	32	35	35	31	36	37	34	38	39

Lighting of Squares and Large Open Places.

183. If, for streets, and particularly for narrow streets, lights of small intensity are preferable to powerful lights, it is quite different for the lighting of public squares and large open spaces. In the first case, the reflection of light on the house fronts increases greatly the illumination in the neighborhood of each light; the contrast may be so strong that the parts situated midway between appear to be in darkness. In the second case, there can be no question of any action of reflection by vertical walls. Thus it is well to employ intense lights placed at a considerable height.

The lights may be arranged quincuncially or at the corners of equal squares, if an illumination as uniform as possible is desired. As to the height of the lamps, following are some figures sanctioned by practice:

Maximum Luminous Intensity.	Height of the Lamp.
240 carcels (10 amperes).	10 meters.
328 " (13 ").	15 "
390 " (15 ").	18 "
490 " (18 ").	20 "

The distance between two consecutive lights is determined according to the minimum admissible for intensity of illumination.

From the point of view of the uniformity of illumination of a great surface, it is evidently advantageous to employ a great number of small lights. For it is then easier to satisfy economically the conditions of minimum illumination. But, as Weissenbruch has remarked, the comparison of two systems of illumination of the same surface is not correct if we observe only the single condition of minimum illumination. Account should also be taken of the mean illumination.

The mean intensity of illumination is always greater with large than with small lamps, for the former give an intense light in the vicinity of their supports. In other words, large lamps always give some superabundant light* in the neighborhood of the lamp. Should account be taken of this superabundant light in the calculation of mean intensity, or should there be assumed, for the intensity of illumination of the circle about the lamp, the value corresponding to the 45° ray?

Some authorities are in favor of the second alternative, among others Wybauw, who made a correction relative to it in the diagrams of Fig. 89. Others, on the contrary, think with reason that so much illumination cannot be neglected. The parts superabundantly lighted are only an advantage, for they form veritable secondary sources of reflected light.

However, if two systems of illumination are to be compared, there should be introduced the mean intensity of illumination calculated by taking into account the total quantity of light received by the surface, giving it an importance at least as great as that of the minimum illumination.

Weissenbruch has demonstrated the necessity of introducing these two elements, mean intensity and minimum intensity of illumination, in calculating the lighting of railway stations; this conclusion applies equally in all cases when we have to do not only with minimum but with the most intense illumination.

It is difficult to give exact values of the minimum intensity of illumination. These values vary too much according to circumstances. The state railways of Belgium allow, for example, an intensity of illumination of $\frac{1}{50}$ of a carcel meter for the lighting of stations. This number is somewhat too small, particularly as compared with the values of the mean illumination of the principal streets of Paris, lighted by gas.

Employment of Reflectors.

184. With arc-lights, the intensity of illumination is very much greater than the minimum required; it would then be advantageous to diminish the quantity of light received just below the lamp in order to carry it to a greater distance. To do this, reflectors of particular form must be employed, which render the illumination of the horizontal ground sensibly uniform. Among apparatus of

* *Lum. Él.*, Vol. XI. pp. 149, 244.

this kind, the dioptric lantern of Trotter* seems to solve the problem in the most perfect manner.

Trotter combined his reflectors with a view to solving the problem of the uniform illumination of the ground. This problem may be stated as follows: It is required to illuminate by a single central light a plane circular area, so that if this area be divided into rings of the same surface, each of these rings may receive the same quantity of light. These successive rings have radii which increase proportionally to $\sqrt{1}$, $\sqrt{2}$, $\sqrt{3}$, etc.

It is sufficient for this that the rays emanating from the source of light and making equal angles with one another, may be so directed that the tangents of their new inclinations with the vertical may increase as $\sqrt{1}$, $\sqrt{2}$, $\sqrt{3}$, etc.

Trotter obtains this result, at least approximately, by aid of a hexagonal lantern whose faces are formed by ribbed strips of glass; the form of these ribs is carefully determined by the preceding, and their section may be obtained graphically.

The results obtained with this apparatus are very satisfactory. Preece, who has tried it, speaks of it in the highest terms. The intensity of illumination is increased at a certain distance from the lamp, but is diminished in a much greater proportion just below it. Unfortunately the shape of the glass is very complicated, which greatly increases the cost. For this reason its use has not become general.

Recourse is, however, often had to reflectors, but without an exact determination of their form. Plain or ribbed ones are chosen, generally of sheet iron enamelled white. But the only object of this apparatus is to throw upon the ground the rays of light directed upward which otherwise would be of no practical use.

Like Jasper at the Electrical Exposition of 1881, we may also use quite large white discs, which are placed horizontally above the lamp, and which play the double part of reflectors and secondary sources of light by means of diffusion. It is in this category that Elster's† new reflectors should be placed.

The following table gives the means of passing rapidly from illumination by lamps of given intensity I to other systems in which there are employed lamps whose intensity is double, triple, etc., that of the former. It is applicable to the lighting of a hori-

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* *Lum. Él.*, Vol. XIV. p. 98.

† *Elektr. Zeitschr.*, 1891, 438.

zontal surface by a single lamp, by a series of lamps in a straight line, by lamps arranged at the corners of squares, and quincuncially. The unit of length is the radius of the circumference of the given minimum illumination.

One Light.			Lights in a Straight Line.		Lights Quincuncially.
Luminous Intensity. I	Height. h	Radius of the Circumference of Minimum Illumination. r	Height. h	Distance between the Lights. $2a = 2r$	Distance between the Lights.
1	0.70	1.0	1.65	3.0	4.2
2	0.98	1.4	2.31	4.2	5.9
3	1.19	1.7	2.86	5.2	7.3
4	1.40	2.0	3.30	6.0	8.4
5	1.54	2.2	3.63	6.6	9.2
6	1.68	2.4	4.07	7.4	10.4
7	1.82	2.6	4.40	8.0	11.2
8	1.96	2.8	4.62	8.4	11.8
9	2.10	3.0	4.95	9.0	12.6
10	2.24	3.2	5.17	9.4	13.2

Lighting of Enclosed Places.

185. If the problems of lighting a horizontal plane by many lamps is difficult, that of lighting enclosed places is particularly complicated. The lighting of a room depends, in fact, on many factors, among which the luminous intensity of the lamps employed does not play so preponderating a part as would at first be supposed.

Outside the effect of lighting, there is another element to be considered, which Wybauw calls the effect of illumination; this is indeed a consequence of lighting properly so called, but is not related to it in any definite proportion, and it often becomes a factor of considerable importance in modifying one's judgments of the lighting of rooms. The numerous flames of a chandelier give the impression of an intensity of light much greater than that of a single flame which might have the same power. Two gas flames or two electric lamps may have very different intensities; and yet when they are not absolutely side by side, they will produce the same effect upon our eyes. The light of a simple candle is seen at night at a considerable distance, at even 500 m., while its effect as a source of light is not appreciable on objects placed at this distance. It is the same with

a white wall, lighted at night. A row of lanterns close together, on the front of a house during a *fête* produces at a certain distance absolutely the same effect of illumination as a row of gas-lights whose jets are, however, much brighter. These effects of illumination have no common measure with the intensity of lighting, and yet they contribute in an important degree to the final effect obtained, and account should certainly be taken of them.

Diffused light is also an important element in the lighting of enclosed places. As a means of illumination it is given an importance which it does not have when measured in comparison with simple direct light.

In a room lighted by gas-burners with opal globes, the eyes deceived by the appearance labor under the impression of an illumination much greater than the real; it is necessary to take up a newspaper or book to appreciate the insufficiency of the light which these globes give.

A diffusing surface when lighted throughout its extent becomes a source of light; it gives out light at all points and in all directions, in contrast to a simple reflecting surface, such as a mirror. If a mirror is placed behind a light, the room will appear to be lighted by two lights; if the mirror is replaced by a white wall at a suitable distance from the light, the effect will be much more satisfactory, although in reality the light reflected by the dull surface may be much inferior as to its intensity to that reflected by the mirror.

All bodies reflect light, but with an exceedingly variable intensity, according to their distance from the source, their color, and finally the degree of roughness or polish of their surfaces. This diffused light is a power auxiliary to the effect of lighting produced by the direct light. It is diffusion which makes daylight so much superior to all artificial light. The latter illuminates objects in a single direction only, leaving lateral or opposite faces in a strong shadow which the light, reflected by surrounding objects, can only slightly diminish.

It is quite difficult to take account of the increase in illumination produced by light diffused by the ceiling and walls of a room. We may, however, obtain an approximate idea, as Mascart has shown, in the following manner. Diffusion is nothing but ordinary reflection on a surface whose irregularities are of the same order as, or are greater than, the wave-length.

We should then assume that the total fraction of light diffused is analogous to the fraction of light which would be regularly

reflected by a polished surface, and which may, under certain circumstances, reach 90 per cent.

Without giving a numerical value to the coefficient f of diffusion, let us suppose that a system of lights placed in a closed room emits a total quantity of light Q. A definite portion of this light is absorbed by the walls, another portion fQ, being diffused, is distributed anew in the room; the second diffusion gives likewise a quantity of light f^2Q, etc., so that the total light used is

$$Q(1+f+f^2+\cdots)=Q\frac{1}{1-f}.$$

The mean brightness of a sheet of paper placed in all possible positions will be, if the walls are black, proportional only to the quantity Q of light emitted by the sources, and with walls of reflecting power f, proportional to the quantity $Q\frac{1}{1-f}$. The increase of illumination is then represented by the ratio $\frac{1}{1-f}$. For instance, if $f=0.95$, the room would appear twenty times as bright as with black walls. We do not arrive at this extreme value with certainty, but the advantage of having white walls must not be very far short of it. [See Appendix H.]

186. When we have to do with installing the lighting of a large hall, architectural and other necessities determine the height and location of the chandeliers and lights; their number is also frequently the result of the same necessities. We may determine the intensity of these lights or the number of them on each chandelier so as to obtain a minimum intensity of illumination, n luxes for instance, in the horizontal plane 1 m. above the floor. It is this plane which is usually chosen in such cases, in preference to the plane of the floor itself.

The minimum intensity of illumination evidently does not apply to the corners of the room; it is clearly not intended, in fixing this limit, to consider individual points of the space to be lighted. The line of minimum illumination will be a curve inscribed in the polygon formed by the sides of the room and lying without the circles of greatest possible radius inscribed in the polygon having the lights for centers.

If the location of the lights is obligatory, it frequently happens that the distribution of light leaves much to be desired with respect to uniformity. In a room whose ceiling is divided into three parts

by two beams, the location of the lights is obligatory; the necessities of decoration and lighting frequently produce similar constraints.

From a practical point of view, the reflecting action of a ceiling may be replaced by that of an imaginary light directly above the real light and whose intensity is equal to a certain fraction k of that of the latter. Wybauw made some experiments to determine k in a medium-sized room. He found that he might assume, without great error, $k = 0.5$.

The introduction of the imaginary light into the calculations allows the problem to be treated mathematically and certain conclusions to be drawn. But the want of exactness of the coefficient k does not allow much value to be given to conclusions from calculations of this kind. We are obliged to keep to empirical indications and data furnished by practice.

Dimensions of the Room in Meters.			Number of Lights.	Height of the Lights above the Floor in Meters.	Number of Square Meters per Light.
Length.	Width.	Height.			
4.6	4.7	3.8	2– 3	2.0–2.2	8.4
5.6	5.6	4.4	5– 6	2.0–2.4	5.7
7.5	7.5	5.3	9– 12	2.5–2.8	5.3
10.0	10.0	6.9	16– 20	2.8–3.1	5.5
12.5	12.5	9.4	25– 30	3.5–3.8	5.6
5.7	15.7	12.5	40– 45	4.0–4.4	5.8
12.8	18.8	14.0	60– 70	4.7–5.3	5.4
22.0	20.0	15.7	100–120	5.6–6.3	4.0

Uppenborn* has given as a *résumé* of his experiments, and from direct measurements, the preceding table which shows the number and height of the lights (16 c.p.) to be employed to illuminate places of different dimensions.

By examining this table, we see that the distribution of illumination corresponds on the average to one light for each 5.5 sq. m., for rooms of very different heights. We should conclude from this that these places may be sufficiently lighted, but not equally lighted, especially as the illumination due to light diffused by the ceiling varies with the height of the place. The rate of one light to 5.5 sq. m. is high enough so that differences of illumination might have passed unnoticed because of the very abundance of light.

* *Centralblatt für Elektrotechnik*, Vol. III. p. 244.

187. It is interesting to discuss how the quantity of light should vary with the geometrical dimensions of a room in order that the illumination may not change.

We shall consider only the cases of closed rooms with ceilings of moderate height.

At first sight it seems as if, in order to give an equal illumination to two rooms geometrically similar, their quantities of light should be in the ratio of their surfaces or of the squares of homologous dimensions.

If we imagine a single source at the center of a sphere, the quantity of light received per unit of surface is inversely proportional to the square of the radius; the illumination will then remain the same if the intensity of the source is proportional to the square of the radius.

It is quite otherwise in practice. Fontaine determined that, in the majority of cases, the quantity of light should be proportional to the volume of the room, and not merely to its surface. For a drawing-room, for instance, whose walls are of medium tint, it has been observed that a quantity of light of 0.5 candle per cubic meter gives a satisfactory illumination, using suitably distributed lamps of from 10 to 16 candle power.

It should be remarked that a room is never entirely empty. It contains furniture and objects of some kind or other which are so many obstacles to the propagation of light; the supports of the lamps, chandeliers, candelabra, etc., intercept also a considerable part of the light; finally, the air itself has not the perfect transparency which the law of inverse squares of the distances assumes.

We may sum up all these causes of loss of light by assuming that the efficacious illumination due to a light ceases at a determinate distance, within which it would have spent its full effect.

This limiting distance varies much with the conditions of practice, the number of obstacles, and the clearness of the air. It is not the same for a theatre, whose central part is entirely empty, as for a drawing-room full of furniture, or a factory crowded with machinery, etc., and it is very much less at the time of a fog.

The following table, prepared by Mascart*, gives some data as to the way in which public halls were illuminated at different epochs, and shows that illumination has pursued a very rapid progressive course, especially in recent years.

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* *Bull. de la Soc. int. des Électr.* Vol. V. p. 103.

	Dimensions.		Total Number of Candles.	Number of Candles.	
	Floor Area. Square Meters.	Volume. Cubic Meters.		Per Horizontal Square Meter.	Per Cubic Meter.
Salle des Glaces of the Palace of Versailles.					
In 1745	720	9360	1800	2.50	0.19
" 1873	720	9360	4000	5.35	0.43
" 1878	720	9360	8000	11.10	0.85
Salle des Fêtes at Compiègne.					
In 1888	440	3520	1000	2.28	0.28
Opera House (used as ball-room).					
Foyer	672	7392	6000	8.93	0.81
Body of the house	400	9200	11140	27.85	1.21
Stage	530	8000	4720	8.90	0.59
City Hall (Balls of 1888).					
Fête-Hall	1295	24000	18720	14.46	0.78
Dining-Hall	300	2460	4320	14.40	1.75
Conservatory	165	1350	720	4.36	0.53
Grand Salon	496	4067	7560	15.24	1.86
Side Gallery	257	3600	3600	13.98	0.56
Reserved Salon	195	1350	720	4.36	0.53
Theaters (Body of the House).					
Odéon	350	5600	2470	7.06	0.44
Gaîté	250	4800	2360	9.44	0.55
Comédie Française	240	3500	2340	9.75	0.67
Palais-Royal	90	1000	1900	21.10	1.90
Porte-Saint-Martin	200	3250	3200	16.00	0.98
Renaissance	96	1400	1970	20.52	1.40

Lighting of Factories.

188. The preceding figures show how vague are the data relative to the lighting of enclosed places. The problem resolves itself into determining in advance the number of candles per square meter of horizontal surface, or per cubic meter of the total volume, and

into distributing the lights of moderate intensity so as to obtain the most uniform distribution of light.

If certain engineers, who have had much to do with the installation of electric light in factories are to be believed, calculations based on the fundamental photometric laws alone should not govern the installation. A correspondent of *l'Électricien* gives, for instance, the following data concerning the lighting of thread mills. These establishments require a great deal of light; the rooms are of large dimensions, and contain numerous machines quite uniformly distributed and of the same dimensions. There is scarcely a plant in which less than one 12-ampere lamp is employed for from 180 to 200 sq. m.; this case supposes a great height of room, and ecru or light-colored thread. The maximum illumination found corresponds to a 10-ampere lamp for 80 or 100 sq. m.

Among the looms, the minimum is one 12-ampere lamp for 120 sq. m. For white, for ecru, and for light colors, a very good illumination is given with one 10-ampere lamp for 75 or 85 sq. m. For dark materials, it is necessary to reckon on at least one 10-ampere lamp for 50 sq. m.

The data which Uppenborn gives on this subject agree practically with the preceding. A *résumé* is given in the following table.

Nature of the Space to be Illuminated.	Number of Square Meters per 10-Ampere Lamp.
Uncovered places	2000 sq. m.
Train houses	1400 " "
Founderies (general lighting)	500–600 " "
" (special lighting)	200–250 " "
Machine shops	200 " "
Thread and cloth mills	200 " "

In a factory, a very simple way of verifying the quality of the lighting consists in asking workmen distributed uniformly in the room whether they see better in one place than in another, when they have been accustomed to the lighting for several months. The quality and quantity of the production of the shop is also an indication of the quality of the light. These indications are hardly scientific; they may have some value, however, but it is necessary to confirm them by direct measurements of the illumination.

189. Let us add that the object of artificial lighting is not always to distribute light in a uniform manner in all directions. For manual work or for reading it is often desired to concentrate the light at certain points. In artistic lighting we seek to produce a harmonious blending of lights and shadows which puts into relief figures, ornaments, and decoration, so that we are obliged to sacrifice a part of the general brightness.

The concentration of light on particular points gives a certain importance to a very simple and useful apparatus, the reflector, employed with all the common radiants.

Almost all commercial reflectors have a very annoying defect as respects the illumination of a room; viz. the top of this apparatus is narrowed to such a point that it hinders in great part lighting the ceiling. Now the ceiling is of great utility in the lighting of a room; the room is better lighted and appears still more so because of its illumination. When fixed lights are employed, reflectors which are sufficiently open above should be used.

The form of the reflecting surface is of little importance when it is desired to have the light reflected downward in all directions; then the conical form, with an angle at the base of from 35° to 45°, is perfectly suitable.

But the generatrix of the surface of the reflector is not a matter of indifference when the flame is to light the surface of a table principally. In this case it is not well to use those conical reflectors which bring the rays together in too great quantities just below the light, to such an extent as to form at the middle of the table an intensely luminous ring, to the detriment of the lighting of the rest. A reflector should be chosen whose surface forms a zone of an ellipsoid of revolution, its axis being coincident with the axis of the light. We may without inconvenience replace the ellipsoidal zone by a spherical one of like dimensions.

Intensity of Illumination required for Reading.

190. The study of the common reflector leads us naturally to give the values of the minimum illumination required for reading. These values are in no respect exact, for they are dependent greatly on the physiological conditions of the eye.

The intensity of the illumination should be proportionately greater as it is desired to read more rapidly and with less fatigue. Javal has determined that on a printed page whose intensity of illu-

mination is one candle-meter, with good sight one can read No. 7 characters at a distance of 70 cm. from the page, No. 8 characters at 80 cm., and No. 9 characters at 90 cm. Léonard Weber found that the rapidity of reading is in direct ratio to the degree of illumination. Thus a person who reads six lines of a book when the intensity of illumination is 2 candles-meter, reads twelve lines of it in the same length of time if the intensity of illumination is doubled. These conclusions are naturally true between certain limits only.

Cohn, a well-known German hygienist, estimates at 50 candles-meter the illumination produced by daylight on a well-exposed table; he further estimates that the minimum hygienically necessary for reading and writing without abnormal fatigue should be 10 candles-meter.

Measurement of Illumination.

191. In case of doubt as to the value of a system of lighting, recourse must be had to measurements of the intensity of illumination at different parts of the space lighted. The methods have been studied in Chapter III.; it is proper, however, to return to them to indicate the modifications which they must undergo so as to be adapted to this particular kind of measurements.

It is evident that two illuminations are equivalent when the same object, submitted alternately to one and the other, appears to have the same brightness and produces the same effect on the retina. We know that the eye is to some extent unfitted to give a photometric judgment in the general case, but the information which it furnishes acquires more precision when the quantity of light is reduced to the minimum necessary for a determined operation.

It is this which happens in the case of reading. For instance, if it is desired to read continuously a text printed in a certain size of type and placed at an invariable distance from the eye, it is necessary that the light diffused by the paper should not fall below a definite minimum for each person's sight.

When the illumination falls below this limit, reading is no longer continuous; one is obliged to read each word separately, and generally seeks to bring his eye nearer the paper, to increase the apparent angle of the characters, always provided that the accommodation of the eye allows them to be seen with clearness.

Such are, as we have seen, the elements of photometric methods based on visual acuteness. These methods are the simplest, and are sufficiently precise for measurements of illumination.

If the experiment is repeated with characters of unequal size, it is readily recognized that, for the same distance of the eye, the illumination should be proportionately more intense for continuous reading as the letters become smaller. A sheet of paper with a series of phrases printed in characters of different types will then furnish a true measure of illumination.

It is very easy to construct an apparatus on this principle. Schutte has invented, for the use of photographers, a very ingenious small apparatus which may also be of service in measurements of illumination, and which it is easy to modify advantageously. The apparatus called a lux-meter by Wybauw is much like that of Schutte.

This apparatus consists in a disc movable about its center and formed of superposed layers of translucent sheets; it is next divided into a series of sectors of which the number of layers increases in a progressive manner, which permits more or less absorption of the light which traverses it at this point. Behind this disc is a screen which has on a like circumference a series of characters of unequal sizes.

The type of the characters which may be read by transparency with a given fraction of light gives an approximate measure of the illumination.

Use of Weber's Photometer.

192. Weber's photometer (§ 53) is also made with a view to measurements of illumination, while using as a standard the acetate of amyl lamp. The apparatus is used in the following way for this particular object:

At the point and in the direction along which we wish to measure the illumination we place a plate of opal glass or a sheet of white cardboard, whitened with white lead; the movable tube *B* of the photometer is then directed at this card. In order that only the light diffused by this cardboard may enter the tube, care must be taken that the angle formed by the generatrix of the cone having its apex at the center of the opalescent disc of the tube *B* and its base on the cardboard, shall not exceed 60°. Further care should be taken that no direct light enters the tube and that the illumination of the cardboard is not modified by the presence of the observer.

Suppose, first, that the diffused light and that of the acetate of amyl standard have the same color. Equality of illumination of the two plates may be produced.

Suppose e to be the intensity of illumination of the white cardboard; because of absorption in the cardboard, a quantity ηe only will reach the dull disc of the tube B, and this having a coefficient of transparency α, the illumination of the field of the movable tube is equal to $\alpha\eta e$ (α should be made equal to 1, if the tube B is used without the dull disc). Let $\frac{d}{100}$ be the distance, expressed in meters, of the dull disc of the fixed tube from the flame of the lamp, and let β be the coefficient of transparency of this plate. The illumination of the field of the fixed tube is then, designating by I the candle power of the lamp,

$$e' = \beta \frac{I}{\left(\frac{d}{100}\right)^2} = \beta I \frac{10000}{d^2}.$$

We have, then, for the setting for equality of illumination of the two plates,

$$\alpha\eta e = e',$$

whence we conclude

$$e = \frac{\beta I}{\alpha\eta} \cdot \frac{10000}{d^2} = C' \frac{10000}{d^2}.$$

If I is expressed in candles, this formula gives the intensity of illumination in terms of the candle-meter.

The constant C' is determined by illuminating the cardboard screen by a light of known intensity placed at a determined distance. We then calculate the intensity of illumination e_0 of the cardboard placed normally to the rays of light, and measure the intensity which corresponds to a reading r of the apparatus.

We have then

$$e_0 = C' \frac{10000}{r^2},$$

whence

$$C' = \frac{e_0 r^2}{10000}.$$

The apparatus may also be used without the white screen by replacing the cap of the tube B by a disc of opal glass designated by the letter μ; we then give to the apparatus and the movable tube such a position that the disc μ occupies the point, and is normal to the direction with respect to which we wish to measure the illumination.

We next determine the reading δ of the apparatus for which the two fields are equally lighted. We have then

$$e = C''' \frac{10000}{\delta^2}.$$

The constant C''' is determined in the same way as C'' by means of a light of known intensity.

These two methods cannot be used when the color of the diffused light differs from that of the light of the standard (acetate of amyl lamp). We should then make two settings d_r and d_g, interposing a red and a green glass in the path of the rays. We then find in the table on page 89 the coefficient k corresponding to $\left(\frac{d_r}{d_g}\right)^2$ and calculate e by means of the formula

$$e' = kC'' \frac{10000}{(d_r')^2},$$

or by means of

$$e' = kC''' \frac{10000}{(\delta_r')^2},$$

according as we employ an independent screen or a fixed disc with a movable tube.

Beside Weber's photometer, we may employ that of Mascart also, of which Pellin has, moreover, constructed a portable form. There are also other forms of portable photometers based on the employment of ordinary Bunsen or Foucault screens. It is very simple to modify this apparatus so as to make it portable without sacrificing too much of its precision.

APPENDIX BY THE TRANSLATORS.

APPENDIX.

A. [See page 39.]

It is difficult to see how this statement can be true. If L, M, and R coincide, all of the three conditions mentioned above must hold: by 1°, the spot must disappear on the left face of the screen; by 2°, the spot must disappear on the right face of the screen; and by 3° there must be equal contrasts on both sides. If all of these conditions hold for the same position of the screen, it would seem that the condition of equal contrasts must be satisfied by an absence of contrast on both sides; that is, the spot can be neither bright on a dark background nor dark on a bright background.

B. [See pages 50 and 95.]

The Lummer-Brodhun photometer may be used to compare the luminous intensities of an arc and an incandescent lamp, but, as the two regions of the luminous field seen in the telescope will appear light blue and light yellow, it is quite difficult to decide exactly when the two regions are equally bright. However, if the mean of several settings is taken, the probable error will not be large.

If now the side of the screen toward the arc-light is covered with light yellow paper, and the other side with light blue-green paper of just the right tint, the two regions in the telescope will appear to be of uniform color, when a balance is obtained. The setting will now probably be different from what it was with the white screen. If calculations of the relative intensities of the two light-sources are made from both sets of readings of the photometer, a coefficient of relative absorption may be obtained. This coefficient will remain practically constant on comparing the arc-light at various inclinations with the incandescent lamp.

As the color of the light emitted by the arc-lamp varies with the inclination, it is not possible to choose tints of paper that will make

the two regions appear of exactly the same color at all inclinations. The variation in color will, however, not be the cause of much error in the observations. This variation in color may be compensated by varying the voltage of the incandescent lamp. If this is done, it is necessary to have previously measured the intensity of the incandescent lamp at various voltages. If these results are plotted in a curve, the intensity of the incandescent lamp at any point within the limits of calibration may be read off directly.

In a particular case, the arc-lamp was suspended as described at the bottom of page 190. The mirror used was found to absorb 17.25 per cent of the light incident on it. The luminous intensities of the arc-lamp measured with the white screen, and with the yellow and blue-green screen were in the ratio of 763 : 1000.

To calculate the true value of the intensity of the arc-lamp, the value calculated from the settings of the photometer with the colored papers must be multiplied by $\frac{0.763}{1-0.1725}=0.92$.

Covering the sides of the screen with the yellow and the blue-green paper can, of course, give no additional *absolute* accuracy to the photometric measurements; it, however, adds greatly to the comparative accuracy of the results, as it makes it possible to measure the *relative* intensities, under various conditions, with great precision.

On reversing the photometer, it is necessary to reverse the screen also, so that the yellow side may still be toward the arc-lamp, otherwise one part of the field will appear a deep yellow and the other a deep blue.

C. [See page 143.]

The following regulations for testing the Hefner lamp are taken from *Schillings Journal f. Gasbeleuchtung u. Wasserversorgung*, 1893: —

"The second (technical) department of the Imperial Physico-Technical Institute undertakes the testing and certification of Hefner lamps according to the following directions consistent with agreements made with the German Gas and Water Association: —

"§ 1.

"The object of the test is to ascertain whether the candle power of the lamp, after being lighted for at least ten minutes, equals the normal value of one Hefner unit as fixed by the standard of the Institute, the lamp burning pure acetate of amyl, and the flame reaching the mark of the gauge furnished with the lamp.

"§ 2.

"Hefner lamps constructed as described in the appendix, are admitted for examination, provided they have one of the flame-gauges there described and the name of the manufacturer as well as the lamp number stamped on the lamp.

"§ 3.

"The test consists —

"1°. In ascertaining the accuracy of the more important dimensions.

"2°. In the photometric comparison of the lamp, using its own flame-gauge, with the standard of the Institute.

"§ 4.

"A certificate will be issued —

"1°. If the test shows that the thickness of the wick-tube is not more than 0.02 mm. larger or 0.01 mm. smaller than the normal, and that its length does not differ by more than 0.5 mm. and its inner radius by more than 0.1 mm. from the normal, and that, after the gauge has been put on, the distance between the top of the wick and the edge of the gauge does not differ by more than 0.1 mm. from the normal.

"2°. If the candle power does not differ from that of the standard by more than 2 per cent.

"§ 5.

"If a certificate is issued, the current number and the Imperial Eagle will be stamped on the following parts of the lamp: —

"1°, the vessel; 2°, the burner; 3°, the wick-tube; 4°, the flame-gauge; 5°, the control-gauge.

"In the certificate will be given the results of the test, showing the deviation of the candle power from the normal within 1 per cent.

"§ 6.

"The fees charged are: —

"1°. For testing and certifying a Hefner lamp with a flame-gauge,	m. 3.00
"2°. For testing and certifying a Hefner lamp with a sight and an optical flame-gauge	4.50
"3°. For testing and certifying a Hefner lamp with a second wick-tube and a flame-gauge	4.50
"4°. For testing and certifying a Hefner lamp with a second wick-tube and both flame-gauges	5.50

"Charlottenburg, March 30th, 1893.

"Imperial Physico-Technical Institute,

"v. Helmholtz."

Next there follows a description of the lamp substantially as given on p. 135, which we omit.

"CERTIFICATE FOR HEFNER LAMP NO.......

"The lamp is marked..............

"It has a v. Hefner-Alteneck vane-sight, a Krüss optical flame-gauge, a second wick-tube, and a control-gauge.

"The dimensions of the wick-tube and the control-gauge differed from the normal within the limits allowed for certification.

"The candle power found by photometric measurement was

	For wick-tube *a*	for wick-tube *b*	
using the vane-sight	....	....	Hefner units.
using the optical gauge	....	....	" "

"As none of the deviations exceeds the limits allowable, the lamp was stamped with the number of the certificate and the Imperial Eagle on all parts mentioned in the regulations.

"A description of the lamp and directions for the use of the lamp, the flame-gauges, and the control-gauge are given with the certificate.

"Charlottenburg,................, 189-.

"Physico-Technical Institute,

"Department II.

"(Signature)."

"The back of the certificate contains extracts from the preceding regulations as well as other information concerning the granting of the certificate."

"DIRECTIONS FOR USE.

"*The Wick.*

"The quality of the wick has in general no influence on the candle power. It is only necessary to take care that it fills the wick-tube entirely, but on the other hand, is not pressed too tight. We therefore find it most convenient to use a sufficient number of thick cotton threads laid together. But since such loose threads are easily displaced and form loops inside the reservoir and clog the gearing, wicks woven on the outside are frequently used. There is no objection to their use as long as they fulfil the condition of filling the tube without being too tight.

"The Acetate of Amyl.

"Care must be taken in procuring acetate of amyl for the Hefner lamp, the commercial article often containing substances which render it useless for photometric use. The acetate of amyl should, therefore, be obtained from a reliable firm, and it should be stated on buying it, that it is to be used for photometric measurements.

"In order to facilitate the purchase of good acetate of amyl, the German Gas and Water Association has undertaken to procure sufficient quantities of suitable acetate of amyl, and after testing it, to sell it at its office (Hofrat Dr. Bunte, Karlsruhe) in sealed bottles containing from 1 liter upward.

"If it is not desired to make use of this opportunity of obtaining tested acetate of amyl, it is best to first examine, as to its usefulness, other acetate of amyl. For this, the following tests, in most part due to Dr. Bannow, are the most useful. According to him acetate of amyl may be used for measurements of candle power if the following conditions are fulfilled:—

"1°. The specific gravity must be from 0.872 to 0.876 at 15° C.

"2°. In distilling the acetate of amyl in a glass retort, at least 0.9 of its quantity should pass over between 137° and 143° C.

"3°. The acetate of amyl should not decidedly redden blue litmus paper.

"4°. If we add an equal quantity of benzine or carbon disulphide to the acetate of amyl, the substances should mix without becoming milky.

"5°. If we shake in a graduate 1 cc. of acetate of amyl with 10 cc. of alcohol 90 per cent (Tralles) and 10 cc. of water, a clear solution should result.

"6°. A drop of acetate of amyl should evaporate on white filter paper without leaving a greasy spot.

"The acetate of amyl should be well corked, and if possible kept in the dark."

Then there follow directions for the use of the lamp.

D. [See page 188.]

This value for the absorption, 1.8 per cent, is altogether too low to be believed. It is so given, however, in the original paper communicated to the (London) *Electrical Review*, of July 13th, 1888. It may be a misprint for 18 per cent.

In § 120, p. 191, will be found values of the absorption for two mirrors. These values range from 15 to 32 per cent for a single reflection. In Appendix B another mirror is mentioned, for which a value of 17.25 per cent was found.

From these values it would appear that 1.8 per cent for a double reflection is undoubtedly in error.

E. [See page 202.]

Theory may indicate (*a*) that the horizontal sections of the photometric surface should be similar curves, or (*b*) that the variations in the various vertical planes should follow the same law, which is a quite different assumption.

In (*a*) there is only one horizontal section, that is, the section passing through the light-source, which is directly concerned with luminous intensity; the other horizontal sections include no *radii vectores*, and it is along these alone that the luminous intensity is laid off.

The assumption made in (*b*) is not supported by the facts. An examination of the figures given in Table III., p. 205, shows that there is no fixed ratio between the variations of the numbers in the two columns for each class of lamps. This may be due to variations in the form of the filament, or to irregularities in the shape of the bulb. But whatever be the reason, it is evident that in practise we may not make this assumption, except as a gross approximation.

F. [See page 228.]

A very interesting and instructive paper on arc-lamps was read before the Electrical Congress at Chicago in 1893, by Professor W. E. Ayrton, F.R.S., etc. In this paper Professor Ayrton threw considerable light on the subject of § 139.

This paper will be found in the Proceedings of the Congress, about to be published by the American Institute of Electrical Engineers. It is not at present available.

G. [See page 236.]

The following extracts from a paper by A. P. Trotter* will be found of interest in considering the form of the polar curve expressing the connection between inclination and candle power. In Mr. Trotter's paper the angles are measured with respect to the axis of the carbons

"It has been assumed by many persons that the hollowing of the crater of the positive carbon tends in some unexplained manner to concentrate and throw the light downward. It is evident that the lower or negative

* *The Electrical Review* (London), May 6, 1892, p. 583.

carbon intercepts a good deal of the light; but there speculations appear to have stopped. A little consideration will show that the effect is precisely and identically the same as though the end of the positive carbon was flat. No tilting of an incandescent or other luminous surface can make it brighter; and, on the other hand, if it is covered with a thin imperfectly transparent layer, as in the case of the atmosphere of the sun, the edge will appear less bright than the middle of the disc. The quantity of light emitted by an incandescent disc in any direction is proportional to the amount of surface visible from that direction. That is to say, candle power varies then as the cosine of the inclination.

"Cosines plotted as a polar curve give a circle passing through the pole. . . . The candle power of the crater of an arc-lamp should, then, if plotted as a polar curve, coincide with part of a circle. Any deviation from the circle must have some cause. Two such deviations are observed and their causes are easily recognized.

"The full curve in Fig. 92 represents the mean of a large number of observations made [by Wybauw], no less than 26 different arcs having been tested. The cosine of 60° being one-half, the area of the crater seen from this direction is one-half of that of the full circle; the candle power is one-half of that emitted by the crater; and the length of the radius vector corresponding to 60° may be taken as the radius of the circle.

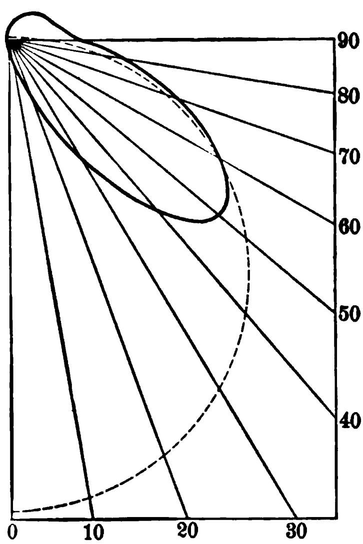

FIG. 92.

"The light due to the negative carbon is clearly shown as an excess above the circular curve; there is indeed nothing else to which it can be due, except the red-hot walls of the crater [and the arc proper].

"At about 60° the curve of candle power begins to fall off, and this is due to nothing else than the shadow of the lower carbon, which intercepts more and more of the light as we pass to smaller angles, until, if the carbons be of the same diameter, no light is thrown in a vertical direction.

"In considering the real meaning of the latter part of the curve, the author drew a number of views of a pair of imaginary carbons, projected at various angles. The elliptical area of the crater in each view was calculated, and he found that these areas, plotted as radii of a polar curve, gave a curve closely resembling the well-known candle-power curve of the arc. It follows that, if this be proved to be true by experiment, the candle power per square millimeter of the crater is constant.

"The author communicated this result to Professor S. P. Thompson, and asked if he would see whether actual experiment would confirm it."

Such actual experiments were carried out by Mr. C. F. Higgins. Concerning the relation between the apparent area of the crater and the luminous intensity Mr. Trotter says:

"A straight line cutting the axis at 100 candle power seems to fit the results. This may be explained by the light which is emitted by the red-hot and glowing parts of the carbon. These were not included in the measurement of area; the true crater only was measured."

Mr. Trotter found the intrinsic intensity of the crater to be 42600 candles per square inch, or 64 candles per square millimeter.

H. [See page 298.]

The following values for the reflecting powers of various surfaces were obtained by Dr. W. E. Sumpner. See *Phil. Mag.*, February, 1893, p. 81.

He says concerning them:

"In the majority of cases the numbers given are approximate only, as there seemed no object in aiming at great accuracy. The first four surfaces referred to in the table, viz. thick white blotting-paper, white (rough) cartridge-paper, tracing-paper, and tracing-cloth, were, however, carefully tested, and the numbers obtained represent the mean of many observations.

"REFLECTING POWERS.

	Per Cent.		Per Cent.
White blotting-paper	82	Deep chocolate paper	4
White cartridge-paper	80	Plane deal (clean)	40 to 50
Tracing-cloth	35	" " (dirty)	20
Tracing-paper	22	Yellow cardboard	30
Ordinary foolscap	70	Parchment (one thickness)	22
Newspapers	50 to 70	" (two thicknesses),	35
Tissue-paper (one thickness),	40	Yellow painted wall (clean),	40
" " (two thicknesses),	55	" " " (dirty),	20
Yellow wall-paper	40	Black cloth	1.2
Blue paper	25	Black velvet	0.4"
Dark brown paper	13		

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Armature Windings of Electric Machines

BY

H. F. PARSHALL,

Consulting Designing Engineer of the General Electric Company; Member of American Society of Mechanical Engineers, and American Institute Electrical Engineers,

AND

H. M. HOBART, S. B.

The treatise discusses exhaustively the principles governing the arrangement of armature conductors of electrical machines, both continuous and alternating current, including various multiphase windings applicable in practice.

The plan of each chapter is—first, a general discussion of the properties of the windings treated, the explanation of the formulæ applicable, illustrated by diagrams showing each useful form of winding. The treatment is so complete that windings can be designed from the information given in the chapters, but in addition to this there are given 60 tables showing exactly how, with a given number of conductors, the windings have to be arranged for a given result. These tables eliminate the necessity for special calculations, so that those uncertain in the use of figures may be able to design armature windings with accuracy.

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